NCERT Solutions for Class-XII Maths

Chapter-10.3

NCERT Chemistry Class 12

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}.\vec{b} = \sqrt{6}$

1. It is given that,

$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and, $\vec{a} \cdot \vec{b} = \sqrt{6}$

Now, we know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$

$\Rightarrow \cos\theta = \dfrac{\sqrt{6}}{\sqrt{3} \times 2}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = \dfrac{\pi}{4}$

Hence, the angle between the given vectors $\vec{a}$ and $\vec{b}$ is $\dfrac{\pi}{4}$

2. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

2. Given vectors are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$

now, $\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$

$\vec{a}.\vec{b} = (\hat{i}.3\hat{i} - \hat{i}.2\hat{j} + \hat{i}.\hat{k} - 2\hat{j}.3\hat{i} + 2\hat{j}.2\hat{j} - 2\hat{j}.\hat{k} + 3\hat{k}.3\hat{i} - 3\hat{k}.2\hat{j} + 3\hat{k}.\hat{k})$

$\vec{a}.\vec{b} = (3 - 0 + 0 - 0 + 4 - 0 + 0 - 0 + 3) = 10$ $\quad [\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0]$

$\therefore\ \vec{a}.\vec{b}=10$ $\quad [\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1]$ We know, $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 10=\sqrt{14}\times\sqrt{14}\times\cos\theta$

$\Rightarrow 10=14\cos\theta$

$\Rightarrow \cos\theta = 10/14$

$\Rightarrow \theta = \cos^{-1}(5/7)$

3. Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

3. Let $\vec{a}=\hat{i}-\hat{j}$ and $\vec{b}=\hat{i}+\hat{j}$

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}(\vec{a}\vec{b})=\frac{1}{\sqrt{1+1}}\{1.1+(-1)(1)\}=\frac{1}{\sqrt{2}}(1-1)=0$$

Hence, the projection of vector $\vec{a}$ on $\vec{b}$ is 0 .

4. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

4. Let $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ and $\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is given by $\frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$

Now $|\vec{b}|=|7\hat{i}-\hat{j}+8\hat{k}|=\sqrt{7^2+(-1)^2+8^2}=\sqrt{49+1+64}=\sqrt{114}$

$\vec{a}.\vec{b}=(\hat{i}+3\hat{j}+7\hat{k}).(7\hat{i}-\hat{j}+8\hat{k})$

$\Rightarrow \vec{a}.\vec{b}=(\hat{i}.7\hat{i}-\hat{i}.\hat{j}+\hat{i}.8\hat{k}+3\hat{j}.7\hat{i}-3\hat{j}.\hat{j}+3\hat{j}.8\hat{k}+7\hat{k}.7\hat{i}-7\hat{k}.\hat{j}+7\hat{k}.8\hat{k})$

$\Rightarrow \vec{a}.\vec{b}=7-0+0+0-3+0+0-0+56$ $\quad [\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0]$

$\Rightarrow \vec{a}.\vec{b}=7-3+56=60$ $\quad [\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1]$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}=\frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$

$=\frac{1}{\sqrt{114}}\times 60$

$=\frac{60}{\sqrt{114}}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ is $\frac{60}{\sqrt{114}}$.

5. Show that each of the given three vectors is a unit vector: $\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}), \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}), \frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$

Also, show that they are mutually perpendicular to each other.

5. Let $\vec{a}=\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$

$\vec{b}=\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$

$\vec{c}=\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})=\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}$

Now we need to find out the magnitude of $\vec{a}, \vec{b}$ & $\vec{c}$

$$|\vec{a}|=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2}=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=\sqrt{\frac{49}{49}}=1$$

$$|\vec{b}|=\sqrt{\left(\frac{3}{7}\right)^2+\left(-\frac{6}{7}\right)^2+\left(\frac{2}{7}\right)^2}=\sqrt{\frac{9}{49}+\frac{36}{49}+\frac{4}{49}}=\sqrt{\frac{49}{49}}=1$$

$$|\vec{c}|=\sqrt{\left(\frac{6}{7}\right)^2+\left(\frac{2}{7}\right)^2+\left(-\frac{3}{7}\right)^2}=\sqrt{\frac{36}{49}+\frac{4}{49}+\frac{9}{49}}=\sqrt{\frac{49}{49}}=1$$

Since, $|\vec{a}|=1, |\vec{b}|=1$ and $|\vec{c}|=1$

$\therefore$ the three given vectors are unit vectors.

To show that each of three vectors are mutually perpendicular to each other

We have to show $\vec{a}.\vec{b}=0, \vec{b}.\vec{c}=0$ and $\vec{c}.\vec{a}=0$

$$\vec{a}.\vec{b}=\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}\right).\left(\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}\right)$$

$$\vec{a}.\vec{b}=\frac{2}{7}\hat{i}.\frac{3}{7}\hat{i}-\frac{2}{7}\hat{i}.\frac{6}{7}\hat{j}+\frac{2}{7}\hat{i}.\frac{2}{7}\hat{k}+\frac{3}{7}\hat{j}.\frac{3}{7}\hat{i}-\frac{3}{7}\hat{j}.\frac{6}{7}\hat{j}+\frac{3}{7}\hat{j}.\frac{2}{7}\hat{k}+\frac{6}{7}\hat{k}.\frac{3}{7}\hat{i}-\frac{6}{7}\hat{k}.\frac{6}{7}\hat{j}+\frac{6}{7}\hat{k}.\frac{2}{7}\hat{k})$$

$$\vec{a}.\vec{b}=\frac{6}{7}-0+0+0-\frac{18}{7}+0+0-0+\frac{12}{7}$$

$$\vec{a}.\vec{b}=\frac{18}{7}-\frac{18}{7}=0 \qquad \left[\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0\right]$$

$$\left[\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1\right]$$

$$\vec{b}.\vec{c}=\left(\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}\right).\left(\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}\right)$$

$$\vec{b}.\vec{c}=\frac{3}{7}\hat{i}.\frac{6}{7}\hat{i}+\frac{3}{7}\hat{i}.\frac{2}{7}\hat{j}-\frac{3}{7}\hat{i}.\frac{3}{7}\hat{k}-\frac{6}{7}\hat{j}.\frac{6}{7}\hat{i}-\frac{6}{7}\hat{j}.\frac{2}{7}\hat{j}+\frac{6}{7}\hat{j}.\frac{3}{7}\hat{k}+\frac{2}{7}\hat{k}.\frac{6}{7}\hat{i}+\frac{2}{7}\hat{k}.\frac{2}{7}\hat{j}-\frac{2}{7}\hat{k}.\frac{3}{7}\hat{k})$$

$$\vec{a}.\vec{b}=\frac{18}{7}+0-0-0-\frac{12}{7}+0+0+0-\frac{6}{7}$$

$\vec{b}.\vec{c}=\frac{18}{7}-\frac{18}{7}=0$

$\vec{c}.\vec{a}=\left(\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}\right).\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}\right)$

$\vec{c}.\vec{a}=\frac{6}{7}\hat{i}.\frac{2}{7}\hat{i}+\frac{6}{7}\hat{i}.\frac{3}{7}\hat{j}+\frac{6}{7}\hat{i}.\frac{6}{7}\hat{k}+\frac{2}{7}\hat{j}.\frac{2}{7}\hat{i}+\frac{2}{7}\hat{j}.\frac{3}{7}\hat{j}+\frac{2}{7}\hat{j}.\frac{6}{7}\hat{k}-\frac{3}{7}\hat{k}.\frac{2}{7}\hat{i}-\frac{3}{7}\hat{k}.\frac{3}{7}\hat{j}-\frac{3}{7}\hat{k}.\frac{6}{7}\hat{k})$

$\vec{c}.\vec{a}=\frac{12}{7}+0+0+0+\frac{6}{7}+0-0-0-\frac{18}{7}$

$\vec{c}.\vec{a}=\frac{18}{7}-\frac{18}{7}=0$

Since, $\vec{a}.\vec{b}=0, \vec{b}.\vec{c}=0 \text{ and } \vec{c}.\vec{a}=0$

$\therefore$ $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other.

6. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b}),(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$.

6. $(\vec{a}\cdot\vec{b})\cdot(\vec{a}-\vec{b})=8$

$\Rightarrow \vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}=8$

$\Rightarrow |\vec{a}|^2-|\vec{b}|^2=8$

$\Rightarrow (8|\vec{b}|)^2-|\vec{b}|^2=8 \quad \left[|\vec{a}|=8|\vec{b}|\right]$

$\Rightarrow 64|\vec{b}|^2-|\vec{b}|^2=8$

$\Rightarrow 63|\vec{b}|^2=8$

$\Rightarrow |\vec{b}|^2=\frac{8}{63}$

$\Rightarrow |\vec{b}|=\sqrt{\frac{8}{63}}$ [Magnitude of a vector is non-negative]

$\Rightarrow |\vec{b}|=\frac{2\sqrt{2}}{3\sqrt{7}}$

$\Rightarrow |\vec{a}|=8|\vec{b}|=\frac{8\times2\sqrt{2}}{3\sqrt{7}}=\frac{16\sqrt{2}}{3\sqrt{7}}$

7. Evaluate the product $(3\vec{a}-5\vec{b}).(2\vec{a}+7\vec{b})$.

7. To find $\left(3\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$

$=3\vec{a}.2\vec{a}+3\vec{a}.7\vec{b}-5\vec{b}.2\vec{a}-5\vec{b}.7\vec{b}$

$= 6\vec{a}.\vec{a} + 21\vec{a}.\vec{b} - 10\vec{a}.\vec{b} - 35\vec{b}.\vec{b}$ $\left[\text{Since}, \vec{a}.\vec{b} = \vec{b}.\vec{a}\right]$

$= 6|\vec{a}|^2 + 11\vec{a}.\vec{b} - 35|\vec{b}|^2$

8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

8. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.
It is given that $|\vec{a}| = |\vec{b}|, \vec{a}\cdot\vec{b} = \frac{1}{2}$, and $\theta = 60^\circ$.
We know that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60^\circ$ [Using (1)]

$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$ $\Rightarrow |\vec{a}|^2 = 1$

$\Rightarrow |\vec{a}| = |\vec{b}| = 1$

9. Find $|\vec{x}|$, if for a unit vector $\vec{a}, (\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$.
9. Given, vector $\vec{a}$ is a unit vector

$\therefore |\vec{a}| = 1$

$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$

$\Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \vec{a}.\vec{a} = 12$

$\Rightarrow |\vec{x}|^2 + \vec{x}.\vec{a} - \vec{x}.\vec{a} - |\vec{a}|^2 = 12$ $\left[\text{Since}, \vec{x}.\vec{a} = \vec{a}.\vec{x}\right]$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - 1^2 = 12$

$\Rightarrow |\vec{x}|^2 = 12 + 1 = 13$

$\Rightarrow |\vec{x}| = \sqrt{13}$

$\therefore |\vec{x}| = \sqrt{13}$

10. If $\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

10. The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.
Now,

$\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$

If $\left(\vec{a}+\lambda\vec{b}\right)$ is perpendicular to $\vec{c}$, then

$\left(\vec{a}+\lambda\vec{b}\right)\cdot\vec{c}=0$

$\Rightarrow\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right]\cdot\left(3\hat{i}+\hat{j}\right)=0$

$\Rightarrow(2-\lambda)3+(2+2\lambda)1+(3+\lambda)0=0$

$\Rightarrow 6-3\lambda+2+2\lambda=0$

$\Rightarrow -\lambda+8=0$

$\Rightarrow \lambda=8$

Hence, the required value of λ is 8 .

11. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.

11. To show $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ for any two non zero vectors $\vec{a}$ and $\vec{b}$, we need to show dot product of $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ is zero.

$\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right).\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$

$=\left(|\vec{a}|\vec{b}\right).\left(|\vec{a}|\vec{b}\right)-\left(|\vec{a}|\vec{b}\right).\left(|\vec{b}|\vec{a}\right)+\left(|\vec{b}|\vec{a}\right).\left(|\vec{a}|\vec{b}\right)-\left(|\vec{b}|\vec{a}\right).\left(|\vec{b}|\vec{a}\right)$

$=|\vec{a}|^2\,\vec{b}.\vec{b}-|\vec{b}|^2\,\vec{a}.\vec{a}$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{b}|^2|\vec{a}|^2$ $\qquad\left[\text{Since, } \vec{a}.\vec{b}=\vec{b}.\vec{a}\right]$

$=0$

Since, dot product of $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ is zero.

$\therefore$ $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$

12. If $\vec{a}.\vec{a}=0$ and $\vec{a}.\vec{b}=0$, then what can be concluded about the vector $\vec{b}$?

12. It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$.

Now,

$\vec{a}\cdot\vec{a}=0\Rightarrow|\vec{a}|^2=0\Rightarrow|\vec{a}|=0$

$\therefore\vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a}\cdot\vec{b}=0$ can be any vector.

13. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, Find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

13. Given $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c}=0$.

$|\vec{a}|=1, |\vec{b}|=1$ and $|\vec{c}|=1$

Now

$\vec{a}.(\vec{a}+\vec{b}+\vec{c})=\vec{a}.\vec{0}$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$

$\Rightarrow 1+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$ ……………(i)

$\vec{b}.(\vec{a}+\vec{b}+\vec{c})=\vec{b}.\vec{0}$

$\Rightarrow \vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}=0$

$\Rightarrow 1+\vec{b}.\vec{a}+\vec{b}.\vec{c}=0$ ……………(ii)

$\vec{c}.(\vec{a}+\vec{b}+\vec{c})=\vec{c}.\vec{0}$

$\Rightarrow \vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$

$\Rightarrow 1+\vec{c}.\vec{a}+\vec{c}.\vec{b}=0$ ………………..(iii)

Adding (i), (ii) and (iii) we get

$1+\vec{a}.\vec{b}+\vec{a}.\vec{c}+1+\vec{b}.\vec{a}+\vec{b}.\vec{c}+1+\vec{c}.\vec{a}+\vec{c}.\vec{b}=0$

$\Rightarrow 3+\vec{a}.\vec{b}+\vec{c}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}+\vec{b}.\vec{c}=0$ $\left[\text{Since}, \vec{a}.\vec{b}=\vec{b}.\vec{a}\right]$

$\Rightarrow 3+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$ $\left[\text{Since}, \vec{b}.\vec{c}=\vec{c}.\vec{b}\right]$

$\Rightarrow 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-3$ $\left[\text{Since}, \vec{a}.\vec{c}=\vec{c}.\vec{a}\right]$

$\therefore \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-3/2$

14. If either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}.\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

14. Consider $\vec{a}=2\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+3\hat{j}-6\hat{k}$.

Then,

$\vec{a}\cdot\vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$

We now observe that:

$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true.

15. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find $\angle ABC$, [$\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$].

15. Given points are A (1, 2, 3), B (-1, 0, 0) and C (0, 1, 2)

$\angle ABC$ is the angle between vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$

$\overrightarrow{BA} = \{1-(-1)\}\hat{i} + (2-0)\hat{j} + (3-0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{BC} = \{0-(-1)\}\hat{i} + (1-0)\hat{j} + (2-0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{BA}.\overrightarrow{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}).(\hat{i} + \hat{j} + 2\hat{k})$

$\Rightarrow \overrightarrow{BA}.\overrightarrow{BC} = 2\hat{i}.\hat{i} + 2\hat{i}.\hat{j} + 2\hat{i}.2\hat{k} + 2\hat{j}.\hat{i} + 2\hat{j}.\hat{j} + 2\hat{j}.2\hat{k} + 3\hat{k}.\hat{i} + 3\hat{k}.\hat{j} + 3\hat{k}.2\hat{k}$

$\Rightarrow \overrightarrow{BA}.\overrightarrow{BC} = 2 + 0 + 0 + 0 + 2 + 0 + 0 + 0 + 6$ $\quad \left[\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0\right]$

$\Rightarrow \overrightarrow{BA}.\overrightarrow{BC} = 10$ $\quad \left[\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1\right]$

$|\overrightarrow{BA}| = |2\hat{i} + 2\hat{j} + 3\hat{k}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4+4+9} = \sqrt{17}$

$|\overrightarrow{BC}| = |\hat{i} + \hat{j} + 2\hat{k}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$

We know

$\overrightarrow{BA}.\overrightarrow{BC} = |\overrightarrow{BA}||\overrightarrow{BC}|\cos\angle ABC$

$\Rightarrow 10 = \sqrt{17} \times \sqrt{6} \times \cos\angle ABC$

$\Rightarrow \cos\angle ABC = \dfrac{10}{\sqrt{17} \times \sqrt{6}}$

$\Rightarrow \cos\angle ABC = \dfrac{10}{\sqrt{102}}$

$\Rightarrow \angle ABC = \cos^{-1}(10/\sqrt{102})$

16. Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear.

16. The given points are $A(1,2,7), B(2,6,3)$, and $C(3,10,-1)$.

$\therefore \overrightarrow{AB} = (2-1)\hat{i} + (6-2)\hat{j} + (3-7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{BC} = (3-2)\hat{i} + (10-6)\hat{j} + (-1-3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{AC} = (3-1)\hat{i} + (10-2)\hat{j} + (-1-7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$

$\left|\overrightarrow{AB}\right| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1+16+16} = \sqrt{33}$

$\left|\overrightarrow{BC}\right| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1+16+16} = \sqrt{33}$

$\left|\overrightarrow{AC}\right| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4+64+64} = \sqrt{132} = 2\sqrt{33}$

$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Hence, the given points A, B, and C are collinear.

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

17. Let vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A, B and C respectively.

Then, $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

Now vectors $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{AC}$ represent sides of ΔABC

$\overrightarrow{AB} = (1-2)\hat{i} + (-3-(-1))\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{BC} = (3-1)\hat{i} + (-4-(-3))\hat{j} + (-4-(-5))\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{AC} = (3-2)\hat{i} + (-4-(-1))\hat{j} + (-4-1)\hat{k} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$\left|\overrightarrow{AB}\right|^2 = 41$

$\left|\overrightarrow{BC}\right| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$

$\left|\overrightarrow{BC}\right|^2 = 6$

$\left|\overrightarrow{AC}\right| = \sqrt{1^2 + (-3)^2 + (-5)^2} = \sqrt{1+9+25} = \sqrt{35}$

$\left|\overrightarrow{AC}\right|^2 = 35$

Now $|\vec{BC}|^2 + |\vec{AC}|^2 = 35 + 6$

$\Rightarrow |\vec{BC}|^2 + |\vec{AC}|^2 = 41$

And $|\vec{AB}|^2 = 41$

$\therefore |\vec{BC}|^2 + |\vec{AC}|^2 = |\vec{AB}|^2$

From Pythagoras theorem we know

If $|\vec{BC}|^2 + |\vec{AC}|^2 = |\vec{AB}|^2$ then ΔABC is a right-angled triangle

$\therefore \Delta$ ABC is a right-angled triangle.

18. If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if

(A) $\lambda = 1$ (B) $\lambda = -1$

(C) $a = |\lambda|$ (D) $a = 1/|\lambda|$

18. Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$

Now,

$|\lambda\vec{a}| = 1$

$\Rightarrow |\lambda||\vec{a}| = 1 [\lambda \neq 0]$

$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} [|\vec{a}| = a]$

$\Rightarrow a = \frac{1}{|\lambda|}$

Hence, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$ The correct answer is D.