Straight Lines

Exercise 20A

Q. 1. Find the distance between the points:

(i) A(2, -3) and B(-6, 3)
(ii) C(-1, -1) and D(8, 11)
(iii) P(-8, -3) and Q(-2, -5)
(iv) R(a + b, a – b) and S(a – b, a + b)

Answer : (i) Formula Used:

Distance between any two points A(x_1, y_1) and B(x_2, y_2)=

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between A(2, -3) and B(-6, 3)

$$= \sqrt{(-6 - 2)^2 + (3 - (-3))^2}$$

$$= \sqrt{64 + 36} = \sqrt{100}$$

= 10 units

Therefore, the distance between points A and B is 10 units.

(ii) Distance between C(-1, -1) and D(8, 11) =

$$\sqrt{(8 - (-1))^2 + (11 - (-1))^2}$$

$$= \sqrt{81 + 144} = \sqrt{225}$$

= 15 units

Therefore, the distance between points C and D is 10 units.

(iii) Distance between P(-8, -3) and Q(-2, -5)=

$\sqrt{(-2 - (-8))^2 + (-5 - (-3))^2}$

$= \sqrt{36 + 4} = \sqrt{40}$

$= 2\sqrt{10}$ units

Therefore, the distance between the points P and Q is $2\sqrt{10}$ units.

(iv) Distance between R(a + b, a - b) and S(a - b, a + b) $\sqrt{((a - b) - (a + b))^2 + ((a + b) - (a - b))^2}$

$= \sqrt{4b^2 + 4b^2}$

$= 2b\sqrt{2}$ units

Therefore, the distance between the points R and S is $2b\sqrt{2}$ units.

Q. 2. Find the distance of the point P(6, -6) from the origin.

Answer : Distance of point P(6, -6) from origin (0, 0) =

$\sqrt{(0 + 6)^2 + (0 - 6)^2}$

$= \sqrt{36 + 36}$

= $6\sqrt{2}$ units

Therefore, the distance of the point P from the origin is $6\sqrt{2}$ units.

Q. 3. If a point P(x, y) is equidistant from the points A(6, -1) and B(2, 3), find the relation between x and y.

Answer : Given: Point P(x, y) is equidistant from points A(6, -1) and B(2, 3)

i.e., distance of P from A = distance of P from B

$\Rightarrow \sqrt{(x - 6)^2 + (y + 1)^2} = \sqrt{(x - 2)^2 + (y - 3)^2}$

Squaring both sides,

$\Rightarrow (x - 6)^2 + (y - 1)^2 = (x - 2)^2 + (y - 3)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$

$\Rightarrow -12x + 36 + 2y + 1 = -4x + 4 - 6y + 9$

$\Rightarrow -8x + 8y = -24$

$\Rightarrow x - y = 3$

Therefore, $x - y = 3$ is the required relation.

Q. 4. Find a point on the x-axis which is equidistant from the points A(7, 6) and B(-3, 4).

Answer : Let the point on x-axis be P(x, 0).

Given: Point P(x, 0) is equidistant from points A(7, 6) and B(-3, 4)

i.e., distance of P from A = distance of P from B

$\Rightarrow \sqrt{(x-7)^2 + 36} = \sqrt{(x+3)^2 + 16}$

Squaring both sides,

$\Rightarrow (x - 7)^2 + 36 = (x + 3)^2 + 16$

$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

$\Rightarrow -20x = -60$

$\Rightarrow x = 3$

Therefore, the point on the x-axis is (3, 0).

Q. 5., Find the distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$, when

(i) AB is parallel to the x-axis
(ii) AB is parallel to the y-axis.

Answer : **(i) Given:** AB is parallel to the x-axis.

When AB is parallel to the x-axis, the y co-ordinate of A and B will be the same.

i.e., $y_1 = y_2$

Distance

$$= \sqrt{(x_2 - x_1)^2 + (y_1 - y_1)^2}$$

⇒ $|x_2 – x_1|$

Therefore the distance between A and B when AB is parallel to x-axis is $|x_2 – x_1|$

(ii) Given: AB is parallel to the y-axis.

When AB is parallel to the y-axis, the x co-ordinate of A and B will be the same.

i.e., $x_2 = x_1$

Distance

$$= \sqrt{(x_1 - x_1)^2 + (y_2 - y_1)^2}$$

⇒ $|y_2 – y_1|$

Therefore the distance between A and B when AB is parallel to y-axis is $|y_2 – y_1|$

Q. 6. A is a point on the x-axis with abscissa -8 and B is a point on the y-axis with ordinate 15. Find the distance AB.

Answer : Given: The two points are A(-8, 0) and B(0, 15)

Distance between A and B

$$= \sqrt{(0+8)^2 + (15-0)^2}$$

$$\Rightarrow \sqrt{64 + 225}$$

⇒ $\sqrt{289}$

⇒ 17 units

Therefore, the distance between A and B is 17 units.

Q. 7. Find a point on the y-axis which is equidistant from A(-4, 3) and B(5, 2).

Answer : Let the point on the y-axis be P(0, y)

Given: P is equidistant from A(-4, 3) and B(5, 2).

i.e., PA = PB

$$\Rightarrow \sqrt{(-4-0)^2+(3-y)^2} = \sqrt{(5-0)^2+(2-y)^2}$$

Squaring both sides, we get

$\Rightarrow (-4 – 0)^2 + (3 – y)^2 = (5 – 0)^2 + (2 – y)^2$

$\Rightarrow 16 + 9 – 6y + y^2 = 25 + 4 – 4y + y^2$

$\Rightarrow 25 – 6y = 29 – 4y$

$\Rightarrow 2y = -4$

$\Rightarrow y = -2$

Therefore, the required point on the y-axis is (0, -2).

Q. 8. Using the distance formula, show that the points A(3, -2), B(5, 2) and C(8,8) are collinear.

Answer : Given: The 3 points are A(3, -2), B(5, 2) and C(8, 8).

$$AB = \sqrt{(5-3)^2+(2+2)^2}$$

$$= \sqrt{4+16}$$

$= 2\sqrt{5}$ units .....(1)

$$BC = \sqrt{(8-5)^2+(8-2)^2}$$

$$= \sqrt{9+36}$$

$= 3\sqrt{5}$ units .....(2)

$$AC = \sqrt{(8-3)^2+(8+2)^2}$$

$$= \sqrt{25+100}$$

$= 5\sqrt{5}$ units .....(3)

From equations 1, 2 and 3, we have

⇒ AC = AB + BC

This is possible only if the points are collinear.

Therefore, the points A, B and C are collinear.

Hence, proved.

Q. 9. Show that the points A(7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right-angled triangle.

Answer : Given: The 3 points are A(7, 10), B(-2, 5) and C(3, -4)

$$AB = \sqrt{(-2-7)^2 + (5-10)^2}$$

$$= \sqrt{81 + 25}$$

$= \sqrt{106}$ units .....(1)

$$BC = \sqrt{(3+2)^2 + (-4-5)^2}$$

$$= \sqrt{25 + 81}$$

$= \sqrt{106}$ units .....(2)

$$AC = \sqrt{(3-7)^2 + (-4-10)^2}$$

$$= \sqrt{16 + 196}$$

$= \sqrt{212}$ units

From equations 1 and 2, we have

⇒ AB = BC

Therefore, Δ ABC is an isosceles triangle .....(3)

Also, $AB^2 = 106$ units .....(4)

BC^2 = 106 units …..(5)

AC^2 = 212 units …..(6)

From equations 4, 5 and 6, we have

$AB^2 + BC^2 = AC^2$

So, it satisfies the Pythagoras theorem.

Δ ABC is right angled triangle …..(7)

From 3 and 7, we have

Δ ABC is an isosceles right angled triangle.

Hence, proved.

Q. 10. Show that the points A(1, 1), B(-1, -1) and C(-√3, √3) are the vertices of an equilateral triangle each of whose sides is 22 units.

Answer : Given: The 3 points are A(1, 1), B(-1, -1) and C(-√3, √3).

$$AB = \sqrt{(-1 - 1)^2 + (-1 - 1)^2}$$

$$= \sqrt{4 + 4}$$

= 2√2 units …..(1)

$$BC = \sqrt{(-\sqrt{3} + 1)^2 + (\sqrt{3} + 1)^2}$$

$$= \sqrt{3 - 2\sqrt{3} + 1 + 3 + 2\sqrt{3} + 1}$$

= 2√2 units …..(2)

$$AC = \sqrt{(-\sqrt{3} - 1)^2 + (\sqrt{3} - 1)^2}$$

$$= \sqrt{3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} + 1}$$

= 2√2 units .....(3)

From equations 1, 2 and 3, we have

AB = BC = AC = 2√2 units.

Therefore, Δ ABC is an equilateral triangle each of whose sides is 2√2 units.

Hence, proved.

Q. 11. Show that the points A(2, -2), B(8, 4), C(5, 7) and D(-1, 1) are the angular points of a rectangle.

Answer : Given: The 4 points are A(2, -2), B(8, 4), C(5, 7) and D(-1, 1).

Note: For a quadrilateral to be a rectangle, the opposite sides of the quadrilateral must be equal and the diagonals must be equal as well.

AB = $\sqrt{36 + 36}$

= 6√2 units .....(1)

BC = $\sqrt{9 + 9}$

= 3√2 units .....(2)

CD = $\sqrt{36 + 36}$

= $6\sqrt{2}$ units .....(3)

AD = $\sqrt{9 + 9}$

= $3\sqrt{2}$ units .....(4)

From equations 1, 2, 3 and 4, we have

AB = CD and BC = AD .....(5)

Also, AC = $\sqrt{9 + 81}$

= $3\sqrt{10}$ units

BD = $\sqrt{81 + 9}$

= $3\sqrt{10}$ units

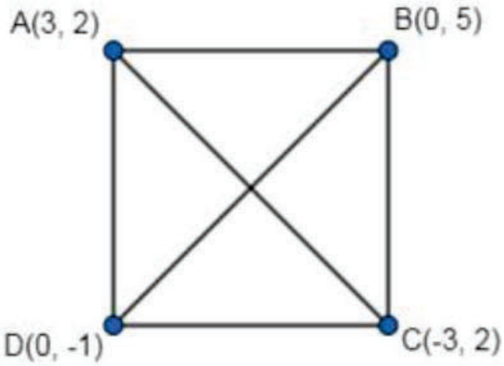
Thus, AC = BD .....(6)

From equations 5 and 6, we can conclude that the opposite sides of quadrilateral ABCD are equal and the diagonals of ABCD are equal as well.

Therefore, point A, B, C and D are the angular points of a rectangle.

Q. 12. Show that A(3, 2), B(0, 5), C(-3, 2) and D(0, -1) are the vertices of a square.

Answer :



Given: The points are A(3, 2), B(0, 5), C(-3, 2) and D(0, -1).

Note: For a quadrilateral to be a square, all the sides of the quadrilateral must be equal in length and the diagonals must be equal in length as well.

$AB = \sqrt{(0 - 3)^2 + (5 - 2)^2} = \sqrt{9 + 9}$

$= 3\sqrt{2}$ units

$BC = \sqrt{(-3 - 0)^2 + (2 - 5)^2} = \sqrt{9 + 9}$

$= 3\sqrt{2}$ units

$CD = \sqrt{(0 + 3)^2 + (-1 - 2)^2} = \sqrt{9 + 9}$

$= 3\sqrt{2}$ units

$DA = \sqrt{(3 - 0)^2 + (2 + 1)^2} = \sqrt{9 + 9}$

$= 3\sqrt{2}$ units

Therefore, AB = BC = CD = DA …..(1)

$$AC = \sqrt{(-3 - 3)^2 + (2 - 2)^2}$$

= 6 units

$$BD = \sqrt{(0 - 0)^2 + (-1 - 5)^2}$$

= 6 units

Therefore, AC = BD …..(2)

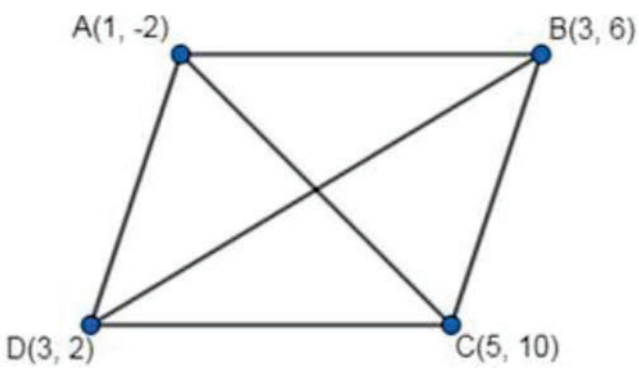
From 1 and 2, we have all the sides of ABCD are equal and the diagonals are equal in length as well.

Therefore, ABCD is a square.

Hence, the points A, B, C and D are the vertices of a square.

Q. 13. Show that A(1, -2), B(3, 6), C(5, 10) and D(3, 2) are the vertices of a parallelogram.

Answer :



Given: Vertices of the quadrilateral are A(1, -2), B(3, 6), C(5, 10) and D(3, 2).

Note: For a quadrilateral to be a parallelogram opposite sides of the quadrilateral must be equal in length, and the diagonals must not be equal.

AB = $\sqrt{(3 - 1)^2 + (6 + 2)^2} = \sqrt{4 + 64}$

= 2√17 units

BC = $\sqrt{(5 - 3)^2 + (10 - 6)^2} = \sqrt{4 + 16}$

= 2√5 units

CD = $\sqrt{(3 - 5)^2 + (2 - 10)^2} = \sqrt{4 + 64}$

= 2√17 units

DA = $\sqrt{(1 - 3)^2 + (-2 - 2)^2} = \sqrt{4 + 16}$

= 2√5 units

Therefore, AB = CD and BC = DA .....(1)

AC = $\sqrt{(5 - 1)^2 + (10 + 2)^2} = \sqrt{16 + 144}$

= 4√10 units

BD = $\sqrt{(3 - 3)^2 + (2 - 6)^2}$

= 4 units

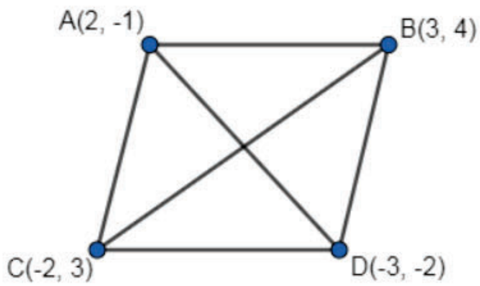
Therefore, AC ≠ BD .....(2)

From 1 and 2, we have

Opposite sides of ABCD are equal, and diagonals are not equal. Hence, points A, B, C and D are the vertices of a parallelogram.

Q. 14. Show that the points A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2) are the vertices of a rhombus.

Answer :



Given: Vertices of the quadrilateral are A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2).
Note: For a quadrilateral to be a rhombus, all the sides must be equal in length and the diagonals must not be equal.

$$AB = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{1+25}$$

$= \sqrt{26}$ units

$$BC = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1}$$

$= \sqrt{26}$ units

$$CD = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{1+25}$$

$= \sqrt{26}$ units

$$DA = \sqrt{(2+3)^2 + (-1+2)^2} = \sqrt{25+1}$$

$= \sqrt{26}$ units

Therefore, AB = BC = CD = DA .....(1)

AC = $\sqrt{(-2 - 2)^2 + (3 + 1)^2} = \sqrt{16 + 16}$

$= 4\sqrt{2}$ units

BD = $\sqrt{(-3 - 3)^2 + (-2 - 4)^2} = \sqrt{36 + 36}$

$= 6\sqrt{2}$ units

Also, AC ≠ BD .....(2)

From 1 and 2, we have all the sides are equal and diagonals are not equal.

Hence, the points A, B, C and D are the vertices of a rhombus.

Q. 15. If the points A (-2, -1), B(1, 0), C(x, 3) and D(1, y) are the vertices of a parallelogram, find the values of x and y.

Answer : Given: Vertices of the parallelogram are A(-2, -1), B(1, 0), C(x, 3) and D(1, y).

To find: values of x and y.

Since, ABCD is a parallelogram, we have AB = CD and BC = DA.

AB = $\sqrt{(1 + 2)^2 + (0 + 1)^2} = \sqrt{9 + 1}$

$= \sqrt{10}$ units

BC = $\sqrt{(x - 1)^2 + 9}$

$CD = \sqrt{(1 - x)^2 + (y - 3)^2}$

$DA = \sqrt{9 + (1 + y)^2}$

Since AB = CD,

$\Rightarrow \sqrt{10} = \sqrt{(1 - x)^2 + (y - 3)^2}$

Squaring both sides, we get

$\Rightarrow 10 = (1 - x)^2 + (y - 3)^2$

$\Rightarrow 10 = 1 - 2x + x^2 + y^2 - 6y + 9$

$\Rightarrow x^2 + y^2 - 2x - 6y = 0$ …..(1)

Since BC = DA,

$\Rightarrow \sqrt{(x - 1)^2 + 9} = \sqrt{9 + (1 + y)^2}$

Squaring both sides,

$\Rightarrow (x - 1)^2 + 9 = 9 + (1 + y)^2$

$\Rightarrow x^2 - 2x + 1 = 1 + 2y + y^2$

$\Rightarrow x^2 - y^2 - 2x - 2y = 0$ …..(2)

Equation 1 – Equation 2 gives us,

$\Rightarrow 2y^2 - 4y = 0$

$\Rightarrow y^2 - 2y = 0$

$\Rightarrow y(y - 2) = 0$

$\Rightarrow y = 0$ or $y = 2$

But $y \neq 0$ because then point D(1, 0) is same as B(1, 0)

Therefore, $y = 2$

When $y = 2$, from equation 1,

$\Rightarrow x^2 + 4 - 2x - 12 = 0$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4) \times (x + 2) = 0$

$\Rightarrow x = 4$ or $x = -2$

So, the possible set of values for x and y are:

$x = 4, y = 2$

$x = -2, y = 2$

But when $x = -2$, then C(-2, 3). Then ABCD does not form a parallelogram.

Therefore, the only solution is $x = 4$ and $y = 2$.

Q. 16. Find the area of ΔABC whose vertices are A(-3, -5), B(5, 2) and C(-9, -3).

Answer : Given: The vertices of the triangle are A(-3, -5), B(5, 2) and C(-9, -3).

Formula: Area of Δ ABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Here,

$x_1 = -3, y_1 = -5$

$x_2 = 5, y_2 = 2$

$x_3 = -9, y_3 = -3$

Putting the values,

Area of Δ ABC $= \frac{1}{2}[-3(2 + 3) + 5(-3 + 5) - 9(-5 - 2)]$

$= \frac{1}{2}[-15 + 10 + 63]$

= 29 square units.

Therefore, the area of Δ ABC is 29 square units.

Q. 17. Show that the points A(-5, 1), B(5, 5) and C(10, 7) are collinear.

Answer : Given: The points are A(-5, 1), B(5, 5) and C(10, 7).

Note: Three points are collinear if the sum of lengths of any sides is equal to the length of the third side.

AB = $\sqrt{(5 + 5)^2 + (5 - 1)^2} = \sqrt{100 + 16}$

= 2√29 units .....(1)

BC = $\sqrt{(10 - 5)^2 + (7 - 5)^2} = \sqrt{25 + 4}$

= √29 units .....(2)

AC = $\sqrt{(10 + 5)^2 + (7 - 1)^2} = \sqrt{225 + 36}$

= 3√29 units .....(3)

From equations 1, 2 and 3, we have

AB + BC = AC

Therefore, the three points are collinear.

Q. 18. Find the value of k for which the points A(-2, 3), B(1, 2) and C(k, 0) are collinear.

Answer : Given: The points are A(-5, 1), B(1, 2) and C(k, 0)

To find: value of k

$AB = \sqrt{(1 + 5)^2 + (2 - 1)^2} = \sqrt{36 + 1}$

$= \sqrt{37}$ units

$BC = \sqrt{(k - 1)^2 + 4}$

$AC = \sqrt{(k + 5)^2 + 1}$

Since the points are collinear, AB + BC = AC

$\Rightarrow \sqrt{37} + \sqrt{(k-1)^2 + 4} = \sqrt{(k+5)^2 + 1}$

Squaring both sides and rearranging,

$\Rightarrow 37 + (k-1)^2 + 4 - (k+5)^2 - 1 = -2\sqrt{37}\sqrt{(k-1)^2 + 4}$

On simplifying,

$\Rightarrow 40 - 2k + 1 - 10k - 25 = -2\sqrt{37}\sqrt{(k-1)^2 + 4}$

$\Rightarrow 16 - 12k = -2\sqrt{37}\sqrt{(k-1)^2 + 4}$

$\Rightarrow 8 - 6k = -\sqrt{37}\sqrt{(k-1)^2 + 4}$

Squaring both sides,

$\Rightarrow 64 - 96k + 36k^2 = 37 \times (k^2 - 2k + 5)$

$\Rightarrow 64 - 96k + 36k^2 = 37k^2 - 74k + 185$

Rearranging,

$\Rightarrow 37k^2 - 74k + 185 = 36k^2 - 96k + 64$

⇒ $k^2 + 22k + 121 = 0$

⇒ $(k + 11)^2 = 0$

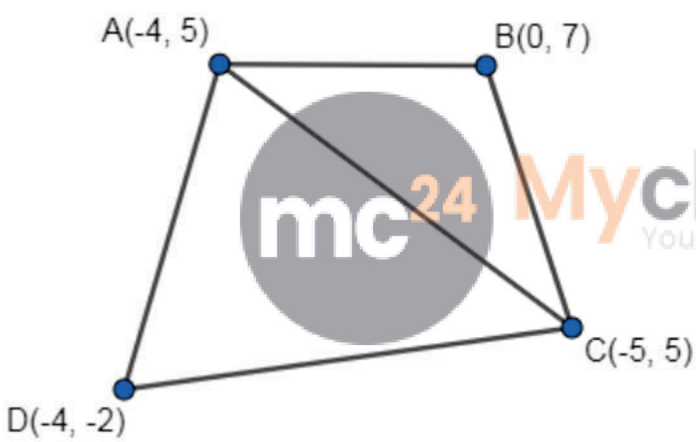
⇒ k = -11

Therefore, the value of k for which the points A, B and C are collinear is –11.

Q. 19. Find the area of the quadrilateral whose vertices are A(-4, 5), B(0, 7), C(5, -5) and D(-4, -2).

Answer : Given: The vertices of the quadrilateral are A(-4, 5), B(0, 7), C(5, -5) and D(-4, -2).

Formula: Area of a triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$



Area of quadrilateral ABCD = Area of Δ ABC + Area of Δ ADC

$= \frac{1}{2}[-4(7 + 5) + 0 + 5(5 - 7)]$

$= \frac{1}{2}[-48 - 10]$

= –29

Taking modulus (∵ area is always positive),

Area of Δ ABC = 29 sq. units …..(1)

Area of Δ ADC $= \frac{1}{2}[-4(-2 + 5) + -4(-5 - 5) + 5(5 + 2)]$

$$= \frac{1}{2}[-12 + 40 + 35]$$

= 31.5 sq. units …..(2)

From 1 and 2,

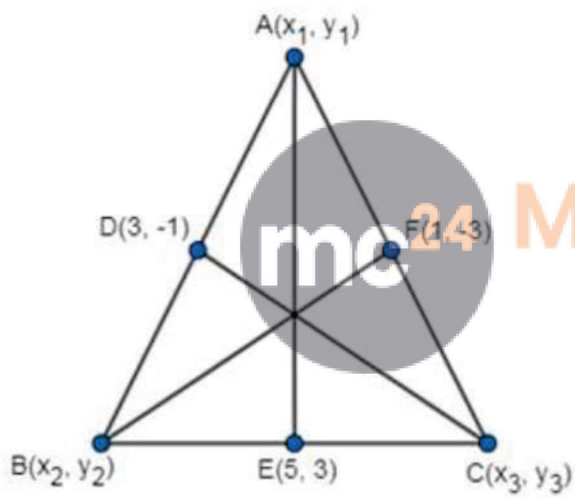
Area of quadrilateral ABCD = 29 + 31.5

= 60.5 square units.

Therefore, the area of quadrilateral ABCD is 60.5 square units.

Q. 20. Find the area of ΔABC, the midpoints of whose sides AB, BC and CA are D(3, -1), E(5, 3) and F(1, -3) respectively.

Answer :



The figure is as shown above.

$x_1 + x_2 = 2 \times 3 = 6$ …..(1)

$x_1 + x_3 = 2 \times 1 = 2$ …..(2)

$x_2 + x_3 = 2 \times 5 = 10$ …..(3)

Equation 1 – Equation 2 gives us

$x_2 - x_3 = 4$ …..(4)

Equation 3 + Equation 4,

$2x_2 = 14 \Rightarrow x_2 = 7$

∴ $x_1 = -1$ and $x_3 = 3$

Similarly,

$y_1 + y_2 = 2 \times -1 = -2$ …..(5)

$y_1 + y_3 = 2 \times -3 = -6$ …..(6)

$y_2 + y_3 = 2 \times 3 = 6$ …..(7)

Equation 5 – Equation 6 gives us

$y_2 - y_3 = 4$ …..(8)

Equation 7 + Equation 8,

$2y_2 = 10 \Rightarrow y_2 = 5$

∴ $y_1 = -7$ and $y_3 = 1$

$$\text{Area of } \Delta \text{ ABC} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-1(5 - 1) + 7(1 + 7) + 3(-7 - 5)]$$

$$= \frac{1}{2}[-4 + 56 - 36]$$

Q. 21. Find the coordinates of the point which divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7.

Answer : Let P(x, y) be the point that divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7

Formula: If $m_1 : m_2$ is the ratio in which the join of two points is divided by another point (x, y), then

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Here, x_1 = -5, x_2 = 4, y_1 = 11 and y_2 = -7

Substituting,

$$x = \frac{2 \times 4 + 7 \times -5}{2 + 7}$$

$$x = \frac{8 - 35}{9}$$

$$x = \frac{-27}{9}$$

⇒ x = -3

$$y = \frac{2 \times -7 + 7 \times 11}{2 + 7}$$

$$y = \frac{-14 + 77}{9}$$

$$y = \frac{63}{9}$$

⇒ y = 8

Therefore, the coordinates of the point which divided the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7 is (-3, 8).

Q. 22. Find the ratio in which the x-axis cuts the join of the points A(4, 5) and B(-10, -2). Also, find the point of intersection.

Answer : Let the point which cuts the join of A(4, 5), and B(-10, -2) in the ratio k : 1 be P(x, 0)

Formula: If k : 1 is the ratio in which the join of two points is divided by another point (x, y), then

$$x = \frac{kx_2 + x_1}{k + 1}$$

$$y = \frac{ky_2 + y_1}{k + 1}$$

Taking for the y co-ordinate,

$$0 = \frac{k \times -2 + 5}{k + 1}$$

⇒ 2k = 5

$$\Rightarrow k = \frac{5}{2}$$

Therefore,

$$x = \frac{\frac{5}{2} \times -10 + 4}{\frac{5}{2} + 1}$$

$$x = \frac{-50 + 8}{5 + 2}$$

$$x = \frac{-42}{7}$$

x = -6

Therefore, the ratio in which x-axis cuts the join of the points A(4, 5) and B(-10, -2) is 5 : 2and the point of intersection is (-6, 0).

Q. 23. In what ratio is the line segment joining the points A(-4, 2) and B(8, 3) divided by the y-axis? Also, find the point of intersection.

Answer : Let the point which cuts the join of A(-4, 2) and B(8, 3) in the ratio k : 1 be P(0, y)

Formula: If k : 1 is the ratio in which the join of two points are divided by another point (x, y), then

$$x = \frac{kx_2 + x_1}{k + 1}$$

$$y = \frac{ky_2 + y_1}{k + 1}$$

Taking for the x co-ordinate,

$$0 = \frac{k \times 8 + (-4)}{k + 1}$$

⇒ 8k = 4

$$\Rightarrow k = \frac{1}{2}$$

Therefore,

$$y = \frac{\frac{1}{2} \times 3 + 2}{\frac{1}{2} + 1}$$

$$y = \frac{3 + 4}{1 + 2}$$

$$y = \frac{7}{3}$$

Therefore, the ratio in which the line segment joining the points A(-4, 2) and B(8, 3) divided by the y-axis is 1 : 2 and the point of intersection is $\left(0, \frac{7}{3}\right)$

Exercise 20B

Q. 1. Find the slope of a line whose inclination is

(i) 30°
(ii) 120°
(iii) 135°
(iv) 90°

Answer : We know that the slope of a given line is given by

Slope = tanθ Where θ = angle of inclination

(i) Given that θ = 30°

$$\text{Slope} = \tan(30°) = \frac{1}{\sqrt{3}}$$

(ii) Given that θ = 120°

Slope = tan(120°) = tan(90°+30°) = -cot(30°) = $-\sqrt{3}$

(iii) Given that θ = 135°

Slope = tan(135°) = tan(90°+45°) = -cot(45°)-1

(iv) Given that θ = 90°

Slope = tan(90°) = ∞

Q. 2. Find the inclination of a line whose slope is

(i) $\sqrt{3}$

(ii) $\frac{1}{\sqrt{3}}$

(iii) 1

(iv) -1

(v) $-\sqrt{3}$

Answer : We know that the slope of a given line is given by

Slope = tanθ Where θ angle of inclination

(i) $\tan\theta = \sqrt{3}$
$\Rightarrow \theta = \tan^{-1}(\sqrt{3})$
$\Rightarrow \theta = 60°$

(ii) $\tan\theta = \frac{1}{\sqrt{3}}$
$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
$\Rightarrow \theta = 30°$

(iii) $\tan\theta = 1$
$\Rightarrow \theta = \tan^{-1}(1)$
$\Rightarrow \theta = 45°$

(iv) $\tan\theta = -1$

$\Rightarrow \theta = \tan^{-1}(-1)$

$\Rightarrow \theta = -45° = 315°$

(v) $\tan\theta = -\sqrt{3}$

$\Rightarrow \theta = \tan^{-1}(-\sqrt{3})$

$\Rightarrow \theta = -60° = 300°$

Q. 3. Find the slope of a line which passes through the points

(i) (0, 0) and (4, -2)
(ii) (0, -3) and (2, 1)
(iii) (2, 5) and (-4, -4)
(iv) (-2, 3) and (4, -6)

Answer :

If a line passing through $(x_1, y_1) \& (x_2, y_2)$ then slope of the line is given by $\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

(i) Given points are (0,0) and (4,-2)

$$\text{slope} = \left(\frac{-2-0}{4-0}\right)$$
$$= \frac{-1}{2}$$

(ii) Given points are (0, -3) and (2, 1)

$$\text{slope} = \left(\frac{1-(-3)}{2-0}\right)$$
$$= 2$$

(iii) Given points are (2, 5) and (-4, -4)

$$\text{slope} = \left(\frac{-4-5}{-4-2}\right)$$
$$= \frac{3}{2}$$
$$= 1.5$$

(iv) Given points are (-2, 3) and (4, -6)

$$\text{slope} = \left(\frac{-6-3}{4+2}\right)$$
$$= \frac{-3}{2}$$
$$= -1.5$$

Q. 4. If the slope of the line joining the points A(x, 2) and B(6, -8) is $\frac{-5}{4}$, find the value of x.

Answer :

If a line passing through $(x_1, y_1)\&(x_2, y_2)$ then slope of the line is given by $\text{slope} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$.

Given points are A(x,2) and B(6,-8), and the slope is

$$\frac{-5}{4}$$

$$\Rightarrow \left(\frac{-8-2}{6-x}\right) = \frac{-5}{4}$$

$$\Rightarrow \left(\frac{-10}{6-x}\right) = \frac{-5}{4} \Rightarrow -40 = -30+5x$$
$\Rightarrow 5x = -10$
$\Rightarrow x = -2$

Q. 5. Show that the line through the points (5, 6) and (2, 3) is parallel to the line through the points (9, -2) and (6, -5)

Answer : We know that for two lines to be parallel, their slope must be the same.

Given points are A(5,6),B(2,3) and C(9,-2),D(6,-5)

$$\text{slope} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$$\Rightarrow \left(\frac{3-6}{2-5}\right) = \left(\frac{-5+2}{6-9}\right)$$

$$\Rightarrow \left(\frac{-3}{-3}\right) = \left(\frac{-3}{-3}\right) \Rightarrow 1 = 1$$

Hence proved.

Q. 6. Find the value of x so that the line through (3, x) and (2, 7) is parallel to the line through (-1, 4) and (0, 6).

Answer : We know that for two lines to be parallel, their slope must be the same. The given points are A(3,x),B(2,7) and C(-1,4),D(0,6)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\Rightarrow \left(\frac{6-4}{0+1}\right) = \left(\frac{7-x}{2-3}\right)$$

$$\Rightarrow \left(\frac{2}{1}\right) = \left(\frac{7-x}{-1}\right) \Rightarrow -2 = 7-x$$

$\Rightarrow x = 9$

Q. 7. Show that the line through the points (-2, 6) and (4, 8) is perpendicular to the line through the points (3, -3) and (5, -9).
Answer : For two lines to be perpendicular, their product of slope must be equal to -1.

Given points are A(-2,6),B(4,8) and C(3,-3),D(5,-9)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of line AB×slope of line CD = -1

$$\Rightarrow \left(\frac{8-6}{4+2}\right) \times \left(\frac{-9+3}{5-3}\right) = -1$$

$$\Rightarrow \left(\frac{2}{6}\right) \times \left(\frac{-6}{2}\right) = -1 \Rightarrow -1 = -1$$

$\Rightarrow$LHS = RHS

Q. 8. If A(2, -5), B(-2, 5), C(x, 3) and D(1, 1) be four points such that AB and CD are perpendicular to each other, find the value of x.

Answer : For two lines to be perpendicular, their product of slope must be equal to -1.

Given points are A(2, -5),B(-2, 5) and C(x, 3),D(1, 1)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

⇒Slope of line AB is equal to

$$\left(\frac{5+5}{-2-2}\right)$$
$$= \left(\frac{10}{-4}\right)$$
$$= \left(\frac{-5}{2}\right)$$
$$= -2.5$$

And the slope of line CD is equal to

$$\left(\frac{1-3}{1-x}\right)$$

$$= \left(\frac{-2}{1-x}\right)$$

Their product must be equal to -1

the slope of line AB×Slope of line CD = -1

$$\Rightarrow -2.5 \times \left(\frac{-2}{1-x}\right) = -1 \Rightarrow 5 = x-1$$

$\Rightarrow x = 6$

Q. 9. Without using Pythagora's theorem, show that the points A(1, 2), B(4, 5) and C(6, 3) are the vertices of a right-angled triangle.

Answer : The **ΔABC** is made up of three lines, AB,BC and CA

For a right angle triangle, two lines must be at 90° so they are perpendicular to each other.

Checking for lines AB and BC

For two lines to be perpendicular, their product of slope must be equal to -1.

Given points A(1, 2), B(4, 5) and C(6, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{5-2}{4-1}\right) = \frac{3}{3} = 1$$

$$\text{Slope of BC} = \left(\frac{3-5}{6-4}\right) = \frac{-2}{2} = -1$$

$$\text{Slope of CA} = \left(\frac{3-2}{6-1}\right) = \frac{1}{5} = 0.2$$

Checking slopes of line AB and BC

1×-1 = -1

So AB is Perpendicular to BC.

So it is a right angle triangle.

Q. 10. Using slopes show that the points A(6, -1), B(5, 0) and C(2, 3) are collinear.

Answer : For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BC = slope of CA

Given points are A(6, -1), B(5, 0) and C(2, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$