Which is less than 0

Means strictly decreasing in its domain i.e R

3. Question

Prove that $f(x) = ax + b$, where a and b are constants and a > 0, is a strictly increasing function on R.

Answer

Domain of the function is R

Finding derivative i.e f'(x)=a

As given in question it is given that a>0

Derivative>0

Means strictly increasing in its domain i.e R

4. Question

Prove that the function $f(x) = e^{2x}$ is strictly increasing on R.

Answer

Domain of the function is R

finding derivative i.e $f'(x)=2e^{x}$

As we know e^{x} is strictly increasing its domain

f'(x)>0

hence f(x) is strictly increasing in its domain

5. Question

Show that the function $f(x) = x^2$ is

a. strictly increasing on [0, ∞[

b. strictly decreasing on [0, ∞[

c. neither strictly increasing nor strictly decreasing on R

Answer

Domain of function is **R.**

f'(x)=2x

for x>0 f'(x)>0 i.e. increasing

for x<0 f'(x)<0 i.e. decreasing

hence it is neither increasing nor decreasing in R

6. Question

Show that the function $f(x) = |x|$ is

a. strictly increasing on]0, ∞[

b. strictly decreasing on] − ∞, 0[

Answer

For x>0

Modulus will open with + sign

f(x)=+x

⇒ f′(x)=+1 which is <0

for x<0

Modulus will open with -ve sign

f(x)=-x => f′(x)=-1 which is >0

hence f(x) is increasing in x>0 and decreasing in x<0

7. Question

Prove that the function $f(x) = \log_e x$ is strictly increasing on]0, ∞[.

Answer

f(x)=ln(x)

$f'(x) = \frac{1}{x}$

for x<0

f′(x)=-ve →increasing

for x>0

f′(x)=+ve →decreasing

f(x) in increasing when x>0 i.e x∈(0,∞)

8. Question

Prove that the function $f(x) = \log_a x$ is strictly increasing on]0, ∞[when a > 1 and strictly decreasing on]0, ∞[when 0 < a < 1.

Answer

Consider f(x)=$\log_a$ x

domain of f(x) is x>0

$f'(x) = \frac{1}{x}\ln(a)$

⇒ for a>1, ln(a)>0,

hence f′(x) >0 which means strictly increasing.

⇒ for 0<a<1, ln(a)<0,

hence f′(x)<0 which means strictly decreasing.

9. Question

Prove that $f(x) = 3^x$ is strictly increasing on R.

Answer

Consider f(x)=3^x

The domain of f(x) is R.

f′(x)=3^xln(3)

3^x is always greater than 0 and ln(3) is also + ve.

Overall f′(x) is >0 means strictly increasing in its domain i.e. R.

10. Question

Show that $f(x)=x^3-15x^2+75x-50$ is increasing on R.

Answer

Consider $f(x)=x^3-15x^2+75x-50$

Domain of the function is R.

$f'(x)=3x^2-30x+75$

$=3(x^2-10+25)$

$=3(x-5)(x-5)$

$=3(x-5)^2$

$f'(x)=0$ for $x=5$

for $x<5$

$f'(x)>5$

and

for $x>5$

$f'(x)>5$

we can see throughout R the derivative is +ve but at x=5 it is 0 so it is increasing.

11. Question

Show that $f(x)=\left(x-\frac{1}{x}\right)$ is increasing all $x \in R$, where $x \neq 0$.

Answer

$f(x) = \left(x-\frac{1}{x}\right)$

domain of function is R-{0}

$f'(x) = 1+\frac{1}{x^2}$

$f'(x)\ \forall\ x \in R$ is greater than 0.

12. Question

Show that $f(x)=\left(\frac{1}{x}+5\right)$ is decreasing for all $x \in R$, where $x \neq 0$.

Answer

$f(x) = \frac{1}{x}+5$

domain of function is R-{0}

$f'(x) = -\frac{1}{x^2}$

for all x, $f'(x)<0$

Hence function is decreasing.

13. Question

Show that $f(x)=\frac{1}{(1+x^2)}$ is decreasing for all $x \geq 0$

Answer

Consider $f(x)=\frac{1}{(1+x^2)}$,

$f'(x)=-\frac{2x}{(1+x^2)^2}$

for $x \geq 0$,

f'(x) is -ve.

hence function is decreasing for $x \leq 0$

14. Question

Show that $f(x)=\left(x^3+\frac{1}{x^3}\right)$ is decreasing on $]-1,1[$.

Answer

$f(x)=x^3+x^{-3}$

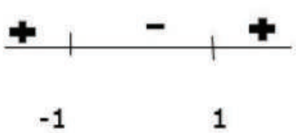
$f'(x)=3x^2-3x^{-4}$

$=3(x^2-1/x^4)$

$=3\left(\frac{x^3-1}{x^2}.\frac{x^3+1}{x^2}\right)$

$=\frac{3(x-1)(x^2+x+1)(x+1)(x^2-x+1)}{x^4}$

Root of f'(x)=1and-1



Here we can clearly see that f'(x) is decreasing in [-1,1]

So, f(x) is decreasing in interval [-1,1]

15. Question

Show that $f(x)=\frac{x}{\sin x}$ is increasing on $\left]0,\frac{\pi}{2}\right[$.

Answer

Consider $f(x)=\frac{x}{\sin x}$,

$f'(x)=\frac{\sin x - x.\cos x}{\sin^2 x}$

$f'(x)=\frac{\cos x(\tan x-x)}{\sin^2 x}$

in $\left]0,\frac{\pi}{2}\right[$ cos>0,

tan x-x>0,

$\sin^2 x > 0$

hence f'(x)>0,

so, function is increasing in the given interval.

16. Question

Prove that the function $f(x) = \log(1+x) - \frac{2x}{(x+2)}$ is increasing for all $x > -1$.

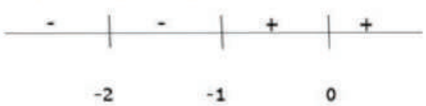
Answer

Consider $f(x) = \log(1+x) - \frac{2x}{(x+2)}$,

$$f'(x) = \frac{1}{1+x} - \frac{4}{(x+2)^2}$$

$$= \frac{(x+2)^2 - 4(x+1)}{(x+1)(x+2)^2}$$

$$= \frac{x^2}{(x+1)(x+2)^2}$$



Clearly we can see that f'(x)>o for x>-1.

Hence function is increasing for all x>-1

17. Question

Let I be an interval disjoint from $]-1,1[$. Prove that the function $f(x) = \left(x + \frac{1}{x}\right)$ is strictly increasing on I.

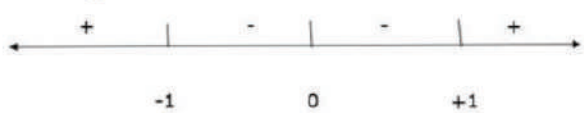
Answer

Consider $f(x) = \left(x + \frac{1}{x}\right)$

$$f'(x) = 1 - \frac{1}{x^2}$$

$$f'(x) = \frac{x^2 - 1}{x^2}$$

$$= \frac{x-1 . x+1}{x^2}$$



We can see f'(x) <0 in [-1,1]

i.e. f(x) is decreasing in this interval.

We can see f'(x) >0 in (-∞, -1) ∪(1, ∞)

i.e. f(x) is increasing in this interval.

18. Question

Show that $f(x)=\frac{(x-2)}{(x+1)}$ is increasing for all $x \in R$, except at $x=-1$.

Answer

Consider $f(x) = \frac{(x-2)}{(x+1)}$,

$$f'(x) = \frac{3}{(x+1)^2}$$

f'(x) at x=-1 is not defined

and for all x ∈ R- {-1}

f'(x)>0

hence f(x) is increasing.

19. Question

Find the intervals on which the function $f(x)=(2x^2-3x)$ is

(a) strictly increasing

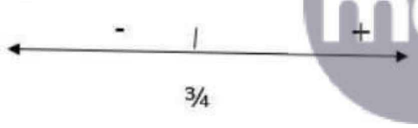
(b) strictly decreasing.

Answer

$f(x)=(2x^2-3x)$

f'(x)=4x-3

f'(x)=0 at x=3/4



Clearly we can see that function is increasing for x∈[3/4, ∞) and is decreasing for x∈(-∞,3/4)

20. Question

Find the intervals on which the function $f(x)=2x^3-3x^2-36x+7$ is

(a) strictly increasing (b) strictly decreasing.

Answer

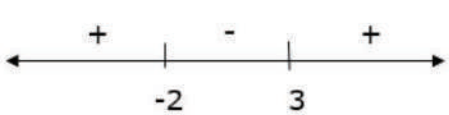
$f(x)=2x^3-3x^2-36x+7$

$f'(x)=6x^2-6x-36$

$f'(x)=6(x^2-x-6)$

f'(x)=6(x-3)(x+2)

f'(x) is 0 at x=3 and x=-2



F'(x)>0 for x ∈ (-∞, -2] ∪ [3, ∞)

hence in this interval function is increasing.

F'(x)<0 for x ∈ (-2 ,3)

hence in this interval function is decreasing.

21. Question

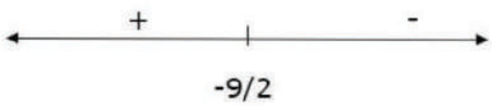
Find the intervals on which the function $f(x)=6-9x-x^2$ is

(a) strictly increasing (b) strictly decreasing.

Answer

$f(x)=6-9x-x^2$

$f'(x)=-(2x+9)$



We can see that f(x) is increasing for $x \in \left(-\infty,-\frac{9}{2}\right]$ and decreasing in $x \in \left(-\frac{9}{2},\infty\right)$

22. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x)=\left(x^4-\frac{x^3}{3}\right)$$

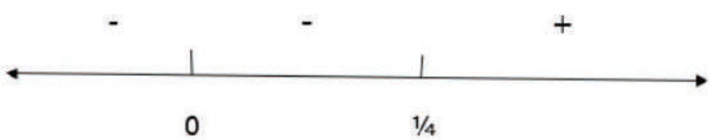
Answer

Consider $f(x)=\left(x^4-\frac{x^3}{3}\right)$

$f'(x)=4x^3-x^2$

$=x^2(4x-1)$

F'(x)=0 for x=0 and x=1/4



Function f(x) is decreasing for x ∈ (-∞,1/4] and increasing in x ∈ (1/4 , ∞)

23. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x)=x^3-12x^2+36x+17$$

Answer

$f(x)=x^3-12x^2+36x+17$

$f'(x)=3x^2-24x+36$

$f'(x)=3(x^2-8x+12)$

$=3(x-6)(x-2)$

Function f(x) is decreasing for x ∈ [2,6] and increasing in x ∈ (-∞,2) ∪ (6, ∞)

24. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = \left(x^3 - 6x^2 + 9x + 10\right)$$

Answer

$f(x)=x^3-6x^2+9x+10$

$f'(x)=3x^2-12x+9$

$f'(x)=3(x^2-4x+3)$

$=3(x-3)(x-1)$

Function f(x) is decreasing for x ∈ [1,3] and increasing in x ∈ (-∞,1) ∪ (3, ∞)

25. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = \left(6 + 12x + 3x^2 - 2x^3\right)$$

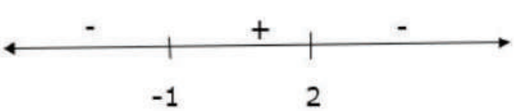
Answer

$f(x)=-2x^3+3x^2+12x+6$

$f'(x)=-6x^2+6x+12$

$f'(x)=-6(x^2-x-2)$

$=-6(x-2)(x+1)$



Function f(x) is increasing for x ∈ [-1,2] and decreasing in x ∈ (-∞,-1) ∪ (2, ∞)

26. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = 2x^3 - 24x + 5$$

Answer

$f(x)=2x^3-24x+5$

$f'(x)=6x^2-24$

$f'(x)=6(x^2-4)$

$=6(x-2)(x+2)$

Function f(x) is decreasing for x ∈ [-2,2] and increasing in x ∈ (-∞,-2) ∪ (2, ∞)

27. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = (x-1)(x-2)^2$$

Answer

f(x)=(x-1)$(x-2)^2$=x2-4x+4 * x-1= x3-4x2+4x-x2+4x-4

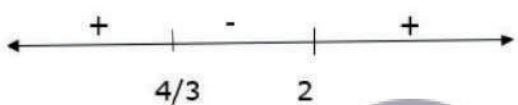
$f(x)=x^3-5x^2+8x-4$

$f'(x)=3x^2-10x+8$

$f'(x)=3x^2-6x-4x+8$

$=3x(x-2)-4(x-2)$

$=(3x-4)(x-2)$



Function f(x) is decreasing for x ∈ [4/3,2] and increasing in x ∈ (-∞,4/3) ∪ (2, ∞)

28. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = (x^4 - 4x^3 + 4x^2 + 15)$$

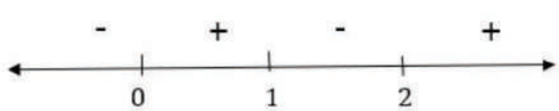
Answer

$f(x)=x^4-4x^3+4x^2+15$

$f'(x)=4x^3-12x^2+8x$

$= 4x(x^2-3x+2)$

$=4x(x-1)(x-2)$



Function f(x) is decreasing for x ∈ (-∞,0] ∪ [1, 2] and increasing in x ∈ (0,1) ∪ (2, ∞)

29. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = 2x^3 + 9x^2 + 12x + 15$$

Answer

$f(x)=2x^3+9x^2+12x+15$

$f'(x)=6x^2+18x+12$

$f'(x)=6(x^2+3x+2)$

$=6(x+2)(x+1)$

Function f(x) is decreasing for $x \in [-1,-2]$ and increasing in $x \in (-\infty,-1) \cup (-2, \infty)$

30. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

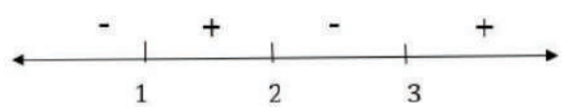
Answer

$f(x)=x^4-8x^3+22x^2-24x+21$

$f'(x)=4x^3-24x^2+44x-24$

$= 4(x^3 -6x^2+11x-6)$

$=4(x-3)(x-1)(x-2)$



Function f(x) is decreasing for $x \in (-\infty,1] \cup [2, 3]$ and increasing in $x \in (1,2) \cup (3, \infty)$

31. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

Answer

$f(x)=3x^4-4x^3-12x^2+5$

$f'(x)=12x^3-12x^2-24x$

$=12x(x^2 -x-2)$

$=12(x)(x+1)(x-2)$

Function f(x) is decreasing for $x \in (-\infty,-1] \cup [0, 2]$ and increasing in $x \in (-1,0) \cup (2, \infty)$

32. Question

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

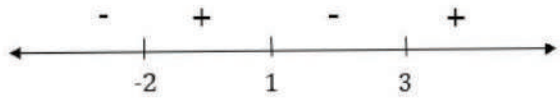
Answer

$f'(x) = \frac{12x^3}{10} - \frac{12x^2}{5} - 6x + \frac{36}{5}$

$f'(x)=(12x^3-24x^2-60x+72)/10$

$=1.2(x^3-2x^2-5x+6)$

$=1.2(x-1)(x-3)(x+2)$



Function f(x) is decreasing for $x \in (-\infty,-2] \cup [1, 3]$ and increasing in $x \in (-2,1) \cup (3, \infty)$

Exercise 11H

1. Question

Find the slope of the tangent to the curve

i. $y = \left(x^3 - x\right)$ at $x = 2$

ii. $y = \left(2x^2 + 3\sin x\right)$ at $x = 0$

iii. $y = \left(\sin 2x + \cot x + 2\right)^2$ at $x = \frac{\pi}{2}$

Answer

i. $\frac{dy}{dx} = 3x^2 - 1$

$\frac{dy}{dx}$ at $(x = 2) = 11$

ii. $\frac{dy}{dx} = 4x + 3\cos x$

$\frac{dy}{dx}$ at $(x = 0) = 3$

iii. $\frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2 x)$

$\frac{dy}{dx}$ at $\left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$

2. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$y = x^3 - 2x + 7$ at $(1, 6)$

Answer

$m : \frac{dy}{dx} = 3x^2 - 2$

m at (1, 6) = 1

Tangent : y – b = m(x – a)

y – 6 = 1(x – 1)

x – y + 5 = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

y – 6 = -1(x – 1)

x + y – 7 = 0

3. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Answer

$m: 2y\frac{dy}{dx} = 4a$

$m \text{ at } \left(\frac{a}{m^2}, \frac{2a}{m}\right) = m$

Tangent : y – b = m(x – a)

$y - \frac{2a}{m} = m\left(x - \frac{a}{m^2}\right)$

$m^2x - my + a = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - \frac{2a}{m} = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$

$m^2x + m^3y - 2am^2 - a = 0$

4. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$

Answer

$m: \frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

$m \text{ at } (a\cos\theta, b\sin\theta) = \frac{-b\cos\theta}{a\sin\theta}$

Tangent : y – b = m(x – a)

$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$

$bx\cos\theta + ay\sin\theta = ab$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

ax sec θ – by cosec θ = $a^2 - b^2$

5. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\sec\theta, b\tan\theta)$

Answer

$m: \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$

$m \text{ at } (a\sec\theta, b\tan\theta) = \frac{b\sec\theta}{a\tan\theta}$

Tangent : y - b = m(x - a)

$$y - b\tan\theta = \frac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$$

$$bx\sec\theta - ay\tan\theta = ab$$

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - b\sin\theta = \frac{-a\sin\theta}{b\cos\theta}(x - a\cos\theta)$$

$$by\,\mathrm{cosec}\theta + ax\sec\theta = (a^2 + b^2)$$

6. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y = x^3 \text{ at } P(1,1)$$

Answer

m : $\frac{dy}{dx} = 3x^2$

m at (1, 1) = 3

Tangent : y - b = m(x - a)

y - 1 = 3(x - 1)

y = 3x - 2

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - 1 = \frac{-1}{3}(x - 1)$$

x + 3y = 4

7. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y^2 = 4ax \text{ at } (at^2, 2at)$$

Answer

m : $2y\frac{dy}{dx} = 4a$

m at (at^2, 2at) = 1/t

Tangent : y - b = m(x - a)

$$y - 2at = \frac{1}{t}(x - at^2)$$

x - ty + at^2 = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

y - 2at = -t(x - at^2)

tx + y = at^3 + 2at

8. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$y=\cot^2 x-2\cot x+2$ at $x=\frac{\pi}{4}$

Answer

$m: \frac{dy}{dx}=2\cot x(-\text{cosec}^2 x)+2\,\text{cosec}^2 x$

m at (x = π/4) = 2(-2) + 2(2) = 0

Tangent : y - b = m(x - a)

y - 1 = 0(x - π/4)

y = 1

Normal : $y-b=\frac{-1}{m}(x-a)$

$y-1=\frac{-1}{0}\left(x-\frac{\pi}{4}\right)$

x = π/4

9. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$16x^2+9y^2=144$ at $(2, y_1)$, where $y_1>0$

Answer

$m: 32x+18y\frac{dy}{dx}=0$

$m \text{ at } (2, y_1)=\frac{-32}{9y_1}$

$16(2)^2+9(y_1)^2=144$

$y_1=\frac{4\sqrt{5}}{3}$

Tangent : y - b = m(x - a)

$y-\frac{4\sqrt{5}}{3}=\frac{-32}{9\frac{4\sqrt{5}}{3}}(x-2)$

$8x+3\sqrt{5}y-36=0$

Normal : $y-b=\frac{-1}{m}(x-a)$

$y-\frac{4\sqrt{5}}{3}=\frac{9\frac{4\sqrt{5}}{3}}{32}(x-2)$

$9\sqrt{5x}-24y+14\sqrt{5}=0$

10. Question

Find the equations of the tangent and the normal to the given curve at the indicated point for

$y=x^4-6x^3+13x^2-10x+5$ at the point where x = 1

Answer

$m: \frac{dy}{dx}=4x^3-18x^2+26x-10$

m at (x = 1) = 2

y at (x = 1) = $(1)^4 - 6(1)^3 + 13(1)^2 - 10(1) + 5 = 3$

Tangent : y – b = m(x – a)

y – 3 = 2(x – 1)

2x – y + 1 = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 3 = \frac{-1}{2}(x - 1)$

x + 2y – 7 = 0

11. Question

Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = a$ at $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$

Answer

$m : \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$m \text{ at } \left(\frac{a^2}{4}, \frac{a^2}{4}\right) = -1$

y – b = m(x – a)

$y - \frac{a^2}{4} = -1\left(x - \frac{a^2}{4}\right)$

$2(x + y) = a^2$

12. Question

Show that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

Answer

$m \text{ at } (x_1, y_1) = \frac{b^2x_1}{a^2y_1}$

At $(x_1, y_1) : \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1 \Rightarrow b^2x_1^2 - a^2y_1^2 = a^2b^2$

y – b = m(x – a)

$y - y_1 = \frac{b^2x_1}{a^2y_1}(x - x_1)$

$a^2y_1y - a^2y_1^2 = b^2x_1x - b^2x_1^2$

$b^2x_1x - a^2y_1y = a^2b^2$

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

13. Question

Find the equation of the tangent to the curve $y = \left(\sec^4 x - \tan^4 x\right)$ at $x = \frac{\pi}{3}$.

Answer

$m : \frac{dy}{dx} = 4\sec^3x(\tan x \sec x) - 4\tan^3x(\sec^2x)$

$m \text{ at } \left(x = \frac{\pi}{3}\right) = 4(2)^3(\sqrt{3} \times 2) - 4(\sqrt{3})^3(2)^2 = 16\sqrt{3}$

At x = π/3 , y = 7

y - b = m(x - a)

$y - 7 = 16\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$3y - 48\sqrt{3}x + 16\sqrt{3}\pi - 21 = 0$

14. Question

Find the equation of the normal to the curve $y = (\sin 2x + \cot x + 2)^2$ at $x = \frac{\pi}{2}$

Answer

$m : \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2x)$

$\frac{dy}{dx} \text{ at } \left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$

At x = π/2, y = 4

$y - b = \frac{-1}{m}(x - a)$

$y - 4 = \frac{1}{12}\left(x - \frac{\pi}{2}\right)$

24y - 2x + π - 96 = 0

15. Question

Show that the tangents to the curve $y = 2x^3 - 4$ at the point x = 2 and x = −2 are parallel.

Answer

$m : \frac{dy}{dx} = 6x^2$

m at (x = 2) = 24

m at (x = -2) = 24

We know that if the slope of curve at two different point is

equal then straight lines are parallel at that points.

16. Question

Find the equation of the tangent to the curve $x^2 + 3y = 3$, where is parallel to the line $y - 4x + 5 = 0$.

Answer

We know that if two straight lines are parallel then their slope

are equal. So, slope of required tangent is also equal to 4.

$m : \frac{dy}{dx} = \frac{-2x}{3} = 4$

x = -6 and y = -11

y – b = m(x – a)

y – (-11) = 4(x – (-6))

4x – y + 13 = 0

17. Question

At what point on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, is the tangent parallel to the y-axis?

Answer

If the tangent is parallel to y-axis it means that it's slope is

not defined or 1/0.

$m : 2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-(2x-2)}{(2y-4)} = \frac{1}{0}$

2y – 4 = 0 ⇒ y = 2

$x^2 + (2)^2 - 2x - 4(2) + 1 = 0$

⇒ $x^2 - 2x - 3 = 0$

⇒ x = 3 and x = -1

So, the requied points are (-1, 2) and (3, 2).

18. Question

Find the point on the curve $x^2 + y^2 - 2x - 3 = 0$ where the tangent is parallel to the x-axis.

Answer

If the tangent is parallel to x-axis it means that it's slope is 0

$m : 2x + 2y\frac{dy}{dx} - 2 = 0$

2x + 2y(0) – 2 = 0

x = 1

$(1)^2 + y^2 - 2(1) - 3 = 0$

⇒ $y^2 = 4$ ⇒ y = 2 and y = -2

So, the requied points are (1, 2) and (1, -2).

19. Question

Prove the tangent to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3, 0) are at right angles.

Answer

We know that if the slope of two tangent of a curve are satisfies a relation $m_1m_2 = -1$, then tangents are at right angles

$m : \frac{dy}{dx} = 2x - 5$

m_1 at (2, 0) = -1

m_2 at (3, 0) = 1

$m_1m_2 = (-1)(1) = -1$

So, we can say that tangent at (2, 0) and (3, 0) are at right angles.

20. Question

Find the point on the curve $y = x^2 + 3x + 4$ at which the tangent passes through the origin.

Answer

If tangent is pass through origin it means that equation of tangent is y = mx

Let us suppose that tangent is made at point (x_1, y_1)

$y_1 = x_1^2 + 3x_1 + 4$ ...(1)

$m : \frac{dy}{dx} = 2x + 3$

m at $(x_1, y_1) = 2x_1 + 3$

Equation of tangent : $y_1 = (2x_1 + 3)x_1$ ...(2)

On comparing eq(1) and eq(2)

$x_1^2 + 3x_1 + 4 = (2x_1 + 3)x_1$

$x_1^2 - 4 = 0 \Rightarrow x_1 = 2$ and -2

At $x_1 = 2$, $y_1 = 14$

At $x_1 = -2$, $y_1 = 2$

So, required points are (2, 14) and (-2, 2)

21. Question

Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$.

Answer

Slope of y = x - 11 is equal to 1

$m : \frac{dy}{dx} = 3x^2 - 11$

$3x^2 - 11 = 1 \Rightarrow x = 2$ and -2

At x = 2

From the equation of curve, $y = (2)^3 - 11(2) + 5 = -9$

From the equation of tangent, y = 2 - 11 = -9

At x = -2

From the equation of curve, $y = (-2)^3 - 11(-2) + 5 = 19$

From the equation of tangent, y = -2 - 11 = -13

So, the final answer is (2, -9) because at x = -2, y is come different from the equation of curve and tangent which is not possible.

22. Question

Find the equation of the tangents to the curve $2x^2 + 3y^2 = 14$, parallel to the line $x = 3y = 4$.

Answer

If tangent is parallel to the line x + 3y = 4 then it's slope is -1/3.

$$m : 4x + 6y\frac{dy}{dx} = 0$$

$$m = \frac{-2x}{3y} = \frac{-2x}{3\sqrt{\frac{14-2x^2}{3}}} = \frac{-1}{3}$$

$$2x = \sqrt{\frac{14-2x^2}{3}}$$

$$4x^2 = \frac{14-2x^2}{3}$$

x = 1 and -1

At x = 1, y = 2 and y = -2 (not possible)

At x = -1, y = -2 and y = 2 (not possible)

y - b = m(x - a)

At (1, 2)

$$y - 2 = \frac{-1}{3}(x-1)$$

3y + x = 7

At (-1, -2)

$$y-(-2) = \frac{-1}{3}(x-(-1))$$

3y + x = -7

23. Question

Find the equation of the tangent to the curve $x^2 + 2y = 8$, which is perpendicular to the line $x - 2y + 1 = 0$.

Answer

.: If tangent is perpendicular to the line x - 2y + 1 = 0 then it's -1/m is -2.

$$m : 2x + 2\frac{dy}{dx} = 0$$

m = -x = 1/2

x = -1/2

At x = -1/2, y = 31/8

$$y - b = \frac{-1}{m}(x-a)$$

At (-1/2, 31/8)

$$y - \frac{31}{8} = \frac{-1}{\frac{1}{2}}\left(x - \left(-\frac{1}{2}\right)\right)$$

16x + 8y - 23 = 0

24. Question

Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis.

Answer

We know that if tangent is parallel to x-axis then it's slope is equal to 0.

$$m : \frac{dy}{dx} = 4x - 6$$

4x – 6 = 0 ⇒ x = 3/2

At x = 3/2 , y = -17/2

So, the required points are $\left(\frac{3}{2}, \frac{-17}{2}\right)$.

25. Question

Find the point on the parabola $y = (x-3)^2$, where the tangent is parallel to the chord joining the point (3, 0) and (4, 1).

Answer

If the tangent is parallel to chord joining the points (3, 0) and (4, 1) then slope of tangent is equal to slope of chord.

$$m = \frac{1-0}{4-3} = 1$$

$$m : \frac{dy}{dx} = 2(x-3)$$

2(x – 3) = 1 ⇒ x = 7/2

At x = 7/2, y = 1/4

So, the required points are $\left(\frac{7}{2}, \frac{1}{4}\right)$.

26. Question

Show that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Answer

If curves cut at right angle if $8k^2 = 1$ then vice versa also true. So, we have to prove that $8k^2 = 1$ if curve cut at right angles.

If curve cut at right angle then the slope of tangent at their intersecting point satisfies the relation $m_1 m_2 = -1$

We have to find intersecting point of two curves.

$x = y^2$ and xy = k then $y = k^{\frac{1}{3}}$ and $x = k^{\frac{2}{3}}$

$$m_1 : \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$m_1 \text{ at } \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \frac{1}{2k^{\frac{1}{3}}}$$

$$m_2 : \frac{dy}{dx} = \frac{-k}{x^2}$$

$$m_2 \text{ at } \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \frac{-k}{k^{\frac{4}{3}}} = -\frac{1}{k^{\frac{1}{3}}}$$

$$m_1 m_2 = -1$$