

## Exercise 7(C)

**Solution:**(i) Given,  $a/b = c/d$ 

$$\frac{5a}{7b} = \frac{5c}{7d} \quad (\text{Multiplying each by } 5/7)$$

$$\frac{5a + 7b}{5a - 7b} = \frac{5c + 7d}{5c - 7d} \quad (\text{By componendo and Dividendo})$$

(ii) Given,  $a/b = c/d$ 

$$\frac{9a}{13b} = \frac{9c}{13d} \quad (\text{Multiplying each by } 9/13)$$

$$\frac{9a + 13b}{9a - 13b} = \frac{9c + 13d}{9c - 13d} \quad (\text{By componendo and Dividendo})$$

On cross-multiplication we have,

$$(9a + 13b)(9c - 13d) = (9c + 13d)(9a - 13b)$$

$$\frac{xa}{yb} = \frac{xc}{yd} \quad (\text{Multiplying each by } x/y)$$

$$\frac{xa + yb}{yb} = \frac{xc + yd}{yd} \quad (\text{By componendo})$$

$$\frac{xa + yb}{xc + yd} = \frac{yb}{yd}$$

$$\frac{xa + yb}{xc + yd} = \frac{b}{d}$$

(iii) Given,  $a/b = c/d$ 

- Hence Proved

**Solution:**Given,  $a/b = c/d$

$$\frac{6a}{7b} = \frac{6c}{7d} \quad (\text{Multiplying each by } 6/7)$$

$$\frac{6a + 7b}{7b} = \frac{6c + 7d}{7d} \quad (\text{By componendo})$$

$$\frac{6a + 7b}{6c + 7d} = \frac{7b}{7d} = \frac{b}{d} \quad \dots\dots\dots (1)$$

Also,  $a/b = c/d$

$$\frac{3a}{4b} = \frac{3c}{4d} \quad (\text{Multiplying each by } 3/4)$$

$$\frac{3a - 4b}{4b} = \frac{3c - 4d}{4d} \quad (\text{By dividendo})$$

$$\frac{3a - 4b}{3c - 4d} = \frac{4b}{4d} = \frac{b}{d} \quad \dots\dots\dots (2)$$

From (1) and (2), we have

$$\frac{6a + 7b}{6c + 7d} = \frac{3a - 4b}{3c - 4d}$$

$$(6a + 7b)(3c - 4d) = (3a - 4b)(6c + 7d)$$

- Hence Proved

**1. Given,  $a/b = c/d$ , prove that:  
 $(3a - 5b)/(3a + 5b) = (3c - 5d)/(3c + 5d)$**

**Solution:**

Given,

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{3a}{5b} = \frac{3c}{5d} \quad (\text{Multiplying both by } 3/5)$$

$$\frac{3a + 5b}{3a - 5b} = \frac{3c + 5d}{3c - 5d} \quad (\text{By componendo and Dividendo})$$

$$\frac{3a - 5b}{3a + 5b} = \frac{3c - 5d}{3c + 5d} \quad (\text{By alternendo})$$

**2. If  $\frac{5x + 6y}{5u + 6v} = \frac{5x - 6y}{5u - 6v}$  ;**

**Then prove that  $x : y = u : v$**

**Solution:**

$$\frac{5x + 6y}{5u + 6v} = \frac{5x - 6y}{5u - 6v} \quad (\text{By alternendo})$$

$$\frac{5x + 6y}{5x - 6y} = \frac{5u + 6v}{5u - 6v}$$

$$\frac{5x + 6y + 5x - 6y}{5x + 6y - 5x + 6y} = \frac{5u + 6v + 5u - 6v}{5u + 6v - 5u + 6v}$$

(By componendo and dividendo)

$$10x/12y = 10u/12v$$

Thus,

$$x/y = u/v \Rightarrow x : y = u : v$$

**3. If  $(7a + 8b)(7c - 8d) = (7a - 8b)(7c + 8d)$ ;**

**Prove that  $a : b = c : d$**

**Solution:**

The given can be rewritten as,

$$\frac{7a + 8b}{7a - 8b} = \frac{7c + 8d}{7c - 8d}$$

Applying componendo and dividendo, we have

$$\frac{7a + 8b + 7a - 8b}{7a + 8b - 7a + 8b} = \frac{7c + 8d + 7c - 8d}{7c + 8d - 7c + 8d}$$

$$\frac{14a}{16b} = \frac{14c}{16d}$$

$$\frac{a}{b} = \frac{c}{d}$$

**4. (i) If  $x = 6ab/(a + b)$ , find the value of:**

$$\frac{x + 3a}{x - 3a} + \frac{x + 3b}{x - 3b}$$

**Solution:**

$$\text{Given, } x = 6ab/(a + b)$$

$$\Rightarrow x/3a = 2b/a + b$$

Now, applying componendo and dividendo we have

$$\frac{x + 3a}{x - 3a} = \frac{2b + a + b}{2b - a - b}$$

$$\frac{x + 3a}{x - 3a} = \frac{3b + a}{b - a} \quad \dots (1)$$

$$\text{Again, } x = 6ab/(a + b)$$

$$\Rightarrow x/3b = 2a/a + b$$

Now, applying componendo and dividendo we have

$$\frac{x+3b}{x-3b} = \frac{2a+a+b}{2a-a-b}$$

$$\frac{x+3b}{x-3b} = \frac{3a+b}{a-b} \quad \dots (2)$$

From (1) and (2), we get

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{-3b-a+3a+b}{a-b}$$

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{2a-2b}{a-b} = 2$$

(ii) If  $a = 4\sqrt{6}/(\sqrt{2} + \sqrt{3})$ , find the value of:

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}}$$

**Solution:**

Given,  $a = 4\sqrt{6}/(\sqrt{2} + \sqrt{3})$

$$a/2\sqrt{2} = 2\sqrt{3}/(\sqrt{2} + \sqrt{3})$$

Now, applying componendo and dividendo we have

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{2\sqrt{3} + \sqrt{2} + \sqrt{3}}{2\sqrt{3} - \sqrt{2} - \sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{3\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \quad \dots (1)$$

Again,  $a = 4\sqrt{6}/(\sqrt{2} + \sqrt{3})$

$$a/2\sqrt{3} = 2\sqrt{2}/(\sqrt{2} + \sqrt{3})$$

Now, applying componendo and dividendo we have

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2} + \sqrt{2} + \sqrt{3}}{2\sqrt{2} - \sqrt{2} - \sqrt{3}}$$

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} \quad \dots (2)$$

From (1) and (2), we have

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} + \frac{3\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{2} + \sqrt{3} - 3\sqrt{3} - \sqrt{2}}{\sqrt{2} - \sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2} - 2\sqrt{3}}{\sqrt{2} - \sqrt{3}} = 2$$

5. If  $(a + b + c + d)(a - b - c + d) = (a + b - c - d)(a - b + c - d)$ , prove that  $a : b = c : d$ .

**Solution:**

Rewriting the given, we have

$$\frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}$$

Now, applying componendo and dividendo

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)} = \frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{2(a+b)}{2(c+d)} = \frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d} = \frac{a-b}{c-d}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Applying componendo and dividendo again, we get

$$\frac{a+b+a-b}{a+b-a+b} = \frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b} = \frac{2c}{2d}$$

$$\frac{a}{b} = \frac{c}{d}$$

- Hence Proved



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