Solution 1: **Exercise 23(B)**

Given $A = 60^0$ and $B = 30^0$

(i)

$$\begin{aligned} LHS &= \sin(A+B) \\ &= \sin(60^0 + 30^0) \\ &= \sin 90^0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} RHS &= \sin A \cos B + \cos A \sin B \\ &= \sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0 \\ &= \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} \\ &= 1 \end{aligned}$$

$LHS = RHS$

(ii)

$$\begin{aligned} LHS &= \cos(A+B) \\ &= \cos(60^0 + 30^0) \\ &= \cos 90^0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} RHS &= \cos A \cos B - \sin A \sin B \\ &= \cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0 \\ &= \frac{1}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\frac{1}{2} \\ &= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\ &= 0 \end{aligned}$$

$LHS = RHS$

(iii)

$$\begin{aligned} LHS &= \cos(A-B) \\ &= \cos(60^0 - 30^0) \end{aligned}$$

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$= \cos 30^0$

$= \frac{\sqrt{3}}{2}$

$RHS = \cos A \cos B + \sin A \sin B$

$= \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

$= \frac{1}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

$LHS = RHS$

(iv)

$LHS = \tan(A - B)$

$= \tan(60^0 - 30^0)$

$= \tan 30^0$

$= \frac{1}{\sqrt{3}}$

$RHS = \frac{\tan A - \tan B}{1 + \tan A . \tan B}$

$= \frac{\tan 60^0 - \tan 30^0}{1 + \tan 60^0 . \tan 30^0}$

$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{2}{2\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$LHS = RHS$

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Solution 2:

Given $A = 30^0$

(i)

$$\sin 2A = \sin 2\left(30^0\right) = \sin 60^0 = \frac{\sqrt{3}}{2}$$

$$2\sin A \cos A = 2\sin 30^0 \cos 30^0$$

$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{2}$$

$$\frac{2\tan A}{1+\tan^2 A} = \frac{2\tan 30^0}{1+\tan^2 30^0}$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$$

(ii)

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$\cos 2A = \cos 2(30^0) = \cos 60^0 = \frac{1}{2}$

$\cos^2 A - \sin^2 A = \cos^2 30^0 - \sin^2 30^0$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$

$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \tan^2 30^0}{1 + \tan^2 30^0}$

$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$\therefore \cos 2A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

(iii)

$2\cos^2 A - 1 = 2\cos^2 30^0 - 1$

$= 2\left(\frac{3}{4}\right) - 1$

$= \frac{3}{2} - 1$

$= \frac{1}{2}$

$1 - 2\sin^2 A = 1 - 2\sin^2 30^0$

$= 1 - 2\left(\frac{1}{4}\right)$

$= \frac{1}{2}$

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$1 - 2\sin^2 A = 1 - 2\sin^2 30^0$

$= 1 - 2\left(\frac{1}{4}\right)$

$= \frac{1}{2}$

$\therefore 2\cos^2 A - 1 = 1 - 2\sin^2 A$

(iv)

$\sin 3A = \sin 3(30^0)$

$= \sin 90^0$

$= 1$

$3\sin A - 4\sin^3 A = 3\sin 30^0 - 4\sin^3 30^0$

$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - \frac{1}{2}$

$= 1$

$\therefore \sin 3A = 3\sin A - 4\sin^3 A$

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Solution 3:

Given that $A = B = 45^0$

(i)

$$LHS = \sin(A - B)$$
$$= \sin(45^0 - 45^0)$$
$$= \sin 0^0$$
$$= 0$$
$$RHS = \sin A \cos B - \cos A \sin B$$
$$= \sin 45^0 \cos 45^0 - \cos 45^0 \sin 45^0$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$= 0$$
$$LHS = RHS$$

(ii)

$$LHS = \cos(A + B)$$
$$= \cos(45^0 + 45^0)$$
$$= \cos 90^0$$
$$= 0$$
$$RHS = \cos A \cos B - \sin A \sin B$$
$$= \cos 45^0 \cos 45^0 - \sin 45^0 \sin 45^0$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$= 0$$
$$LHS = RHS$$

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Solution 4:

Given that $A = 30^{\circ}$

(i)

$$LHS = \sin 3A$$
$$= \sin 3(30^{0})$$
$$= \sin 90^{0}$$
$$= 1$$
$$RHS = 4 \sin A \sin (60^{\circ} - A) \sin (60^{\circ} + A)$$
$$= 4 \sin 30^{0} \sin (60^{\circ} - 30^{0}) \sin (60^{\circ} + 30^{0})$$
$$= 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1)$$
$$= 1$$
$$LHS = RHS$$

(ii)

$$LHS = (\sin A - \cos A)^{2}$$
$$= (\sin 30^{0} - \cos 30^{0})^{2}$$
$$= \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^{2}$$
$$= \frac{1}{4} + \frac{3}{4} - \frac{\sqrt{3}}{2}$$
$$= 1 - \frac{\sqrt{3}}{2}$$
$$= \frac{2 - \sqrt{3}}{2}$$

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$$RHS = 1 - \sin 2A$$
$$= 1 - \sin 2(30^0)$$
$$= 1 - \sin 60^0$$
$$= 1 - \frac{\sqrt{3}}{2}$$
$$= \frac{2 - \sqrt{3}}{2}$$
$$LHS = RHS$$

(iii)

$$LHS = \cos 2A$$
$$= \cos 2(30^0)$$
$$= \cos 60^0$$
$$= \frac{1}{2}$$
$$RHS = \cos^4 A - \sin^4 A$$
$$= \cos^4 30^0 - \sin^4 30^0$$
$$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$$
$$= \frac{9}{16} - \frac{1}{16}$$
$$= \frac{1}{2}$$
$$LHS = RHS$$

(iv)

$$LHS = \frac{1 - \cos 2A}{\sin 2A}$$
$$= \frac{1 - \cos 2(30^0)}{\sin 2(30^0)}$$

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$$= \frac{1-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}}$$

$RHS = \tan A$

$$= \tan 30^0$$

$$= \frac{1}{\sqrt{3}}$$

$LHS = RHS$

(v)

$$LHS = \frac{1+\sin 2A+\cos 2A}{\sin A+\cos A}$$

$$= \frac{1+\sin 2(30^0)+\cos 2(30^0)}{\sin 30^0+\cos 30^0}$$

$$= \frac{1+\frac{\sqrt{3}}{2}+\frac{1}{2}}{\frac{1}{2}+\frac{\sqrt{3}}{2}}$$

$$= \frac{3+\sqrt{3}}{\sqrt{3}+1}\left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$$

$$= \frac{3\sqrt{3}-3+3-\sqrt{3}}{2}$$

$$= \frac{2\sqrt{3}}{2}$$

$$= \sqrt{3}$$

$RHS = 2\cos A$

$$= 2\cos\left(30^0\right)$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}$$

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(vi)

$$LHS = 4\cos A\cos\left(60^\circ - A\right).\cos\left(60^\circ + A\right)$$

$$= 4\cos 30^0\cos\left(60^\circ - 30^0\right).\cos\left(60^\circ + 30^0\right)$$

$$= 4\cos 30^0\cos 30^0\cos 90^0$$

$$= 4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)(0)$$

$$= 0$$

$$RHS = \cos 3A$$

$$= \cos 3\left(30^0\right)$$

$$= \cos 90^0$$

$$= 0$$

$$LHS = RHS$$

(vii)

$$LHS = \frac{\cos^3 A - \cos 3A}{\cos A} + \frac{\sin^3 A + \sin 3A}{\sin A}$$

$$= \frac{\cos^3 30^0 - \cos 3\left(30^0\right)}{\cos 30^0} + \frac{\sin^3 30^0 + \sin 3\left(30^0\right)}{\sin 30^0}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)^3 - 0}{\frac{\sqrt{3}}{2}} + \frac{\left(\frac{1}{2}\right)^3 + 1}{\frac{1}{2}}$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{\frac{9}{8}}{\frac{1}{2}}$$

$$= \frac{3}{4} + \frac{9}{4}$$

$$= \frac{12}{4}$$

$$= 3$$

$$= RHS$$

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