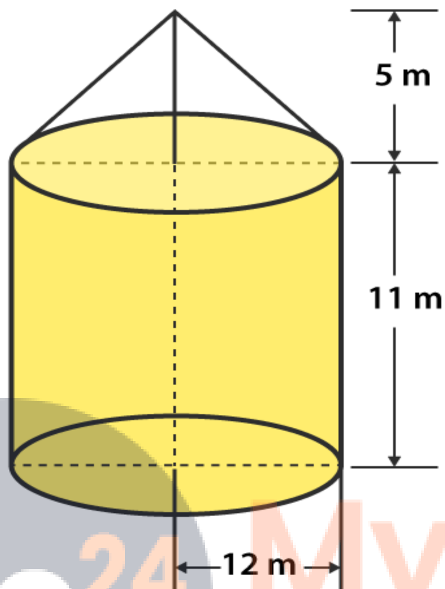
Exercise 16.2

1. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.

Solution:



Given,

The diameter of the cylinder (also the same for cone) = 24 m.

So, its radius (R) = 24/2 = 12 m

The height of the Cylindrical part (H_1) = 11m

So, Height of the cone part (H_2) = 16 – 11 = 5 m

Now,

Vertex of the cone above the ground = 11 + 5 = 16 m

Curved Surface area of the Cone (S_1) = $\pi RL = 22/7 \times 12 \times L$

The slant height (L) is given by,

$L = \sqrt{(R^2 + H_2^2)} = \sqrt{(12^2 + 5^2)} = \sqrt{169}$

L = 13 m

So,

Curved Surface Area of Cone (S_1) = $22/7 \times 12 \times 13$

And,

Curved Surface Area of Cylinder (S_2) = $2\pi RH_1$

$S_2 = 2\pi(12)(11)\ m^2$

Thus, the area of Canvas required for tent

$S = S_1 + S_2 = (22/7 \times 12 \times 13) + (2 \times 22/7 \times 12 \times 11)$

S = 490 + 829.38

$S = 1319.8\ m^2$

$S = 1320\ m^2$

Therefore, the area of canvas required for the tent is $1320\ m^2$

2. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area and the volume of the rocket.
Solution:

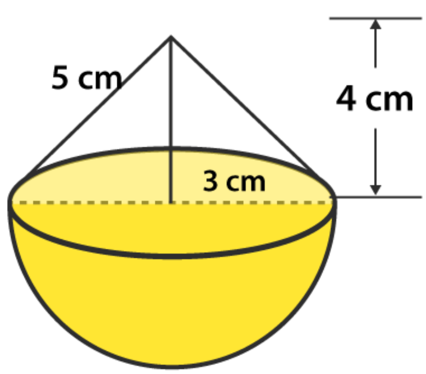
Given,
Radius of the cylindrical portion of the rocket (R) = 2.5 m
Height of the cylindrical portion of the rocket (H) = 21 m
Slant Height of the Conical surface of the rocket (L) = 8 m
Curved Surface Area of the Cone (S_1) = $\pi RL = \pi(2.5)(8) = 20\pi$
And,
Curved Surface Area of the Cone (S_2) = $2\pi RH + \pi R^2$
$S_2 = (2\pi \times 2.5 \times 21) + \pi (2.5)^2$
$S_2 = (\pi \times 105) + (\pi \times 6.25)$
Thus, the total curved surface area S is
$S = S_1 + S_2$
$S = (\pi 20) + (\pi 105) + (\pi 6.25)$
$S = (22/7)(20 + 105 + 6.25) = 22/7 \times 131.25$
$S = 412.5\ m^2$
Therefore, the total Surface Area of the Conical Surface = $412.5\ m^2$
Now, calculating the volume of the rocket
Volume of the conical part of the rocket (V_1) = $1/3 \times 22/7 \times R^2 \times h$
$V_1 = 1/3 \times 22/7 \times (2.5)^2 \times h$
Let, h be the height of the conical portion in the rocket.
We know that,
$L^2 = R^2 + h^2$
$h^2 = L^2 - R^2 = 8^2 - 2.5^2$
h = 7.6 m
Using the value of h, we will get
Volume of the conical part (V_1) = $1/3 \times 22/7 \times 2.5^2 \times 7.6\ m^2 = 49.67\ m^2$
Next,
Volume of the Cylindrical Portion (V_2) = $\pi R^2 h$
$V_2 = 22/7 \times 2.5^2 \times 21 = 412.5\ m^2$
Thus, the total volume of the rocket = $V_1 + V_2$
$V = 412.5 + 49.67 = 462.17\ m^2$
Hence, the total volume of the Rocket is $462.17\ m^2$

3. A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m^2
Solution:

Given,
Height of the tent = 77 dm
Height of a surmounted cone = 44 dm
Height of the Cylindrical Portion = Height of the tent – Height of the surmounted Cone

= 77 - 44
= 33 dm = 3.3 m
And, given diameter of the cylinder (d) = 36 m
So, its radius (r) of the cylinder = 36/2 = 18 m
Let's consider L as the slant height of the cone.
Then, we know that
$L^2 = r^2 + h^2$
$L^2 = 18^2 + 3.3^2$
$L^2 = 324 + 10.89$
$L^2 = 334.89$
L = 18.3 m
Thus, slant height of the cone (L) = 18.3 m
Now, the Curved Surface area of the Cylinder (S_1) = 2πrh
$S_1 = 2\pi\ (184.4)\ m^2$
And, the Curved Surface area of the cone (S_2) = πrL
$S_2 = \pi \times 18 \times 18.3\ m^2$
So, the total curved surface of the tent (S) = $S_1 + S_2$
$S = S_1 + S_2$
$S = (2\pi 18 \times 4.4) + (\pi 18 \times 18.3)$
$S = 1533.08\ m^2$
Hence, the total Curved Surface Area (S) = $1533.08\ m^2$
Next,
The cost of 1 m^2 canvas = Rs 3.50
So, 1533.08 m^2 of canvas will cost = Rs (3.50 x 1533.08)
= Rs 5365.8

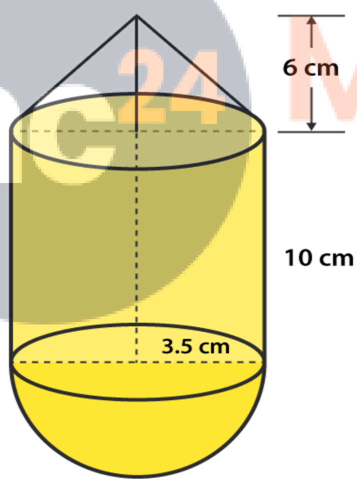
4. A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm, respectively. Determine the surface area of the toy.
Solution:



Given that,
The height of the cone (h) = 4 cm
Diameter of the cone (d) = 6 cm
So, its radius (r) = 3
Let, 'l' be the slant height of cone.

Then, we know that
$l^2 = r^2 + h^2$
$l^2 = 3^2 + 4^2 = 9 + 16 = 25$
l = 5 cm
Hence, the slant height of the cone (l) = 5 cm
So, the curved surface area of the cone (S_1) = πrl
$S_1 = \pi(3)(5)$
$S_1 = 47.1 \text{ cm}^2$
And, the curved surface area of the hemisphere (S_2) = $2\pi r^2$
$S_2 = 2\pi(3)^2$
$S_2 = 56.23 \text{ cm}^2$
So, the total surface area (S) = $S_1 + S_2$
S = 47.1 + 56.23
$S = 103.62 \text{ cm}^2$
Therefore, the curved surface area of the toy is 103.62 cm^2

5. A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the height of the cylindrical and conical portions are 10 cm and 6 cm, respectively. Find the total surface area of the solid. (Use π = 22/7).
Solution:



Given,
Radius of the common base (r) = 3.5 cm
Height of the cylindrical part (h) = 10 cm
Height of the conical part (H) = 6 cm
Let, 'l' be the slant height of the cone
Then, we know that
$l^2 = r^2 + H^2$
$l^2 = 3.5^2 + 6^2 = 12.25 + 36 = 48.25$
l = 6.95 cm
So, the curved surface area of the cone (S_1) = πrl
$S_1 = \pi(3.5)(6.95)$
$S_1 = 76.38 \text{ cm}^2$

And, the curved surface area of the hemisphere (S_2) = $2\pi r^2$
$S_2 = 2\pi(3.5)^2$
$S_2 = 77\ cm^2$
Next, the curved surface area of the cylinder (S_3) = $2\pi rh$
$S_2 = 2\pi(3.5)\ (10)$
$S_2 = 220\ cm^2$
Thus, the total surface area (S) = $S_1 + S_2 + S_3$
$S = 76.\ 38 + 77 + 220 = 373.38\ cm^2$
Therefore, the total surface area of the solid is $373.38\ cm^2$

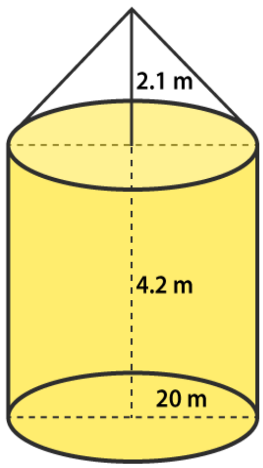
6. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical parts are 5cm and 13 cm, respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.
Solution:

Given,
Height of the Cylindrical portion (H) = 13 cm
Radius of the Cylindrical portion (r) = 5 cm
Height of the whole solid = 30 cm
Then,
The curved surface area of the cylinder (S_1) = $2\pi rh$
$S_1 = 2\pi(5)(13)$
$S_1 = 408.2\ cm^2$
Let, 'L' be the slant height of the cone
And, the curved surface area of the cone (S_2) = πrL
$S_2 = \pi(6)L$
For conical part, we have
h = 30 - 13 - 5 = 12 cm
Then, we know that
$L^2 = r^2 + h^2$
$L^2 = 5^2 + 12^2$
$L^2 = 25 + 144$
$L^2 = 169$
L = 13 m
So,
$S_2 = \pi(5)(13)\ cm^2$
$S_2 = 204.28\ cm^2$
Now, the curved surface area of the hemisphere (S_3) = $2\pi r^2$
$S_3 = 2\pi(5)^2$
$S_3 = 157.14\ cm^2$
Thus, the total curved surface area (S) = $S_1 + S_2 + S_3$
$S = (408.2 + 204.28 + 157.14)$
$S = 769.62\ cm^2$
Therefore, the surface area of the toy is $770\ cm^2$

7. Consider a cylindrical tub having radius as 5 cm and its length 9.8 cm. It is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed in tub. If the radius of the hemisphere is 3.5 cm and the height of the cone outside the hemisphere is 5 cm, find the volume of water left in the tub.
Solution:

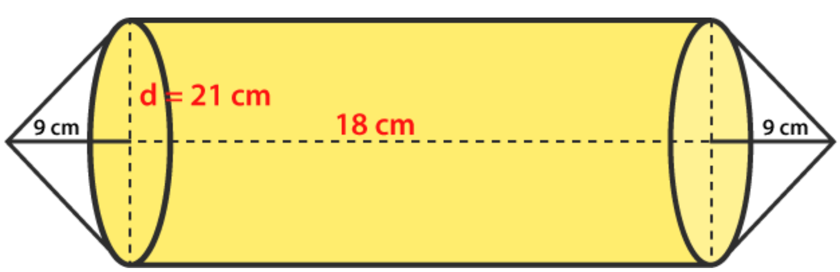
Given,
The radius of the Cylindrical tub (r) = 5 cm
Height of the Cylindrical tub (H) = 9.8 cm
Height of the cone outside the hemisphere (h) = 5 cm
Radius of the hemisphere = 3.5 cm
Now, we know that
The volume of the Cylindrical tub (V_1) = $\pi r^2 H$
$V_1 = \pi(5)^2\ 9.8$
$V_1 = 770\ cm^3$
And, the volume of the Hemisphere (V_2) = $2/3 \times \pi \times r^3$
$V_2 = 2/3 \times 22/7 \times 3.5^3$
$V_2 = 89.79\ cm^3$
And, the volume of the Hemisphere (V_3) = $23 \times \pi \times r \times 2h$
$V_3 = 2/3 \times 22/7 \times 3.5^2 \times 5$
$V_3 = 64.14\ cm^3$
Thus, total volume (V) = Volume of the cone + Volume of the hemisphere
= $V_2 + V_3$
V = $89.79 + 64.14\ cm^3$
= $154\ cm^3$
So, the total volume of the solid = $154\ cm^3$
In order to find the volume of the water left in the tube, we have to subtract the volume of the hemisphere and the cone from the volume of the cylinder.
Hence, the volume of water left in the tube = $V_1 - V_2$
= 770 - 154
= $616\ cm^3$
Therefore, the volume of water left in the tube is $616\ cm^3$.

8. A circus tent has a cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 cm. The heights of the cylindrical and conical portions is 4.2 cm and 2.1 cm respectively. Find the volume of that tent.
Solution:



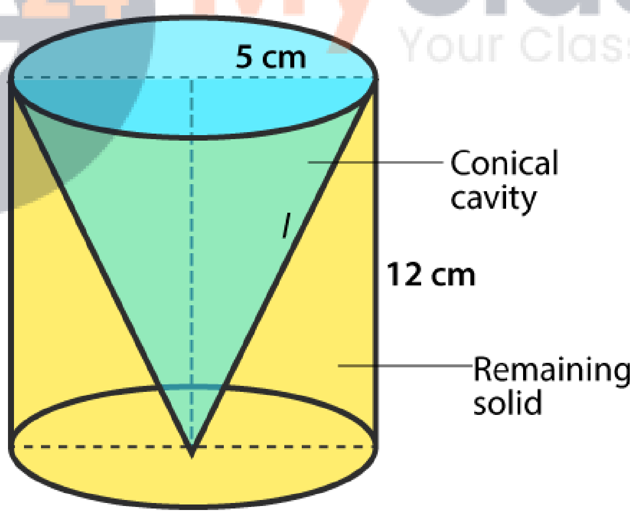
Given,
Radius of the cylindrical portion (R) = 20 m
Height of the cylindrical portion (h_1) = 4.2 m
Height of the conical portion (h_2) = 2.1 m
Now, we know that
Volume of the Cylindrical portion (V_1) = $\pi r^2 h_1$
$V_1 = \pi(20)^2 4.2$
$V_1 = 5280\ m^3$
And, the volume of the conical part (V_2) = $1/3 \times 22/7 \times r^2 \times h^2$
$V_2 = 13 \times 22/7 \times 20^2 \times 2.1$
$V_2 = 880\ m^3$
Thus, the total volume of the tent (V) = volume of the conical portion + volume of the Cylindrical portion
$V = V_1 + V_2$
$V = 6160\ m^3$
Therefore, volume of the tent is 6160 m^3

9. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with the conical ends each of axis length 9 cm. Determine the capacity of the tank.
Solution:



Given,
Base diameter of the cylindrical base of the petrol tank = 21 cm
So, its radius (r) = diameter/2 = 21/2 = 10.5 cm
Height of the Cylindrical portion of the tank (h_1) = 18 cm
Height of the Conical portion of the tank (h_2) = 9 cm
Now, we know that
The volume of the Cylindrical portion (V_1) = $\pi r^2 h_1$
$V_1 = \pi(10.5)^2\ 18$
$V_1 = 6237\ cm^3$
The volume of the Conical portion (V_2) = $1/3 \times 22/7 \times r^2 \times h_2$
$V_2 = 1/3 \times 22/7 \times 10.5^2 \times 9$
$V_2 = 1039.5\ cm^3$
Therefore, the total volume of the tank (V) = 2 x volume of a conical portion + volume of the Cylindrical portion
$V = V_1 + V_2 = 2 \times 1039.5 + 6237$
$V = 8316\ cm^3$
So, the capacity of the tank = V = $8316\ cm^3$

10. A conical hole is drilled in a circular cylinder of height 12 cm and base radius 5 cm. The height and base radius of the cone are also the same. Find the whole surface and volume of the remaining Cylinder.
Solution:



Given,
Height of the circular Cylinder (h_1) = 12 cm
Base radius of the circular Cylinder (r) = 5 cm
Height of the conical hole = Height of the circular cylinder, i.e., $h_1 = h_2 = 12$ cm
And, Base radius of the conical hole = Base radius of the circular Cylinder = 5 cm
Let's consider, L as the slant height of the conical hole.
Then, we know that

$L = \sqrt{r^2 + h^2}$

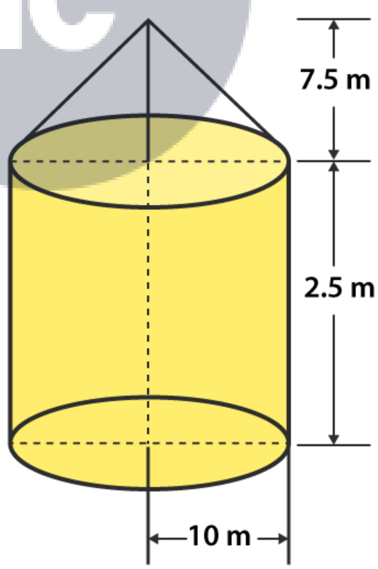
$L = 5^2 + 12^2$

$L = \sqrt{25 + 144}$

$L = 13$ cm

Now,
The total surface area of the remaining portion in the circular cylinder (V_1) = $\pi r^2 + 2\pi rh + \pi rl$
$V_1 = \pi(5)^2 + 2\pi(5)(12) + \pi(5)(13)$
$V_1 = 210\ \pi$ cm^2
And, the volume of the remaining portion of the circular cylinder = Volume of the cylinder – Volume of the conical hole
$V = \pi r^2 h - 1/3 \times 22/7 \times r^2 \times h$
$V = \pi(5)^2(12) - 1/3 \times 22/7 \times 5^2 \times 12$
$V = 200\ \pi$ cm^2

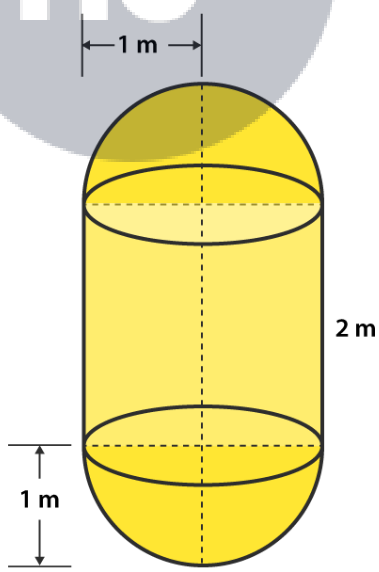
11. A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m. Find the capacity of tent and the cost of the canvas at Rs 100 per square meter.
Solution:



Given,
Diameter of the cylinder = 20 m
So, its radius of the cylinder (R) = 10 m
Height of the cylinder (h_1) = 2.5 m
Radius of the cone = Radius of the cylinder (r) = 15 m
Height of the Cone (h_2) = 7.5 m

Let us consider L as the slant height of the Cone, then we know that
$L^2 = r^2 + h_2^2$
$L^2 = 15^2 + 7.5^2$
$L^2 = 225 + 56.25$
$L^2 = 281.25$
L = 12.5 m
Now,
Volume of the cylinder = $\pi R^2 h_1 = V_1$
$V_1 = \pi(10)^2\ 2.5$
$V_1 = 250\ \pi m^3$
Volume of the Cone = $1/3 \times 22/7 \times r^2 \times h_2^2 = V_2$
$V_2 = 1/3 \times 22/7 \times 10^2 \times 7.5$
$V_2 = 250\pi\ m^3$
So, the total capacity of the tent = volume of the cylinder + volume of the cone = $V_1 + V_2$
$V = 250\ \pi + 250\ \pi$
$V = 500\pi\ m^3$
Hence, the total capacity of the tent is $500\pi\ m^3$
And, the total area of canvas required for the tent is $S = 2\ \pi R h_1 + \pi r L$
$S = 2(\pi)(10)(2.5) + \pi(10)(12.5)$
$S = 550\ m^2$
Therefore, the total cost for canvas is (100) (550) = Rs. 55000

12. A boiler which is in the form of a cylinder 2 m long with hemispherical ends each of 2 m diameter. Find the volume of the boiler.
Solution:



Given,
Diameter of the hemisphere = 2 m
So, the radius of the hemisphere (r) = 1 m
Height of the cylinder (h_1) = 2 m
And, the volume of the Cylinder = $\pi r^2 h_1 = V_1$

$V_1 = \pi(1)^2 2$
$V_1 = 22/7 \times 2 = 44/7\ m^3$
As at each of the ends of the cylinder, hemispheres are attached.
So, totally there are 2 hemispheres.
Then the volume of two hemispheres = $2 \times 2/3 \times 22/7 \times r^3 = V_2$
$V_2 = 2 \times 2/3 \times 22/7 \times 1^3$
$V_2 = 22/7 \times 4/3 = 88/21\ m^3$
Thus,
The volume of the boiler (V) = volume of the cylindrical portion + volume of the two hemispheres
$V = V_1 + V_2$
$V = 44/7 + 88/21$
$V = 220/21\ m^3$
Therefore, the volume of the boiler 220/21 m^3

13. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is 14/3 and the diameter of the hemisphere is 3.5 m. Calculate the volume and the internal surface area of the solid.
Solution:

Given,
Diameter of the hemisphere = 3.5 m
So, the radius of the hemisphere (r) = 1.75 m
Height of the cylinder (h) = 14/3 m
We know that, volume of the Cylinder = $\pi r^2 h_1 = V_1$
$V_1 = \pi(1.75)^2 \text{ x } 14/3\ m^3$
The volume of the hemispherical bottom = $2 \times 2/3 \times 22/7 \times r^3 = V_2$
$V_2 = 2/3 \times 22/7 \times 1.75^3\ m^3$
Therefore,
The total volume of the vessel (V) = volume of the cylinder + volume of the hemisphere
$V = V_1 + V_2$
$V = \pi(1.75)^2 \text{ x } 14/3 + 2/3 \times 22/7 \times 1.75^3$
$V = \pi(1.75)^2 (14/3 + 2/3 \text{ x } 1.75)$
$V = 56.15\ m^2$
Hence, the volume of the vessel = V = 56.15 m^3
Now,
Internal surface area of solid (S) = Surface area of the cylinder + Surface area of the hemisphere
$S = 2\ \pi r\ h_1 + 2\ \pi r^2$
$S = 2\ \pi(1.75)(143) + 2\ \pi(1.75)^2$
$S = 70.51\ m^3$
Therefore, the internal surface area of the solid is 70.51 m^3

14. A solid is composed of a cylinder with hemispherical ends. If the complete length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Rs.10 per dm^2.
Solution:

Given,
Radius of the hemispherical end (r) = 7 cm
Height of the solid = (h + 2r) = 104 cm
⇒ h + 2r = 104
⇒ h = 104 − (2 × 7)
So, h = 90 cm
We know that,
The curved surface area of the cylinder (S_1) = 2 πr h
S_1= 2 π(7)(90)
S_1 = 3960 cm^2
Next,
Curved surface area of the two hemisphere (S_2) = 2 ($2\pi r^2$)
$S_2 = 2 \times 2\pi(7)^2$
S_2 = 616 cm^2
Therefore,
The total curved surface area of the solid (S) = Curved surface area of the cylinder + Curved surface area of the two hemispheres
$S = S_1 + S_2$
S = 3960 + 616
S = 4576 cm^2 = 45.76 dm^2
Given that the cost of polishing the 1 dm^2 surface of the solid is Rs. 10
So, the cost of polishing the 45.76 dm^2 surface of the solid = Rs (10 x 45.76) = Rs. 457.6
Therefore,
The cost of polishing the whole surface of the solid is Rs. 457.60

15. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16cm and height of 42 cm. The total space between the two vessels is filled with cork dust for heat insulation purposes. How many cubic cms of the cork dust will be required?
Solution:
Given,
Depth of the cylindrical vessel = Height of the cylindrical vessel = h = 42 cm (common for both)
Inner diameter of the cylindrical vessel = 14 cm
So, the inner radius of the cylindrical vessel = r_1 = 14/2 = 7 cm
Outer diameter of the cylindrical vessel = 16 cm
So, the outer radius of the cylindrical vessel = r_2 = 16/2 = 8 cm
Now,
The volume of the cylindrical vessel
$V = \pi(r_2^2 - r_1^2) \times h$
$= \pi(8^2 - 7^2) \times 42$
= 22/7 x 15 x 42
V = 1980 cm^3
Therefore, the total space between the two vessels is 1980 cm^3, which is also the amount of cork dust required.

16. A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54 cm and the thickness of the iron sheet used in making roller is 9 cm. Find the mass of the road roller, if 1

cm^3 of the iron has 7.8 gm mass.
Solution:

Given,
Height/length of the cylindrical road roller = h = 1 m = 100 cm
Internal Diameter of the cylindrical road roller = 54 cm
So, the internal radius of the cylindrical road roller = 27 cm = r
Also given, the thickness of the road roller (T) = 9 cm
Let us assume that the outer radii of the cylindrical road roller be R.
T = R - r
9 = R - 27
R = 27 + 9
R = 36 cm
Now,
The volume of the iron sheet (V) = $\pi \times (R^2 - r^2) \times h$
V = $\pi \times (36^2 - 27^2) \times 100$
V = 1780.38 cm^3
Hence, the volume of the iron sheet = 1780.38 cm^3
It's given that, mass of 1 cm^3 of the iron sheet = 7.8 gm
So, the mass of 1780.38 cm^3 of the iron sheet = 1388696.4gm = 1388.7 kg
Therefore, the mass of the road roller is 1388.7 kg

17. A vessel in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13cm. Find the inner surface area of the vessel.
Solution:

Given,
Diameter of the hemisphere = 14 cm
So, the radius of the hemisphere = 7 cm
Total height of the vessel = 13 cm = h + r
Now,
Inner surface area of the vessel = $2 \pi r (h + r)$
= 2 (22/7)(7) (13)
= 572 cm^2
Therefore, the inner surface area of the vessel is 572 cm^2

18. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.
Solution:

Given,
Radius of the conical portion of the toy = 3.5 cm = r
Total height of the toy = 15.5 cm = H
If H is the length of the conical portion
Then,

Length of the cone (h) = H – r = 15.5 – 3.5 = 12 cm
Now, we know that
The curved surface area of the cone (S_1) = πrL, where L is the slant height of the cone.
$L^2 = r^2 + h^2$
$L^2 = 3.5^2 + 12^2$
$L^2 = 12.25 + 144 = 156.25$
L = 12.5
So,
$S_1 = \pi (3.5)(12.5)$
$S_1 = 137.5\ cm^2$
Next, the curved surface area of the hemisphere (S_2) = $2\pi r^2$
$S_2 = 2\pi (3.5)^2$
$S_2 = 77\ cm^2$
Therefore,
The total surface area of the toy (S) = Curved surface area of the cone + curved surface area of the hemisphere
$S = S_1 + S_2$
$S = 137.5 + 77$
$S = 214.5\ cm^2$
Hence, the total surface area of the children's toy is 214.5 cm^2