Chapter 3

Exercise 3.5

Q1 :Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0$
$3x-9y-2=0$

(ii) $2x+y=5$
$3x+2y=8$

(iii) $3x-5y=20$
$6x-10y=40$

(iv) $x-3y-7=0$
$3x-3y-15=0$

Answer:

(i) x - 3y - 3 =0
3x - 9y - 2 = 0

$$\frac{a_1}{a_2} = \frac{1}{3},$$

$$\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3},$$

$$\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii) 2x+y = 5
3x+2y = 8

$$\frac{a_1}{a_2} = \frac{2}{3},$$

$$\frac{b_1}{b_2} = \frac{1}{2},$$

$$\frac{c_1}{c_2} = \frac{-5}{-8}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-8-(-10)} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

$$\frac{x}{2} = 1,$$
$$\frac{y}{1} = 1$$

∴ $x = 2$, $y = 1$

(iii) 3x-5y = 20
6x-10y = 40

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2},$$

$$\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) X-3y-7 = 0
3x-3y-15 = 0

$$\frac{a_1}{a_2} = \frac{1}{3},$$

$$\frac{b_1}{b_2} = \frac{-3}{-3} = 1$$

$$\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\frac{x}{45-(21)} = \frac{y}{-21-(-15)} = \frac{1}{-3-(-9)}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\frac{x}{24} = \frac{1}{6} \; and \; \frac{y}{-6} = \frac{1}{6}$$

x = 4 and y = -1

∴ x = 4, y = -1

Q.2(i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$
$$(a-b)x + (a+b)y = 3a + b - 2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$
$$(2k-1)x + (k-1)y = 2k + 1$$

Answer:

(i) $2x+3y-7=0$

$(a-b)x + (a+b)y - (3a+b-2) = 0$

$$\frac{a_1}{a_2} = \frac{2}{a-b}$$

$$\frac{b_1}{b_2} = \frac{3}{a+b}$$

$$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$6a + 2b - 4 = 7a - 7b$

$a - 9b = -4$.............. (i)

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$2a+2b = 3a - 3b$

$a - 5b = 0$...............(ii)

Subtracting (i) from (ii), we obtain

$4b = 4$

$b = 1$

Substituting this in equation (ii), we obtain

$a-5\times1 = 0$

$a = 5$

Hence, $a = 5$ and $b = 1$ are the values for which the given equations give infinitely many solutions.

(ii) $3x+y-1 = 0$

$(2k-1)x+(k-1)y-2k-1 = 0$

$$\frac{a_1}{a_2} = \frac{3}{2k-1},$$

$$\frac{b_1}{b_2} = \frac{1}{k-1},$$

$$\frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

3k-3 = 2k-1

k=2

Hence, for k = 2, the given equation has no solution.

Q3 :Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x + 5y = 9$
$3x + 2y = 4$

Answer :

8x+5y = 9........(i)

3x+2y = 4........(ii)

From equation (ii), we get

x = $\frac{4-2y}{3}$ .... (iii)

Substituting this value in equation (*i*), we obtain

$$8\left(\frac{4-2y}{3}\right)+5y=9$$

32-16y+15y = 27

-y = -5

Y = 5............ (iv)

Substituting this value in equation (*ii*), we obtain

3x+10 = 4

x = -2

Hence, x = -2, y =5

Again, by cross-multiplication method, we obtain

8x+5y -9 =0

3x+2y-4 = 0

$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$

$$\frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$

$$\frac{x}{-2} = 1 \; and \; \frac{y}{5} = 1$$

x = -2 and y = 5

Q4 :Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i)A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii)A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii)Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v)The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer :

(i) Let *x* be the fixed charge of the food and *y* be the charge for food per day. According to the given information,

X+20y = 1000 .... (1)

X+26y = 1180 .... (2)

Subtracting equation (1) from equation (2), we obtain

6y = 180

Y = 30

Substituting this value in equation (1), we obtain

x+20×30 = 1000

x = 1000-600 = 400

x = 400

Hence, fixed charge = Rs 400 And charge per day = Rs 30

(ii) Let the fraction be $\frac{x}{y}$.

According to the given information,

$\frac{x-1}{y} = \frac{1}{3} \rightarrow 3x - y = 3$.......(i)

$\frac{x}{y+8} = \frac{1}{4} \rightarrow 4x - y = 8$ ....(ii)

Subtracting equation (i) from equation (ii), we obtain

x = 5 .... (iii)

Putting this value in equation (i), we obtain

$15 - y = 3$

$y = 12$

Hence, the fraction is $\frac{5}{12}$.

(iii) Let the number of right answers and wrong answers be *x* and *y* respectively.

According to the given information,

$3x - y = 40$

$4x - 2y = 50$ ....(i)

$2y - y = 25$ ....(ii)

Subtracting equation (ii) from equation (i),

we obtain $x = 15$ (iii)

Substituting this in equation (ii), we obtain

$30 - y = 25$

$y = 5$

Therefore,

number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of 1st car and 2nd car be *u* km/h and *v* km/h.
Respective speed of both cars while they are travelling in same direction = (u-v) km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = (u+v) km/h

According to the given information,

5(u-v) = 100
= u-v = 20.........(i)
1(u+v) = 100....(ii)

Adding both the equations, we obtain

2u = 120

u = 60 km/h ...... (iii)

Substituting this value in equation (ii), we obtain *v* = 40 km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be *x* unit and *y* unit respectively.

Area = *xy*

According to the question,

(x-5)(y+3) = xy-9

= 3x – 5y – 6 = 0 ....(i)

(x+3)(y+2) = xy + 67

= 2x+3y – 61 = 0 ....(ii)

By cross-multiplication method, we obtain

$$\frac{x}{305—18} = \frac{y}{-12-(-183)} = \frac{1}{9-(\,-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

x = 17, y =9