Exercise 20(B)

1. Find the area of a quadrilateral one of whose diagonals is 30 cm long and the perpendiculars from the other two vertices are 19 cm and 11 cm respectively.
Solution:

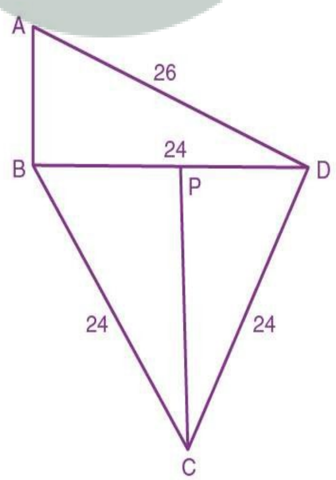
We know that,
Area of quadrilateral = ½ x one diagonal x (sum of the lengths of the perpendiculars drawn from it on the remaining two vertices)
= ½ x 30 x (11 + 19)
= 15 x 30
= 450 sq. cm

2. The diagonals of a quadrilateral are 16 cm and 13 cm. If they intersect each other at right angles; find the area of the quadrilateral.
Solution:

We know that,
Area of the quadrilateral = ½ x the product of the diagonals
= ½ x 16 x 13
= 8 x 13
= 104 cm^2

3. Calculate the area of quadrilateral ABCD, in which ∠ABD = 90º, triangle BCD is an equilateral triangle of side 24 cm and AD = 26 cm.
Solution:

Let's consider the below figure:



From the right triangle ABD, we have ∠ABD = 90º
So, by Pythagoras Theorem
AB = $\sqrt{(26^2 - 24^2)}$
= $\sqrt{(676 - 576)}$
= $\sqrt{100}$

= 10 cm

Now, the area of right triangle ABD is

Ar (∆ABD) = ½ x AB x BD

= ½ x 10 x 24

= 120 cm^2

Again, in the equilateral triangle BCD we have, CP ⊥ BD

So, by Pythagoras Theorem

PC = $\sqrt{(24^2 - 12^2)}$

= √(576 – 144)

= √432

= √(144 x 3)

= 12√3 cm

Now, the area of the triangle BCD is

Ar (∆BCD) = ½ x BD x PC

= ½ x 24 x 12√3

= 144√3 cm^2

Therefore, the area of the quadrilateral is given by

Ar(ABCD) = Ar (∆ABD) + Ar (∆BCD)

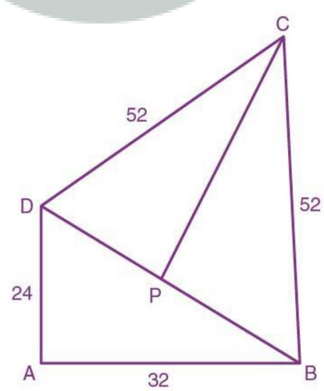
= (120 + 144√3) cm^2

= 369.41 cm^2

4. Calculate the area of quadrilateral ABCD in which AB = 32 cm, AD = 24 cm, ∠A = 90º and BC = CD = 52 cm.

Solution:

The figure can be drawn as follows:



We have, quadrilateral ABCD in which AB = 32 cm, AD = 24 cm, ∠A = 90º and BC = CD = 52 cm

Now, as ABD is a right triangle, it's the area is given as

$\Delta ABD = \frac{1}{2} \times 24 \times 32$
$= 12 \times 32$
$= 384\ cm^2$
Again, by Pythagoras Theorem
$BD = \sqrt{(24^2 + 32^2)}$
$= 8\sqrt{(3^2 + 4^2)}$
$= 8\sqrt{25}$
$= 8 \times 5$
$= 40$ cm
Now, as BCD is an isosceles triangle and $BP \perp BD$, we have
$DP = \frac{1}{2} BD$
$= \frac{1}{2} \times 40$
$= 20$ cm
Then,
From the right triangle DPC, we have
$PC = \sqrt{(52^2 - 20^2)}$ [By Pythagoras Theorem]
$= 4\sqrt{(13^2 - 5^2)}$
$= 4\sqrt{(169 - 25)}$
$= 4 \times \sqrt{144}$
$= 4 \times 12$
$= 48$ cm
So, the area of $\Delta DPC = \frac{1}{2} \times 40 \times 48$
$= 20 \times 48$
$= 960\ cm^2$

Therefore, the area of the quadrilateral is given by
$Ar(\Delta ABD) + Ar(\Delta DPC) = 960 + 384$
$= 1344\ cm^2$

5. The perimeter of a rectangular field is 3/5 km. If the length of the field is twice its width; find the area of the rectangle in sq. metres.
Solution:

Let's assume the width of the rectangular field to be x km and length to be 2x km
Now, according to the question, we have
$2(x + 2x) = 3/5$
$3x = 3/10$
$x = 1/10$ km
i.e., $x = 1000/10$ m $= 100$ m

Thus, the width is 100 m and the length is 200m of the rectangular field
Therefore, the area of the rectangular field is
A = length x width
$= 100 \times 200$
$= 20,000$ sq. m