

Exercise 20(A)

The height of a circular cylinder is 20 cm and the radius of its base is 7 cm. Find:

- (i) the volume
 (ii) the total surface area.

Solution:

Given, a circular cylinder whose

Height, $h = 20$ cm and base radius, $r = 7$ cm

$$(i) \text{ Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 7^2 \times 20 \text{ cm}^3 \\ = 3080 \text{ cm}^3$$

$$(ii) \text{ Total surface area of a cylinder} = 2\pi r (h + r) \\ = 2 \times \frac{22}{7} \times 7(20 + 7) \text{ cm}^2 \\ = 2 \times 22 \times 27 \text{ cm}^2 \\ = 1188 \text{ cm}^2$$

2. The inner radius of a pipe is 2.1 cm. How much water can 12 m of this pipe hold?

Solution:

Given,

The inner radius of the pipe = 2.1 cm

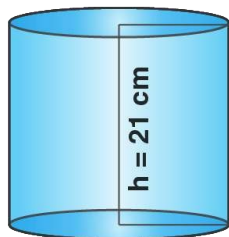
Length of the pipe = 12 m = 1200 cm

$$\text{Volume of the pipe} = \pi r^2 h = \frac{22}{7} \times 2.1^2 \times 1200 \\ = 16632 \text{ cm}^3$$

3. A cylinder of circumference 8 cm and length 21 cm rolls without sliding for $4\frac{1}{2}$ seconds at the rate of 9 complete rounds per second. Find:

- (i) distance travelled by the cylinder in $4\frac{1}{2}$ seconds, and
 (ii) the area covered by the cylinder in $4\frac{1}{2}$ seconds

Solution:



Given,

Base circumference of cylinder (c) = 8 cm

So, the radius = $c/2\pi = (8 \times 7)/(2 \times 22) = 14/11$ cm

Length of the cylinder (h) = 21 cm

(i) If distance covered in one revolution is 8 cm, then distance covered in 9 revolutions = $9 \times 8 = 72$ cm
 or distance covered in 1 second = 72 cm.

Thus, distance covered in $4\frac{1}{2}$ seconds = $72 \times (9/2) = 324$ cm

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$$\begin{aligned} \text{(ii) Curved surface area} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times \frac{14}{11} \times 21 \\ &= 168 \text{ cm}^2 \end{aligned}$$

So, the area covered in one revolution = 168 cm^2

Then,

The area covered in 9 revolutions = $168 \times 9 = 1512 \text{ cm}^2$

Which is also the area covered in 1 second = 1512 cm^2

Therefore, the area covered in $4\frac{1}{2}$ seconds = $1512 \times \frac{9}{2} = 6804 \text{ cm}^2$

4. How many cubic meters of earth must be dug out to make a well 28 m deep and 2.8 m in diameter? Also, find the cost of plastering its inner surface at Rs 4.50 per sq meter.

Solution:

Given,

Radius of the well = $2.8/2 = 1.4 \text{ m}$

Depth of the well = 28 m

$$\begin{aligned} \text{Hence, the volume of earth dug out} &= \pi r^2 h \\ &= \left(\frac{22}{7}\right) \times 1.4 \times 1.4 \times 28 \\ &= \frac{17248}{100} = 172.48 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Area of curved surface} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 1.4 \times 28 \\ &= 246.40 \text{ m}^2 \end{aligned}$$

Now, the cost of plastering at the rate of Rs 4.50 per sq m

= Rs 246.40×4.50

= Rs 1108.80

5. What length of solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of external diameter 20 cm, 0.25 cm thick and 15 cm long?

Solution:

Given,

External diameter of hollow cylinder = 20 cm

So, its radius = 10 cm = R

Thickness = 0.25 cm

Hence, the internal radius = $(10 - 0.25) = 9.75 \text{ cm} = r$

Length of cylinder (h) = 15 cm

Now,

$$\begin{aligned} \text{Volume} &= \pi h (R^2 - r^2) = \pi \times 15 (10^2 - 9.75^2) = 15 \pi (100 - 95.0625) \\ &= 15 \pi \times 4.9375 \text{ cm}^3 \end{aligned}$$

Now,

Diameter of the solid cylinder = 2 cm

so, radius (r) = 1 cm

Let h be the length of the solid cylinder,

$$\text{Volume} = \pi r^2 h = \pi 1^2 h = \pi h \text{ cm}^3$$

Then, according to given condition in the question

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$$\pi h = 15 \pi \times 4.9375$$

$$h = 15 \times 4.9375$$

$$h = 74.0625$$

Thus, the length of solid cylinder = 74.0625 cm

6. A cylinder has a diameter of 20 cm. The area of curved surface is 100 sq cm. Find:

(i) the height of the cylinder correct to one decimal place.

(ii) the volume of the cylinder correct to one decimal place.

Solution:

Given,

The diameter of the cylinder = 20 cm

So, the radius (r) = 10 cm

And the curved surface area = 100 cm²

Height = h cm

(i) Curved surface area = $2\pi rh$

So,

$$2\pi rh = 100 \text{ cm}^2$$

$$2 \times \frac{22}{7} \times 10 \times h = 100$$

$$h = (100 \times 7) / (22 \times 10 \times 2) = 35/22$$

$$h = 1.6 \text{ cm}$$

(ii) Volume of the cylinder = $\pi r^2 h$
 $= \frac{22}{7} \times 10 \times 10 \times 1.6$
 $= 502.9 \text{ cm}^3$

7. A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the weight, in kilogram, of 2 metres of the pipe if 1 cm³ of the metal weights 7.7 g.

Solution:

Given,

Inner radius of the pipe = $r = 5/2 = 2.5 \text{ cm}$

External radius of the pipe = $R = \text{Inner radius of the pipe} + \text{Thickness of the pipe}$
 $= 2.5 \text{ cm} + 0.5 \text{ cm} = 3 \text{ cm}$

Length of the pipe = $h = 2 \text{ m} = 200 \text{ cm}$

Volume of the pipe = External Volume – Internal Volume

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi(R^2 - r^2)h$$

$$= \pi(R + r)(R - r)h$$

$$= \frac{22}{7} (3 - 2.5)(3 + 2.5) \times 200$$

$$= \frac{22}{7} (0.5) (5.5) \times 200 = 1728.6 \text{ cm}^3$$

Since, 1 cm³ of the metal weight 7.7g,

Hence, weight of the pipe = $(1728.6 \times 7.7) \text{ g} = (1728.6 \times 7.7) / 1000 \text{ kg} = 13.31 \text{ kg}$

8. A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm x 14 cm x 10.5 cm. Find the rise in level of the

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water when the solid is submerged.

Solution:

Given,

Diameter of cylindrical container = 42 cm

So, it's radius (r) = 21 cm

Dimensions of rectangular solid = 22cm x 14cm x 10.5cm

Hence,

The volume of solid = $22 \times 14 \times 10.5 \text{ cm}^3$ (i)

Let the height of water = h

So, the volume of water in the container will be = $\pi r^2 h$
 $= 22/7 \times 21 \times 21 \times h \text{ cm}^3$ (ii)

According to the question, from (i) and (ii) we have

$$22/7 \times 21 \times 21 \times h = 22 \times 14 \times 10.5$$

$$22 \times 3 \times 21 \times h = 22 \times 14 \times 10.5$$

$$h = (22 \times 14 \times 10.5) / (22 \times 3 \times 21)$$

$$= 7/3 = 2.33 \text{ cm}$$

Therefore, the water level will be raised to a level of 2.33 cm when the solid is submerged.

9. A cylindrical container with internal radius of its base 10 cm, contains water up to a height of 7 cm. Find the area of wetted surface of the cylinder.

Solution:

Given,

Internal radius of the cylindrical container = 10 cm = r

Height of water = 7 cm = h

So, the surface area of the wetted surface = $2\pi rh + \pi r^2 = \pi r(2h + r) = 22/7 \times 10 \times (2 \times 7 + 10)$
 $= 220/7 \times 24$
 $= 754.29 \text{ cm}^2$

10. Find the total surface area of an open pipe of length 50 cm, external diameter 20 cm and internal diameter 6 cm.

Solution:

Given,

Length of the open pipe = 50 cm

Its external diameter = 20 cm

So, it's external radius (R) = 10 cm

And, its internal diameter = 6 cm

So, it's internal radius (r) = 3 cm

Then,

Surface area of pipe open from both sides = $2\pi Rh + 2\pi rh = 2\pi h(R + r)$
 $= 2 \times 22/7 \times 50 \times (10 + 3)$
 $= 4085.71 \text{ cm}^2$

Area of upper and lower part = $2\pi(R^2 - r^2)$
 $= 2 \times 22/7 \times (10^2 - 3^2)$

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$$\begin{aligned} &= 2 \times 22/7 \times 91 \\ &= 572 \text{ cm}^2 \end{aligned}$$

Hence, the total surface area = $4085.71 + 572 = 4657.71 \text{ cm}^3$

11. The height and the radius of the base of a cylinder are in the ratio 3:1. If its volume is $1029\pi \text{ cm}^3$; find its total surface area.

Solution:

Given,

The ratio between height and radius of a cylinder = 3:1

$$\text{Volume} = 1029\pi \text{ cm}^3 \dots\dots(i)$$

Let the radius of the base = r

Then, it's height will be = $3r$

Hence,

$$\text{Volume} = \pi r^2 h = \pi \times r^2 \times 3r = 3\pi r^3 \dots\dots(ii)$$

Equating (i) and (ii), we get

$$3\pi r^3 = 1029\pi$$

$$r^3 = 1029\pi / 3\pi$$

$$r^3 = 343$$

$$r = 7$$

Thus, radius = 7 cm and height = $3 \times 7 = 21 \text{ cm}$

Now,

$$\begin{aligned} \text{Total surface area} &= 2\pi r(h + r) = 2 \times 22/7 \times 7 \times (21 + 7) \\ &= 2 \times 22/7 \times 7 \times 28 \\ &= 44 \times 28 \\ &= 1232 \text{ cm}^2 \end{aligned}$$

12. The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume.

Solution:

Let the radius of a solid right cylinder (r) = 100 cm

And, let the height of a solid right circular cylinder (h) = 100 cm

$$\begin{aligned} \text{So, the volume (original) of a solid right circular cylinder} &= \pi r^2 h \\ &= \pi \times (100)^2 \times 100 \\ &= 10000 \pi \text{ cm}^3 \end{aligned}$$

Now, the new radius = $r' = 120 \text{ cm}$

New height = $h' = 80 \text{ cm}$

$$\begin{aligned} \text{So, the volume (new) of a solid right circular cylinder} &= \pi r'^2 h' = \pi \times (120)^2 \times 80 \\ &= 1152000 \pi \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Thus, the increase in volume} &= \text{New volume} - \text{Original volume} \\ &= 1152000 \pi \text{ cm}^3 - 1000000 \pi \text{ cm}^3 \\ &= 152000 \pi \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Therefore, percentage change in volume} &= \text{Increase in volume} / \text{Original volume} \times 100\% \\ &= 152000 \pi \text{ cm}^3 / 1000000 \pi \text{ cm}^3 \times 100\% \\ &= 15.2 \% \end{aligned}$$

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13. The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%.

Find the percentage change in its:

(i) volume (ii) curved surface area

Solution:

Let the original dimensions of the solid right cylinder be radius (r) and height (h) in cm.

Then its volume = $\pi r^2 h$ cm³

And, curved surface area = $2\pi r h$

Now, after the changes the new dimensions are:

New radius (r') = $r - 0.2r = 0.8r$ and

New height (h') = $h + 0.1h = 1.1h$

So,

$$\begin{aligned}\text{The new volume} &= \pi r'^2 h' \text{ cm}^3 \\ &= \pi (0.8r)^2 (1.1h) \text{ cm}^3 \\ &= 0.704 \pi r^2 h \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{And, the new curved surface area} &= 2\pi r' h' = 2\pi (0.8r)(1.1h) \\ &= (0.88) 2\pi r h\end{aligned}$$

$$\begin{aligned}\text{(i) Percentage change in its volume} &= \text{Decrease in volume/ original volume} \times 100 \% \\ &= (\text{Original volume} - \text{new volume}) / \text{original volume} \times 100 \% \\ &= (\pi r^2 h - 0.704 \pi r^2 h) / \pi r^2 h \times 100 \\ &= 0.296 \times 100 = 29.6 \%\end{aligned}$$

$$\begin{aligned}\text{(ii) Percentage change in its curved surface area} &= \text{Decrease in CSA/ original CSA} \times 100 \% \\ &= (\text{Original CSA} - \text{new CSA}) / \text{original CSA} \times 100 \% \\ &= (2\pi r h - (0.88) 2\pi r h) / 2\pi r h \times 100 \\ &= 0.12 \times 100 = 12 \%\end{aligned}$$

14. Find the minimum length in cm and correct to nearest whole number of the thin metal sheet required to make a hollow and closed cylindrical box of diameter 20 cm and height 35 cm. Given that the width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of Rs. 56 per m. Find the area of metal sheet required, if 10% of it is wasted in cutting, overlapping, etc.

Solution:

Given,

Height of the cylinder box = $h = 35$ cm

Base radius of the cylinder box = $r = 10$ cm

Width of metal sheet = $1\text{m} = 100$ cm

Area of metal sheet required = total surface area of the box

Length \times width = $2\pi r(r + h)$

Length $\times 100 = 2 \times 22/7 \times 10(10 + 35)$

Length $\times 100 = 22 \times 22/7 \times 10 \times 45$

Length = $(2 \times 22 \times 10 \times 45) / (100 \times 7) = 28.28$ cm = 28 cm (correcting to the nearest whole number)

Thus,

Area of metal sheet = length \times width = $28 \times 100 = 2800$ cm² = 0.28 m²

So, the cost of the sheet at the rate of Rs 56 per m² = Rs $(56 \times 0.28) =$ Rs 15.68

Let the total sheet required be x .

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Then, $x - 10\%$ of $x = 2800 \text{ cm}^2$

$$x - \frac{10}{100} \times x = 2800 \text{ cm}^2$$

$$9x = 2800$$

$$x = 3111 \text{ cm}^2$$

Therefore, a metal sheet of area 3111 cm^2 is required.



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