Exercise 13(B)

1. Find the mid-point of the line segment joining the points:
(i) (-6, 7) and (3, 5)
(ii) (5, -3) and (-1, 7)
Solution:

(i) Let A (-6, 7) and B (3, 5)
So, the mid-point of AB = (-6+3/2, 7+5/2) = (-3/2, 6)

(ii) Let A (5, -3) and B (-1, 7)
So, the mid-point of AB = (5-1/2, -3+7/2) = (2, 2)

2. Points A and B have co-ordinates (3, 5) and (x, y) respectively. The mid-point of AB is (2, 3). Find the values of x and y.
Solution:

Given, mid-point of AB = (2, 3)
Thus,
(3+x/2, 5+y/2) = (2, 3)
3 + x/2 = 2 and 5 + y/2 = 3
3 + x = 4 and 5 + y = 6
x = 1 and y = 1

3. A (5, 3), B (-1, 1) and C (7, -3) are the vertices of triangle ABC. If L is the mid-point of AB and M is the mid-point of AC, show that LM = ½ BC.
Solution:

It's given that, L is the mid-point of AB and M is the mid-point of AC.
Co-ordinates of L are,

$$\left(\frac{5-1}{2}, \frac{3+1}{2}\right) = (2, 2)$$

Co-ordinates of M are,

$$\left(\frac{5+7}{2}, \frac{3-3}{2}\right) = (6, 0)$$

Using distance formula, we have:

$$BC = \sqrt{(7+1)^2 + (-3-1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$LM = \sqrt{(6-2)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

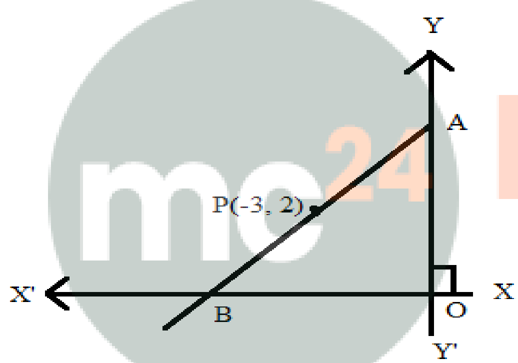
Thus, LM = ½ BC

4. Given M is the mid-point of AB, find the co-ordinates of:
(i) A; if M = (1, 7) and B = (-5, 10)
(ii) B; if A = (3, -1) and M = (-1, 3).
Solution:

(i) Let's assume the co-ordinates of A to be (x, y).
So, (1, 7) = (x-5/2, y+10/2)
1 = x-5/2 and 7 = y+10/2
2 = x – 5 and 14 = y + 10
x = 7 and y = 4
Thus, the co-ordinates of A are (7, 4).

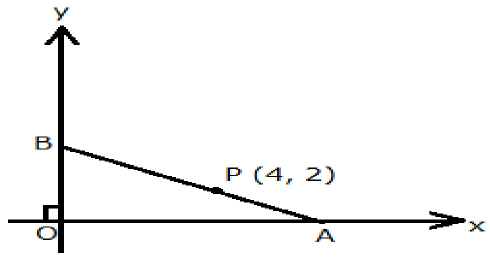
(ii) Let's assume the co-ordinates of B be (x, y).
So, (-1, 3) = (3+x/2, -1+y/2)
-1 = 3+x/2 and 3 = -1+y/2
-2 = 3 + x and 6 = -1 + y
x = -5 and y = 7
Thus, the co-ordinates of B are (-5, 7).

5. P (-3, 2) is the mid-point of line segment AB as shown in the given figure. Find the co-ordinates of points A and B.
Solution:



It's seen that,
Point A lies on y-axis, hence its co-ordinates is taken to be (0, y).
Point B lies on x-axis, hence its co-ordinates is taken to be (x, 0).
Given, P (-3, 2) is the mid-point of line segment AB.
So, by the mid-point section we have
(-3, 2) = (0+x/2, y+0/2)
(-3, 2) = (x/2, y/2)
-3 = x/2 and 2 = y/2
x = -6 and y = 4
Therefore, the co-ordinates of points A and B are (0, 4) and (-6, 0) respectively.

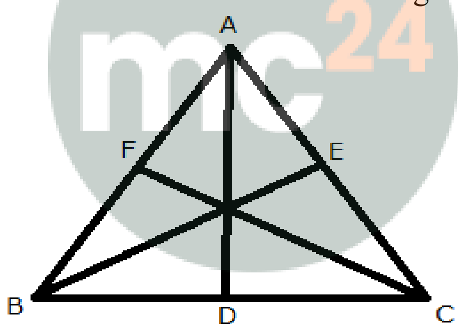
6. In the given figure, P (4, 2) is mid-point of line segment AB. Find the co-ordinates of A and B.
Solution:



Let point A lies on x-axis, hence its co-ordinates can be (x, 0).
And, Point B lies on y-axis, hence its co-ordinates can be (0, y).
Given, P (4, 2) is the mid-point of line segment AB.
So,
(4, 2) = (x+0/2, 0+y/2)
4 = x/2 and 2 = y/2
8 = x and 4 = y
Thus, the co-ordinates of points A and B are (8, 0) and (0, 4) respectively.

7. (-5, 2), (3, -6) and (7, 4) are the vertices of a triangle. Find the length of its median through the vertex (3, -6).
Solution:

Let A (-5, 2), B (3, -6) and C (7, 4) be the vertices of the given triangle.
And let AD be the median through A, BE be the median through B and CF be the median through C.



We know that, median of a triangle bisects the opposite side.
So, the co-ordinates of point F are

$$\left(\frac{-5+3}{2}, \frac{2-6}{2}\right) = \left(\frac{-2}{2}, \frac{-4}{2}\right) = (-1, -2)$$

The co-ordinates of point D are

$$\left(\frac{3+7}{2}, \frac{-6+4}{2}\right) = \left(\frac{10}{2}, \frac{-2}{2}\right) = (5, -1)$$

And, co-ordinates of point E are

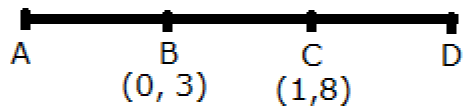
$$\left(\frac{-5+7}{2}, \frac{2+4}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$

The median of the triangle through the vertex B (3, -6) is BE.

Therefore, by distance formula we get

$$BE = \sqrt{(1-3)^2 + (3+6)^2} = \sqrt{4+81} = \sqrt{85} = 9.22$$

8. Given a line ABCD in which AB = BC = CD, B = (0, 3) and C = (1, 8). Find the co-ordinates of A and D.
Solution:



Given, AB = BC = CD
So, B is the mid-point of AC. Let the co-ordinates of point A be (x, y).
(0, 3) = (x+1/2, y+8/2)
0 = (x+1)/2 and 3 = (y+8)/2
0 = x + 1 and 6 = y + 8
x = -1 and y = -2
Hence, the co-ordinates of point A are (-1, -2).

Also given, C is the mid-point of BD. And, let the co-ordinates of point D be (p, q).
(1, 8) = (0+p/2, 3+q/2)
1= 0+p/2 and 8 = 3+q/2
2 = 0 + p and 16 = 3 + q
p = 2 and q = 13
Hence, the co-ordinates of point D are (2, 13).

9. One end of the diameter of a circle is (-2, 5). Find the co-ordinates of the other end of it, if the centre of the circle is (2, -1).
Solution:

We know that,
The centre is the mid-point of any diameter of a circle.
Let assume the required co-ordinates of the other end of mid-point to be (x, y).

$$\therefore (2, -1) = \left(\frac{-2+x}{2}, \frac{5+y}{2}\right)$$

$$\Rightarrow 2 = \frac{-2+x}{2} \text{ and } -1 = \frac{5+y}{2}$$

$$\Rightarrow 4 = -2+x \text{ and } -2 = 5+y$$

$$\Rightarrow 6 = x \text{ and } -7 = y$$

2 = (-2 + x)/2 and -1 = (5 + y)/2
4 = -2 + x and -2 = 5 + y
x = 6 and y = -7
Therefore, the required co-ordinates of the other end of the diameter are (6, -7).