Q. 29

Find the direction cosines of a vector which is equally inclined to the x - axis, y - axis and z - axis.

Answer :

Let the inclination with:

x - axis = α

y - axis = β

z - axis = γ

The vector is equally inclined to the three axes.

$\Rightarrow \alpha = \beta = \gamma$

Direction cosines: $cos\alpha, cos\beta, cos\gamma$

We know that: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$ ...($\alpha = \beta = \gamma$)

$\Rightarrow 3 \cos^2 \alpha = 1$

$$cos\alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow cos\alpha = \frac{1}{\sqrt{3}}$$

$$cos\beta = \frac{1}{\sqrt{3}}$$

$$cos\gamma = \frac{1}{\sqrt{3}}$$

Ans: $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Q. 30

If P(1, 5, 4) and Q(4, 1, - 2) be the position vectors of two points P and Q, find the direction ratios of $\overline{PQ}$.

Answer :

Let $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ be the two points then Direction ratios of line joining P and Q i.e. PQ are $x_2 - x_1, y_2 - y_1, z_2 - z$

Here, P is(1, 5, 4) and Q is (4, 1, - 2)

Direction ratios of PQ are:(4 - 1),(1 - 5),(- 2 - 4) = 3, - 4, - 6

Ans: the direction ratios of $\overrightarrow{PQ}$. are: 3, - 4, - 6

Q. 31

Find the direction cosines of the vector $\vec{a} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

Answer :

$\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Let the inclination with:

x - axis = α

y - axis = β

z - axis = γ

Direction cosines: $cos\alpha, cos\beta, cos\gamma = l, m, n$

For a vector $\vec{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, l = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore l = \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{1 + 4 + 9}} = \frac{1}{\sqrt{14}}.$$

$$\therefore m = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{1 + 4 + 9}} = \frac{2}{\sqrt{14}}$$

$$\therefore n = \frac{3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{3}{\sqrt{1 + 4 + 9}} = \frac{3}{\sqrt{14}}$$

Ans: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Q. 32

If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left(\hat{a}+\hat{b}\right)$ is a unit vector, what is the angle between $\hat{a}$ and $\hat{b}$?

Answer :

It is given that $\hat{a}$ *and* $\hat{b}$ are unit vectors ,as well as $(\hat{a} + \hat{b})$ is also a unit vector

$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{a} + \hat{b}| = 1$

Since the modulus of a unit vector is unity.

Now,

$|\hat{a} + \hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$

$\Rightarrow 1^2 = 1^2 + 1^2 + 2\times1\times1\times \cos\theta$

$\Rightarrow \cos\theta = (1 - 1 - 1)/2$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = \cos^{-1}\frac{-1}{2} = \frac{2\pi}{3}$

Ans: $\frac{2\pi}{3}$

Objective Questions

Q. 1

Mark (√) against the correct answer in each of the following:

A unit vector in the direction of the vector $\vec{a} = \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ is

A. $\left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

B. $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$

C. $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

D. none of these

Answer :

Tip – A vector in the direction of another vector $a\hat{i}+b\hat{j}+c\hat{k}$ is given by $\lambda(a\hat{i}+b\hat{j}+c\hat{k})$ and the unit vector is given by $\frac{\lambda(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{(a\lambda)^2+(b\lambda)^2+(c\lambda)^2}}$

So, a vector parallel to $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$ is given by $\lambda(2\hat{i}-3\hat{j}+6\hat{k})$ where λ is an arbitrary constant.

Now, $|\vec{a}|=\sqrt{2^2+3^2+6^2}=7$

Hence, the required unit vector

$$=\frac{\lambda(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{(2\lambda)^2+(3\lambda)^2+(6\lambda)^2}}$$

$$=\frac{\lambda(2\hat{i}-3\hat{j}+6\hat{k})}{\lambda\sqrt{2^2+3^2+6^2}}$$

$$=\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$$

Q. 2

Mark (√) against the correct answer in each of the following:

The direction cosines of the vector $\vec{a}=\left(-2\hat{i}+\hat{j}-5\hat{k}\right)$ **are**

A. -2, 1, -5

B. $\frac{1}{3},\frac{-1}{6},\frac{-5}{6}$

C. $\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}$

D. $\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$

Answer :

Formula to be used – The direction cosines of a vector $a\hat{i}+b\hat{j}+c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i}+\hat{j}-5\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2+1^2+5^2}}, \frac{1}{\sqrt{2^2+1^2+5^2}}, \frac{-5}{\sqrt{2^2+1^2+5^2}}\right)$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

Q. 3

Mark (√) against the correct answer in each of the following:

If A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$ then the direction cosines of $\overrightarrow{AB}$ are

A. -2, -4, 4

B. $\frac{-1}{2}, -1, 1$

C. $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

D. none of these

Answer :

Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$

Tip – If P(a_1,b_1,c_1) and Q(a_2,b_2,c_2) be two points then the vector $\overrightarrow{PQ}$ is represented by $(a_2-a_1)\hat{i}+(b_2-b_1)\hat{j}+(c_2-c_1)\hat{k}$

Hence, $\overrightarrow{AB}=(-1-1)\hat{i}+(-2-2)\hat{j}+(1+3)\hat{k}=-2\hat{i}-4\hat{j}+4\hat{k}$

Formula to be used – The direction cosines of a vector $a\hat{i}+b\hat{j}+c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i}-4\hat{j}+4\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2+4^2+4^2}}, \frac{-4}{\sqrt{2^2+4^2+4^2}}, \frac{4}{\sqrt{2^2+4^2+4^2}}\right)$$

$= \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right)$

$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

Q. 4

Mark (√) against the correct answer in each of the following:

If a vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively then the value of ($\sin^2\alpha + \sin^2\beta + \sin^2\gamma$) is
A. 1

B. 2
C. 0
D. 3

Answer :

Given - A vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively.

To Find - ($\sin^2\alpha + \sin^2\beta + \sin^2\gamma$)

Formula to be used - $\cos^2\alpha + \cos^2\beta + \cos^2\gamma=1$

Hence,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$=(1-\cos^2\alpha) +(1-\cos^2\beta) +(1-\cos^2\gamma)$

$= 3-(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=3-1$

$=2$

Q. 5

Mark (√) against the correct answer in each of the following:

The vector $(\cos\alpha\cos\beta)\hat{i}+(\cos\alpha\cos\beta)\hat{j}+ (\sin\alpha)\hat{k}$ **is a**
A. null vector

B. unit vector
C. a constant vector
D. none of these

Answer :

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

A unit vector is a vector whose magnitude = 1.

Formula to be used - $\sin^2\theta + \cos^2\theta = 1$

Hence, magnitude of $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$ will be given by $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

= 1 i.e a unit vector

Q. 6

Mark (√) against the correct answer in each of the following:

What is the angle which the vector $\left(\hat{i} + \hat{j} + \sqrt{2}\hat{k}\right)$ makes with the z-axis?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

Answer :

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ is given by

$$\left(\frac{1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}}, \frac{1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}}, \frac{\sqrt{2}}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}}\right)$$

$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$

$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

The direction cosine of z-axis = $\frac{1}{\sqrt{2}}$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ where θ is the angle the vector makes with the z-axis.

$\therefore \theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

Q. 7

Mark (√) against the correct answer in each of the following:

f $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3},\ |\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer :

Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $\sqrt{6} = 2\sqrt{3}\cos\theta$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \therefore \theta = \frac{\pi}{4}$

Q. 8

Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}\cdot\vec{b} = -1$ then the angle between

$\vec{a}$ **and** $\vec{b}$ **is**

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer :

Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b} = -1$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $-1 = \sqrt{2}\sqrt{2}\cos\theta$ i.e. $\cos\theta = \frac{1}{2}$ $\therefore \theta = \frac{\pi}{3}$

Q. 9

Mark (√) against the correct answer in each of the following:

The angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ **and** $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ **is**

A. $\cos^{-1}\frac{5}{7}$

B. $\cos^{-1}\frac{3}{5}$

C. $\cos^{-1}\frac{3}{\sqrt{14}}$

D. none of these

Answer :

Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})=3+4+3=10$

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

Hence, $10 = \sqrt{14}\sqrt{14}\cos\theta$ i.e. $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\frac{5}{7}$

Q. 10

Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ **and** $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ **then the angle between** $(\vec{a} + \vec{b})$ **and** $(\vec{a} - \vec{b})$ **is**

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer :

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

Hence, $0 = \sqrt{18}\sqrt{38}\cos\theta$ i.e. $\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

Q. 11

Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ **and** $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ **then the angle between** $(2\vec{a} + \vec{b})$ **and** $(\vec{a} + 2\vec{b})$ **is**

A. $\cos^{-1}\left(\frac{21}{40}\right)$

B. $\cos^{-1}\left(\frac{31}{50}\right)$

C. $\cos^{-1}\left(\frac{11}{30}\right)$

D. none of these

Answer :

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 5\hat{i} + 3\hat{j} - 4\hat{k}$

and $\vec{a} + 2\vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + 2(3\hat{i} - \hat{j} + 2\hat{k}) = 7\hat{i} + \hat{k}$

$\therefore (2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k}).(7\hat{i} + \hat{k}) = 35 - 4 = 31$

$|2\vec{a} + \vec{b}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50}$

$|\vec{a} - 2\vec{b}| = \sqrt{7^2 + 1^2} = \sqrt{50}$

Hence, $31 = \sqrt{50}\sqrt{50}\cos\theta$ i.e. $\cos\theta = \frac{31}{50}$

$\therefore \theta = \cos^{-1}\frac{31}{50}$

Q. 12

Mark (√) against the correct answer in each of the following:

If $\vec{a} = (2\hat{i} + 4\hat{j} - \hat{k})$ **and** $\vec{b} = (3\hat{i} - 2\hat{j} + \lambda\hat{k})$ **be such that** $\vec{a} \perp \vec{b}$ **then λ = ?**

A. 2

B. -2

C. 3

D. -3

Answer :

Given - $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

i.e. $\lambda = -2$

Q. 13

Mark (√) against the correct answer in each of the following:

What is the projection of $\vec{a}=\left(2\hat{i}-\hat{j}+\hat{k}\right)$ **on** $\vec{b}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$ **?**

A. $\frac{2}{\sqrt{3}}$

B. $\frac{4}{\sqrt{5}}$

C. $\frac{5}{\sqrt{6}}$

D. none of these

Answer :

Given - $\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k},\ \vec{b}=\hat{\imath}-2\hat{\jmath}+\hat{k}$

To find – Projection of $\vec{a}$ on $\vec{b}$ i.e. $\vec{a}\cos\theta$

Formula to be used - $\vec{p}.\vec{q}=|\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – If $\vec{p}$ and $\vec{q}$ are two vectors, then the projection of $\vec{p}$ on $\vec{q}$ is defined as $\vec{p}\cos\theta$

Magnitude of a vector $\vec{p}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ is given by $|\vec{p}|=\sqrt{x^2+y^2+z^2}$

So,

$\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow (2\hat{\imath}-\hat{\jmath}+\hat{k}).(\hat{\imath}-2\hat{\jmath}+\hat{k})=\sqrt{1^2+2^2+1^2}|\vec{a}|\cos\theta$

$\Rightarrow |\vec{a}|\cos\theta=\frac{2+2+1}{\sqrt{6}}$

$\Rightarrow |\vec{a}|\cos\theta=\frac{5}{\sqrt{6}}$

Q. 14

Mark (√) against the correct answer in each of the following:

If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|,$ **then**

A. $\left|\vec{a}\right|=\left|\vec{b}\right|$

B. $\vec{a}\parallel\vec{b}$

C. $\vec{a} \perp \vec{b}$

D. none of these

Answer :

Given - $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

Tip – If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

$\Rightarrow 4ab\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

i.e. $\theta = \frac{\pi}{2}$

So, $\vec{a} \perp \vec{b}$

Q. 15

Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $\left(3\vec{a}+2\vec{b}\right)\cdot\left(5\vec{a}-6\vec{b}\right)=?$

A. 3

B. 5

C. 6

D. 12

Answer :

Given - $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors i.e. $|\vec{a}| = |\vec{b}| = 1$

To Find – $(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip - $\vec{a} \perp \vec{b}$

$\therefore |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

Hence,

$(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

$= 15|\vec{a}|^2 + 10\vec{b}.\vec{a} - 18\vec{a}.\vec{b} - 12|\vec{b}|^2$

$= 15 - 12$

$= 3$

Q. 16

Mark (√) against the correct answer in each of the following:

If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ **and** $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ **are perpendicular to each other then λ = ?**

A. -3

B. -6

C. -9

D. -1

Answer :

Given - $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\vec{b} = \hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (3\hat{\imath} + \hat{\jmath} - 2\hat{k}).(\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}) = 0$

$\Rightarrow 3 + \lambda + 6 = 0$

i.e. $\lambda = -9$

Q. 17

Mark (√) against the correct answer in each of the following:

If θ is the angle between two unit vectors $\hat{a}$ **and** $\hat{b}$ **then** $\frac{1}{2}|\hat{a} - \hat{b}| = ?$

A. $\cos\frac{\theta}{2}$

B. $\sin\frac{\theta}{2}$

C. $\tan\frac{\theta}{2}$

D. none of these

Answer :

Given - $\hat{a}$ and $\hat{b}$ are two unit vectors with an angle θ between them

To find - $\frac{1}{2}|\hat{a}-\hat{b}|$

Formula used - If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

Tip - $|\hat{a}|^2 = |\hat{b}|^2 = 1$ & $\hat{a}.\hat{b} = 1$

Hence,

$\frac{1}{2}|\hat{a}-\hat{b}|$

$= \frac{1}{2}\sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2ab\cos\theta}$

$= \frac{1}{2}\sqrt{2 + 2\cos\theta}$

$= \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$

$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2\frac{\theta}{2}}$

$= \sin\frac{\theta}{2}$

Q. 18

Mark (√) against the correct answer in each of the following:

If $\vec{a}=(\hat{i}-\hat{j}+2\hat{k})$ **and** $\vec{b}=(2\hat{i}+3\hat{j}-4\hat{k})$ **then** $|\vec{a}\times\vec{b}|=?$

A. $\sqrt{174}$

B. $\sqrt{87}$

C. $\sqrt{93}$

D. none of these

Answer :

Given - $\vec{a}=\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\vec{b}=2\hat{\imath}+3\hat{\jmath}-4\hat{k}$ are two vectors.

To find - $|\vec{a}\times\vec{b}|$

Formula to be used - $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$ where $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ is given by $|\vec{p}|=\sqrt{x^2+y^2+z^2}$

So,

$\vec{a}\times\vec{b}$

$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & 2\\ 2 & 3 & -4\end{vmatrix}$

$=\hat{\imath}(4-6)+\hat{\jmath}\,(4+4)+\hat{k}(3+2)$

$=-2\hat{\imath}+8\hat{\jmath}+5\hat{k}$

$\therefore |\vec{a}\times\vec{b}|=\sqrt{2^2+8^2+5^2}=\sqrt{93}$

Q. 19

Mark (√) against the correct answer in each of the following:

If $\vec{a}=(\hat{i}-2\hat{j}+3\hat{k})$ **and** $\vec{b}=(\hat{i}-3\hat{k})$ **then** $|\vec{b}\times 2\vec{a}|=?$

A. $10\sqrt{3}$

B. $5\sqrt{17}$

C. $4\sqrt{19}$

D. $2\sqrt{23}$

Answer :

Given - $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{k}$ are two vectors.

To find - $|\vec{b} \times 2\vec{a}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{b} \times 2\vec{a}$

$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$

$= \hat{\imath}(12) + \hat{\jmath}\,(-6 - 6) + \hat{k}(-4)$

$= 12\hat{\imath} - 12\hat{\jmath} - 4\hat{k}$

$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$

Q. 20

Mark (√) against the correct answer in each of the following:

If $|\vec{a}| = 2, |\vec{b}| = 7$ **and** $(\vec{a} \times \vec{b}) = (3\hat{i} + 2\hat{j} + 6\hat{k})$ **then the angle between** $\vec{a}$ **and** $\vec{b}$ **is**

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer :

Given - $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

Tip – $|\vec{p} \times \vec{q}| = ||\vec{p}||\vec{q}|\sin\theta\hat{n}| = |\vec{p}||\vec{q}|\sin\theta$ & magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence, $|\vec{a} \times \vec{b}| = |3\hat{i} + 2\hat{j} + 6\hat{k}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\therefore 7 = 2 \times 7\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Q. 21

Mark (√) against the correct answer in each of the following:

If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ **and** $|\vec{a} \times \vec{b}| = 35$ **then** $\vec{a} \cdot \vec{b} = ?$

A. 5

B. 7

C. 13

D. 12

Answer :

Given – $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$

To find - $\vec{a}.\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$ & $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ & $\vec{q}$ are any two vectors

Tip – $|\vec{p} \times \vec{q}| = ||\vec{p}||\vec{q}|\sin\theta\hat{n}| = |\vec{p}||\vec{q}|\sin\theta$

So,

$|\vec{a} \times \vec{b}| = 35$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 35$

$\Rightarrow \sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

Q. 22

Mark (√) against the correct answer in each of the following:

Two adjacent sides of a || gm are represented by the vectors $\vec{a} = (3\hat{i} + \hat{j} + 4\hat{k})$ **and** $\vec{b} = (\hat{i} - \hat{j} + \hat{k})$. **The area of the || gm is**

A. $\sqrt{42}$ **sq units**

B. 6 sq units

C. $\sqrt{35}$ **sq units**

D. none of these

Answer :

Given - Two adjacent sides of a || gm are represented by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of ||gm $= |\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}(-4-1) + \hat{j}(4-3) + \hat{k}(-3-1)$

$= -5\hat{i} + \hat{j} - 4\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + 4^2} = \sqrt{42}$

i.e. the area of the parallelogram = $\sqrt{42}$ sq. units

Q. 23

Mark (√) against the correct answer in each of the following:

The diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right)$ **and** $\overrightarrow{d_2} = \left(\hat{i} - 3\hat{j} + 4\hat{k}\right)$. **The area of the || gm is**

A. $7\sqrt{3}$ **sq units**

B. $5\sqrt{3}$ **sq units**

C. $3\sqrt{5}$ **sq units**

D. none of these

Answer :

Given - Two diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of ||gm $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$ and magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\overrightarrow{d_1} \times \overrightarrow{d_2}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{i}(4-6) + \hat{j}(-2-12) + \hat{k}(-9-1)$

$= -2\hat{i} - 14\hat{j} - 10\hat{k}$

$\therefore |\vec{d_1} \times \vec{d_2}| = \sqrt{2^2 + 14^2 + 10^2} = \sqrt{300}$

i.e. the area of the parallelogram = $\frac{1}{2} \times \sqrt{300} = 5\sqrt{3}$ sq. units

Q. 24

Mark (√) against the correct answer in each of the following:

Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$. The area of the triangle is

A. 41 sq units

B. 37 sq units

C. $\frac{41}{2}$ sq units

D. none of these

Answer :

Given - Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$

To find – Area of the triangle

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}(21 + 20)$

$= 41\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{41^2} = 41$

i.e. the area of the parallelogram = $\frac{41}{2}$ sq. units

Q. 25

Mark (√) against the correct answer in each of the following:

The unit vector normal to the plane containing $\vec{a} = (\hat{i} - \hat{j} - \hat{k})$ **and** $\vec{b} = (\hat{i} + \hat{j} + \hat{k})$ **is**

A. $(\hat{j} - \hat{k})$

B. $(-\hat{j} + \hat{k})$

C. $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

D. $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

Answer :

Given - $\vec{a} = \hat{i} - \hat{j} - \hat{k}$ & $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

To find – A unit vector perpendicular to the two given vectors.

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{a^2+b^2+c^2}}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$= -2\hat{j} + 2\hat{k}$, which the vector perpendicular to the two given vectors.

The required unit vector $= \frac{-2\hat{j}+2\hat{k}}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

Q. 26

Mark (√) against the correct answer in each of the following:

If $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are unit vectors such that** $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ **then** $\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer :

Given - $\vec{a},\vec{b},\vec{c}$ are three unit vectors and $(\vec{a}+\vec{b}+\vec{c})=0$

To find - $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Tip – $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

So,

$(\vec{a}+\vec{b}+\vec{c})^2=0$

⇨ $|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

⇨ $3+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

⇨ $(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=\frac{-3}{2}$

Q. 27

Mark (√) against the correct answer in each of the following:

If $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are mutually perpendicular unit vectors then** $\left[\vec{a}+\vec{b}+\vec{c}\right]=?$

A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$
D. 2

Answer :

Given - $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors

To find - $[\vec{a} + \vec{b} + \vec{c}]$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

So,

$(\vec{a} + \vec{b} + \vec{c})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

=3

$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$

Q. 28

Mark (√) against the correct answer in each of the following:

$\left[\hat{i}\ \hat{j}\ \hat{k}\right] = ?$

A. 0

B. 1
C. 2
D. 3

Answer :

To find - $[\hat{i}\ \ \hat{j}\ \ \hat{k}]$

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\therefore [\hat{i}\ \ \hat{j}\ \ \hat{k}]$

$= \hat{i}\,.(\hat{j} \times \hat{k})$

$= \hat{i}.\hat{i}$

$= |\hat{i}|^2$

$= 1$

Q. 29

Mark (√) against the correct answer in each of the following:

If $\vec{a}=\left(2\hat{i}-3\hat{j}+4\hat{k}\right), \vec{b}=\left(\hat{i}+2\hat{j}-\hat{k}\right)$ **and** $\vec{c}=\left(3\hat{i}-\hat{j}-2\hat{k}\right)$ **be the coterminous edges of a parallelepiped then its volume is**

A. 21 cubic units

B. 14 cubic units

C. 7 cubic units

D. none of these

Answer :

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$,

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

To find – The volume of the parallelepiped

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped = $|\ [\hat{a}\ \ \hat{b}\ \ \hat{c}]|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}.(\hat{b} \times \hat{c})$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\{(\hat{i} + 2\hat{j} - \hat{k}) \times (3\hat{i} - \hat{j} - 2\hat{k})\}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).(-5\hat{i} - \hat{j} - 7\hat{k})$

$= -10 + 3 - 28$

$= -35$

The volume = 35 sq units

Q. 30

Mark (√) against the correct answer in each of the following:

If the volume of a parallelepiped having $\vec{a} = \left(5\hat{i} - 4\hat{j} + \hat{k}\right), \vec{b} = \left(4\hat{i} + 3\hat{j} + \lambda\hat{k}\right)$ **and** $\vec{c} = \left(\hat{i} - 2\hat{j} + 7\hat{k}\right)$ **as conterminous edges, is 216 cubic units then the value of λ is**

A. $\frac{5}{3}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer :

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + \lambda\hat{k}$ & $\vec{c} = \hat{i} - 2\hat{j} + 7\hat{k}$

To find – The value of λ

Formula to be used - $[\hat{a}\ \hat{b}\ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped = $|\,[\hat{a}\ \hat{b}\ \hat{c}]|$

Hence,

$[\hat{a}\ \hat{b}\ \hat{c}]$

$= \hat{a}.(\hat{b} \times \hat{c})$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\{(4\hat{i} + 3\hat{j} + \lambda\hat{k}) \times (\hat{i} - 2\hat{j} + 7\hat{k})\}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\left((21 + 2\lambda)\hat{i} + (\lambda - 28)\hat{j} - 11\hat{k}\right)$

=5(21+2λ)-4(λ-28)-11

=206+6λ

The volume =206+6λ

But, the volume = 216 sq units

So, 206+6λ=216 ⇨λ= $\frac{10}{6} = \frac{5}{3}$

Q. 31

Mark (√) against the correct answer in each of the following:

It is given that the vectors $\vec{a} = \left(2\hat{i} - 2\hat{k}\right)$, $\vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ **and** $\vec{c} = \left(4\hat{i} + 2\hat{k}\right)$ **are coplanar. Then, the value of λ is**

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. 1

Answer :

Given – The vectors $\vec{a} = 2\hat{i} - 2\hat{k}, \vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ & $\vec{c} = 4\hat{i} + 2\hat{k}$ are coplanar

To find – The value of λ

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – For vectors to be coplanar, $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

$\Rightarrow \hat{a}.(\hat{b} \times \hat{c}) = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).\{(\hat{i} + (\lambda + 1)\hat{j}) \times (4\hat{i} + 2\hat{k})\} = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda+1 & 0 \\ 4 & 0 & 2 \end{vmatrix} = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).\left(2(\lambda+1)\hat{i} - 2\hat{j} - 4(\lambda+1)\hat{k}\right) = 0$

⇨ 4(λ-1)+8(λ-1)=0

⇨ 12(λ-1)=0 i.e. λ= 1

Q. 32

Mark (√) against the correct answer in each of the following:

Which of the following is meaningless?

A. $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$

B. $\vec{a} \times \left(\vec{b} \cdot \vec{c}\right)$

C. $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$

D. none of these

Answer :

Tip - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c}) = \hat{b}.(\hat{c} \times \hat{a}) = \hat{c}.(\hat{a} \times \hat{b}) = (\hat{a} \times \hat{b}).\hat{c}$ since, dot product is commutative

Hence, $\hat{a} \times (\hat{b}.\hat{c})$ is meaningless.

Q. 33

Mark (√) against the correct answer in each of the following:

$\vec{a} \cdot \left(\vec{a} \times \vec{b}\right) = ?$

A. 0

B. 1

C. a^2b

D. meaningless

Answer :

Tip – The cross product of two vectors is the vector perpendicular to both the vectors.

$\therefore \vec{a} \times \vec{b}$ gives a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Now,

$\vec{a}.(\vec{a} \times \vec{b})$

$= |\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}||\vec{b}|\cos\frac{\pi}{2}$

$= 0$

Q. 34

Mark (√) against the correct answer in each of the following:

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ **the value of** $\left[\vec{a}-\vec{b}\ \vec{b}-\vec{c}\ \vec{c}-\vec{a}\right]$ **is**

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. 1
C. 0
D. none of these

Answer :

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b}\times\hat{c}) = \hat{b}.(\hat{c}\times\hat{a})$ for any three arbitrary vectors

$\therefore [\hat{a}-\hat{b}\ \ \hat{b}-\hat{c}\ \ \hat{c}-\hat{a}]$

$= (\hat{a}-\hat{b}).\{(\hat{b}-\hat{c})\times(\hat{c}-\hat{a})\}$

$= (\hat{a}-\hat{b}).(\hat{b}\times\hat{c}-\hat{c}\times\hat{c}-\hat{b}\times\hat{a}+\hat{c}\times\hat{a})$

$= (\hat{a}-\hat{b}).(\hat{b}\times\hat{c}-\hat{b}\times\hat{a}+\hat{c}\times\hat{a})$

$= [\hat{a}.(\hat{b}\times\hat{c})-\hat{b}(\hat{b}\times\hat{c})-\hat{a}.(\hat{b}\times\hat{a})+\hat{b}(\hat{b}\times\hat{a})+\hat{a}.(\hat{c}\times\hat{a})-\hat{b}.(\hat{c}\times\hat{a})]$

$= [\hat{a}\ \hat{b}\ \hat{c}]-[\hat{a}\ \hat{b}\ \hat{c}] = 0$