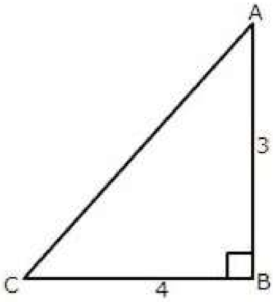
Chapter 22. Trigonometrical Ratios [Sine, Consine, Tangent of an Angle and their Reciprocals]

Exercise 22(A)

Solution 1:

Given angle $ABC = 90^\circ$



$\Rightarrow AC^2 = AB^2 + BC^2$ (AC is hypotenuse)

$\Rightarrow AC^2 = 3^2 + 4^2$

$\therefore AC^2 = 9 + 16 = 25$ and $AC = 5$

(i)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4}{5}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{3}{5}$$

(iii)

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3}{4}$$

(iv)

$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{BC} = \frac{5}{4}$$

(v)

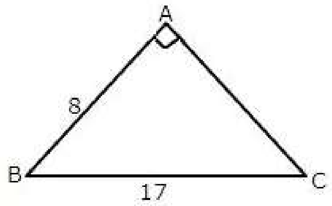
$$\operatorname{cosec} C = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{AB} = \frac{5}{3}$$

(vi)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{3}{4}$$

Solution 2:

Given angle $BAC = 90^\circ$



$\Rightarrow BC^2 = AB^2 + AC^2$ (BC is hypotenuse)

$\Rightarrow 17^2 = 8^2 + AC^2$

$\therefore \quad AC^2 = 289 - 64 = 225$ and $AC = 15$

(i)

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{AC} = \frac{8}{15}$$

(iii)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\sin^2 B + \cos^2 B = \left(\frac{15}{17}\right)^2 + \left(\frac{8}{17}\right)^2$$
$$= \frac{225+64}{289}$$
$$= \frac{289}{289}$$
$$= 1$$

(iv)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

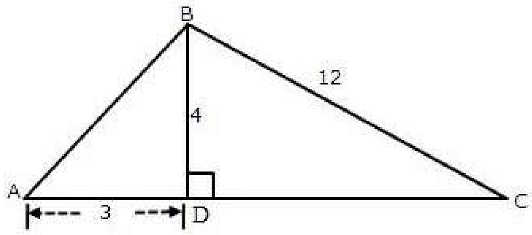
$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\sin B \cdot \cos C + \cos B \cdot \sin C = \frac{15}{17} \cdot \frac{15}{17} + \frac{8}{17} \cdot \frac{8}{17}$$
$$= \frac{225+64}{289}$$
$$= \frac{289}{289}$$
$$= 1$$

Solution 3:

Consider the diagram as



Given angle $ADB = 90^\circ$ and $BDC = 90^\circ$

$\Rightarrow AB^2 = AD^2 + BD^2$ (AB is hypotenuse in ΔABD)

$\Rightarrow AB^2 = 3^2 + 4^2$

$\therefore AB^2 = 9 + 16 = 25$ and $AB = 5$

$\Rightarrow BC^2 = BD^2 + DC^2$ (BC is hypotenuse in ΔBDC)

$\Rightarrow DC^2 = 12^2 - 4^2$

$\therefore DC^2 = 144 - 16 = 128$ and $DC = 8\sqrt{2}$

(i)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{3}{5}$$

(ii)

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AB}{BD} = \frac{5}{4}$$

(iii)

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{BD}{AD} = \frac{4}{3}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{AD} = \frac{5}{3}$$

$$\tan^2 A - \sec^2 A = \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$$

$$= \frac{16}{9} - \frac{25}{9}$$

$$= \frac{-9}{9}$$

$$= -1$$

(iv)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{4}{12} = \frac{1}{3}$$

(v)

$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{BC}{DC} = \frac{12}{8\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

(vi)

$$\cot C = \frac{\text{base}}{\text{perpendicular}} = \frac{DC}{BD} = \frac{8\sqrt{2}}{4} = 2\sqrt{2}$$

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{4}{12} = \frac{1}{3}$$

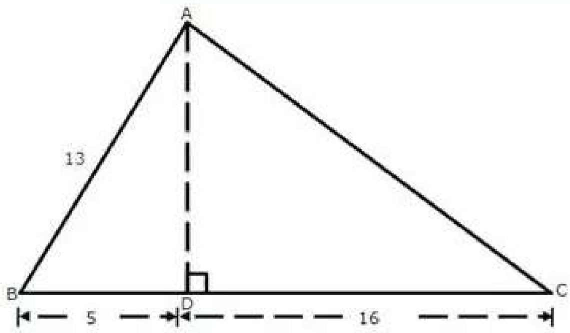
$$\cot^2 C - \frac{1}{\sin^2 C} = \left(2\sqrt{2}\right)^2 - \frac{1}{\left(\frac{1}{3}\right)^2}$$

$$= 8 - 9$$

$$= -1$$

Solution 4:

Given angle $ADB = 90^\circ$ and $ADC = 90^\circ$



$\Rightarrow AB^2 = AD^2 + BD^2$ (AB is hypotenuse in ΔABD)

$\Rightarrow 13^2 = AD^2 + 5^2$

$\therefore \ AD^2 = 169 - 25 = 144$ and $AD = 12$

$\Rightarrow AC^2 = AD^2 + DC^2$ (AC is hypotenuse in ΔADC)

$\Rightarrow AC^2 = 12^2 + 16^2$

$\therefore \ AC^2 = 144 + 256 = 400$ and $AC = 20$

(i)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{12}{13}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$$

(iii)

$$\sec B = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{BD} = \frac{13}{5}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{12}{5}$$

$$\sec^2 B - \tan^2 B = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2$$

$$= \frac{169 - 144}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

(iv)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$$

$$\sin^2 C + \cos^2 C = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

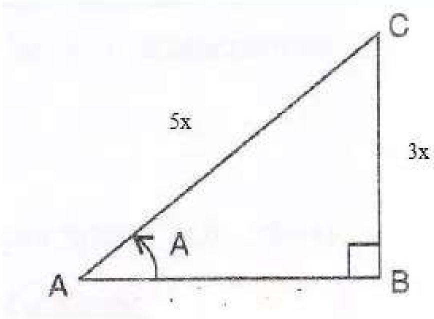
$$= \frac{9 + 16}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

Solution 5:

Consider the diagram below:



$$\sin A = \frac{3}{5}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{BC}{AC} = \frac{3}{5}$$

Therefore if length of $BC = 3x$, length of $AC = 5x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$AB^2 + (3x)^2 = (5x)^2$$

$$AB^2 = 25x^2 - 9x^2 = 16x^2$$

$$\therefore AB = 4x(\text{base})$$

Now

(i)

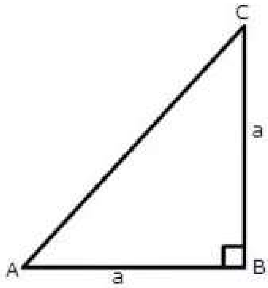
$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3x}{4x} = \frac{3}{4}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

Solution 6:

Given angle $ABC = 90^{\circ}$ in the figure



$\Rightarrow AC^2 = AB^2 + BC^2$ (AC is hypotenuse in ΔABC)

$\Rightarrow AC^2 = a^2 + a^2$

$\therefore \; AC^2 = 2a^2$ and $AC = \sqrt{2}a$

Now

(i) $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

(ii) $\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{\sqrt{2}a}{a} = \sqrt{2}$

(iii) $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

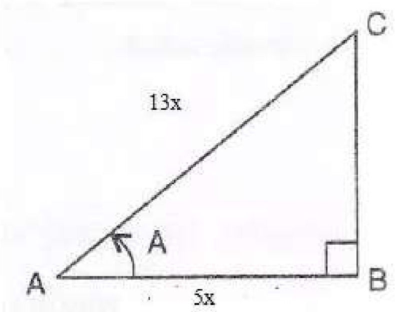
$$\cos^2 A + \sin^2 A = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

Solution 7:

Consider the diagram below:



$$\cos A = \frac{5}{13}$$

$$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{5}{13} \Rightarrow \frac{AB}{AC} = \frac{5}{13}$$

Therefore if length of $AB = 5x$, length of $AC = 13x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 + BC^2 = (13x)^2$$

$$BC^2 = 169x^2 - 25x^2 = 144x^2$$

$$\therefore BC = 12x(\text{perpendicular})$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{12x}{5x} = \frac{12}{5}$$

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{12x}{13x} = \frac{12}{13}$$

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{5x}{12x} = \frac{5}{12}$$

(i)

$$\frac{\sin A - \cot A}{2\tan A}$$

$$= \frac{\frac{12}{13} - \frac{5}{12}}{2\left(\frac{12}{5}\right)}$$

$$= \frac{79}{156} \cdot \frac{5}{24}$$

$$= \frac{395}{3744}$$

(ii)

$$\cot A + \frac{1}{\cos A}$$

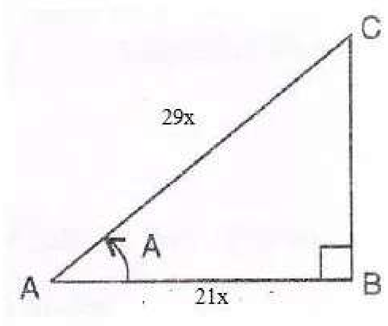
$$= \frac{5}{12} + \frac{1}{\frac{5}{13}}$$

$$= \frac{5}{12} + \frac{13}{5}$$

$$= \frac{181}{60}$$

Solution 8:

Consider the diagram below:



$\sec A = \frac{29}{21}$

$i.e. \frac{\text{hypotenuse}}{\text{base}} = \frac{29}{21} \Rightarrow \frac{AC}{AB} = \frac{29}{21}$

Therefore if length of $AB = 21x$, length of $AC = 29x$

Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(21x)^2 + BC^2 = (29x)^2$

$BC^2 = 841x^2 - 441x^2 = 400x^2$

$\therefore BC = 20x(\text{perpendicular})$

Now

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{20x}{29x} = \frac{20}{29}$

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{20x}{21x} = \frac{20}{21}$

Therefore

$\sin A - \frac{1}{\tan A}$

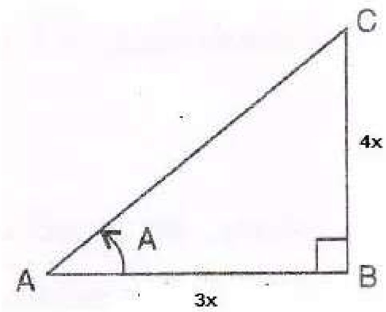
$= \frac{20}{29} - \frac{1}{\frac{20}{21}}$

$= \frac{20}{29} - \frac{21}{20}$

$= -\frac{209}{580}$

Solution 9:

Consider the diagram below:



$\tan A = \frac{4}{3}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3}$

Therefore if length of $AB = 3x$, length of $BC = 4x$

Since

$AB^2 + BC^2 = AC^2 \quad$ [Using Pythagoras Theorem]

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x(\text{hypotenuse})$

Now

$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$

$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3x}{4x} = \frac{3}{4}$

$\text{cosec } A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$

Therefore

$\frac{\text{cosec} A}{\cot A - \sec A}$

$= \frac{\frac{5}{4}}{\frac{3}{4} - \frac{5}{3}}$

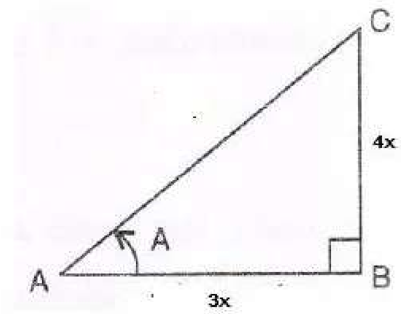
$= \frac{\frac{5}{4}}{\frac{-11}{12}}$

$= -\frac{60}{44}$

$= -\frac{15}{11}$

Solution 10:

Consider the diagram below:



$$4\cot A = 3$$

$$\cot A = \frac{3}{4}$$

$$i.e. \frac{\text{base}}{\text{perpendicular}} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Therefore if length of AB = 3x, length of BC = 4x

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = AC^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore AC = 5x (\text{hypotenuse})$$

(i)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

(ii)

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$$

(iii)

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$$

$$\cot A = \frac{3}{4}$$

$$\operatorname{cosec}^2 A - \cot^2 A$$

$$= \left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2$$

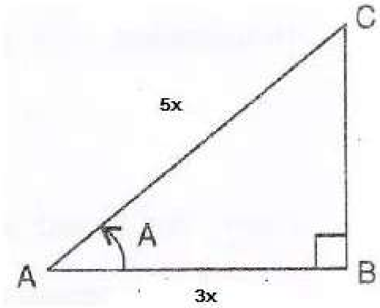
$$= \frac{25 - 9}{16}$$

$$= \frac{16}{16}$$

$$= 1$$

Solution 11:

Consider the diagram below:



$\cos A = 0.6$

$\cos A = \frac{6}{10} = \frac{3}{5}$

$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$

Therefore if length of AB = 3x, length of AC = 5x

Since

$AB^2 + BC^2 = AC^2 \quad$ [Using Pythagoras Theorem]

$(3x)^2 + BC^2 = (5x)^2$

$BC^2 = 25x^2 - 9x^2 = 16x^2$

$\therefore BC = 4x(\text{perpendicular})$

Now all other trigonometric ratios are

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

$\text{cosec } A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$

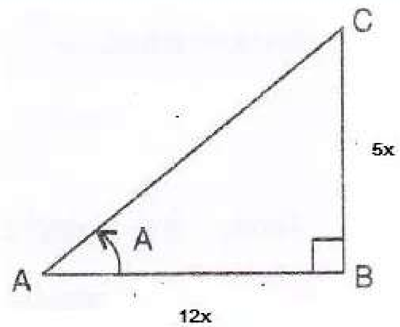
$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{4x}{3x} = \frac{4}{3}$

$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{3x}{4x} = \frac{3}{4}$

Solution 12:

Consider the diagram below:



$\tan A = \frac{5}{12}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{5}{12} \Rightarrow \frac{BC}{AB} = \frac{5}{12}$

Therefore if length of AB = 12x, length of BC = 5x

Since

$AB^2 + BC^2 = AC^2 \quad$ [Using Pythagoras Theorem]

$(12x)^2 + (5x)^2 = AC^2$

$AC^2 = 144x^2 + 25x^2 = 169x^2$

$\therefore AC = 13x(\text{hypotenuse})$

(i)

$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12x}{13x} = \frac{12}{13}$

(ii)

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{5x}{13x} = \frac{5}{13}$

(iii)

$\frac{\cos A + \sin A}{\cos A - \sin A}$

$= \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$

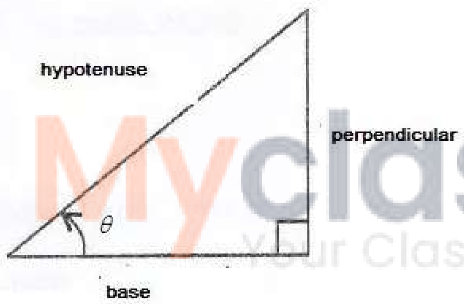
$= \frac{\frac{17}{13}}{\frac{7}{13}}$

$= \frac{17}{7}$

$= 2\frac{3}{7}$

Solution 13:

Consider the diagram below:



$\sin\theta = \frac{p}{q}$

$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{p}{q}$

Therefore if length of perpendicular = px, length of hypotenuse = qx

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$

$\text{base}^2 + (px)^2 = (qx)^2$

$\text{base}^2 = q^2x^2 - p^2x^2 = (q^2 - p^2)x^2$

$\therefore \text{base} = \sqrt{q^2 - p^2}\,x$

Now

$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{q^2 - p^2}}{q}$

Therefore

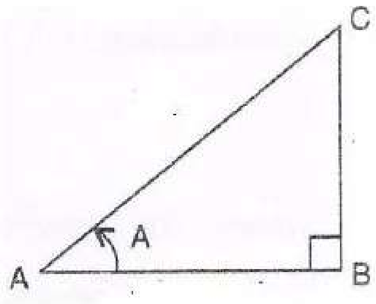
$\cos\theta + \sin\theta$

$= \frac{\sqrt{q^2 - p^2}}{q} + \frac{p}{q}$

$= \frac{p + \sqrt{q^2 - p^2}}{q}$

Solution 14:

Consider the diagram below:



$\cos A = \frac{1}{2}$

$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{1}{2} \Rightarrow \frac{AB}{AC} = \frac{1}{2}$

Therefore if length of AB = x, length of AC = 2x

Since

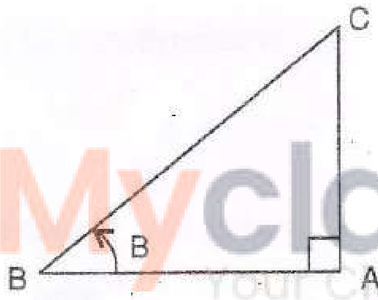
$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(x)^2 + BC^2 = (2x)^2$

$BC^2 = 4x^2 - x^2 = 3x^2$

$\therefore BC = \sqrt{3}x(\text{perpendicular})$

Consider the diagram below:



$\sin B = \frac{1}{\sqrt{2}}$

$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AC}{BC} = \frac{1}{\sqrt{2}}$

Therefore if length of AC = x, length of $BC = \sqrt{2}\,x$

Since

$AB^2 + AC^2 = BC^2 \quad [\text{Using Pythagoras Theorem}]$

$AB^2 + x^2 = (\sqrt{2}x)^2$

$AB^2 = 2x^2 - x^2 = x^2$

$\therefore AB = x(\text{base})$

Now

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$

$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$

Therefore

$\frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\sqrt{3}-1}{1+\sqrt{3}}$

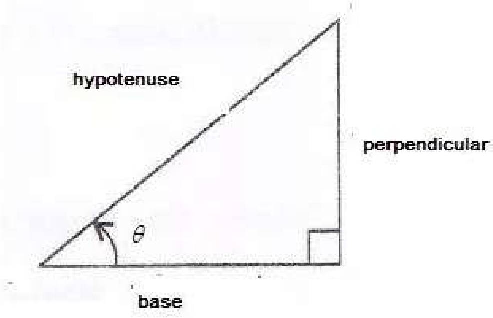
$= \frac{\sqrt{3}-1}{1+\sqrt{3}} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{4-2\sqrt{3}}{2}$

$= 2-\sqrt{3}$

Solution 15:

Consider the diagram below:



$5\cot\theta = 12$

$\cot\theta = \frac{12}{5}$

$i.e. \frac{\text{base}}{\text{perpendicular}} = \frac{12}{5}$

Therefore if length of base = 12x, length of perpendicular = 5x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$

$(12x)^2 + (5x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = 144x^2 + 25x^2 = 169x^2$

$\therefore \text{hypotenuse} = 13x$

Now

$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{13x}{5x} = \frac{13}{5}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{13x}{12x} = \frac{13}{12}$

Therefore

$\text{cosec}\,\theta + \sec\theta$

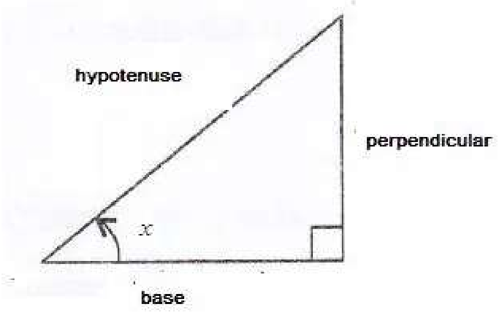
$= \frac{13}{5} + \frac{13}{12}$

$= \frac{221}{60}$

$= 3\frac{41}{60}$

Solution 16:

Consider the diagram below:



$\tan x = 1\frac{1}{3}$

$\tan x = \frac{4}{3}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3}$

Therefore if length of base = 3x, length of perpendicular = 4x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2$ [Using Pythagoras Theorem]

$(3x)^2 + (4x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore$ hypotenuse $= 5x$

Now

$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

$\cos x = \frac{\text{base}}{\text{hypotenuse}} = \frac{3x}{5x} = \frac{3}{5}$

Therefore

$4\sin^2 x - 3\cos^2 x + 2$

$= 4\left(\frac{4}{5}\right)^2 - 3\left(\frac{3}{5}\right)^2 + 2$

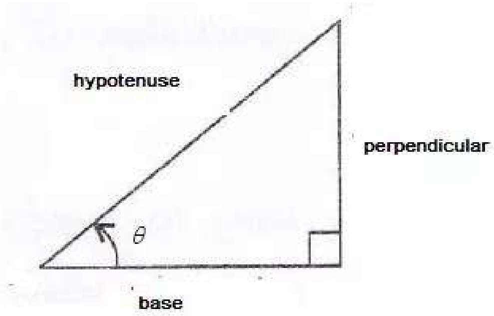
$= \frac{64}{25} - \frac{27}{25} + 2$

$= \frac{87}{25}$

$= 3\frac{12}{25}$

Solution 17:

Consider the diagram below:



$\operatorname{cosec}\theta = \sqrt{5}$

$i.e. \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\sqrt{5}}{1}$

Therefore if length of hypotenuse $= \sqrt{5}x$, length of perpendicular = x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2$ [Using Pythagoras Theorem]

$\text{base}^2 + (x)^2 = (\sqrt{5}x)^2$

$\text{base}^2 = 5x^2 - x^2 = 4x^2$

$\therefore \text{base} = 2x$

Now

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}}$

(i)

$2 - \sin^2\theta - \cos^2\theta$

$= 2 - \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$

$= 2 - \frac{1}{5} - \frac{4}{5}$

$= \frac{5}{5}$

$= 1$

(ii)

$2 + \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$

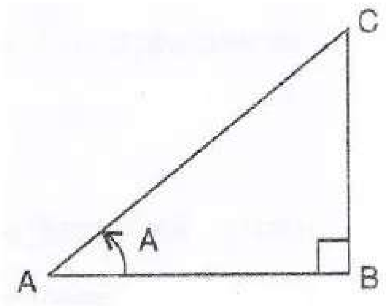
$= 2 + \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2} - \frac{\left(\frac{2}{\sqrt{5}}\right)^2}{\left(\frac{1}{\sqrt{5}}\right)^2}$

$= 2 + 5 - 4$

$= 3$

Solution 18:

Consider the diagram below:



$\sec A = \sqrt{2}$

$i.e. \frac{\text{hypotenuse}}{\text{base}} = \frac{\sqrt{2}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{2}}{1}$

Therefore if length of AB = x, length of $AC = \sqrt{2}x$

Since

$AB^2 + BC^2 = AC^2 \quad$ [Using Pythagoras Theorem]

$(x)^2 + BC^2 = (\sqrt{2}x)^2$

$BC^2 = 2x^2 - x^2 = x^2$

$\therefore BC = x(\text{perpendicular})$

Now

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

Therefore

$\frac{3\cos^2 A + 5\tan^2 A}{4\tan^2 A - \sin^2 A}$

$= \frac{3\left(\frac{1}{\sqrt{2}}\right)^2 + 5(1)^2}{4(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$

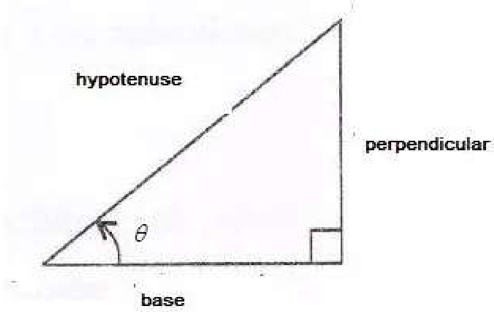
$= \frac{\frac{13}{2}}{\frac{7}{2}}$

$= \frac{13}{7}$

$= 1\frac{6}{7}$

Solution 19:

Consider the diagram below:



$\cot\theta = 1$

$i.e. \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{1}{1}$

Therefore if length of base = x, length of perpendicular = x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2$ [Using Pythagoras Theorem]

$(x)^2 + (x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = x^2 + x^2 = 2x^2$

$\therefore \text{hypotenuse} = \sqrt{2}x$

Now

$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{x}{\sqrt{2}x} = \dfrac{1}{\sqrt{2}}$

$\tan\theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{x}{x} = 1$

Therefore

$5\tan^2\theta + 2\sin^2\theta - 3$

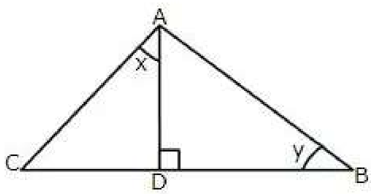
$= 5(1)^2 + 2\left(\dfrac{1}{\sqrt{2}}\right)^2 - 3$

$= 5 + 1 - 3$

$= 3$

Solution 20:

Given angle $DAC = 90^\circ$ and $\angle ADB = 90^\circ$ in the figure



$\Rightarrow AC^2 = AD^2 + DC^2$ (AC is hypotenuse in ΔADC)

$\Rightarrow AD^2 = 26^2 - 10^2$

$\therefore \quad AD^2 = 576$ and $AD = 24$

Again

$\Rightarrow AB^2 = AD^2 + BD^2$ (AB is hypotenuse in ΔABD)

$\Rightarrow AB^2 = 24^2 + 32^2$

$\therefore \quad AB^2 = 1600$ and $AB = 40$

Now

(i)

$\cot x = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{AD}{CD} = \dfrac{24}{10} = 2.4$

(ii)

$\sin y = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AD}{AB} = \dfrac{24}{40} = \dfrac{3}{5}$

$\tan y = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{AD}{BD} = \dfrac{24}{32} = \dfrac{3}{4}$

Therefore

$$\frac{1}{\sin^2 y}-\frac{1}{\tan^2 y}$$
$$=\frac{1}{\left(\frac{3}{5}\right)^2}-\frac{1}{\left(\frac{3}{4}\right)^2}$$
$$=\frac{25}{9}-\frac{16}{9}$$
$$=\frac{9}{9}$$
$$=1$$

(iii)

$$\tan y=\frac{\text{perpendicular}}{\text{base}}=\frac{AD}{BD}=\frac{24}{32}=\frac{3}{4}$$
$$\cos x=\frac{\text{base}}{\text{hypotenuse}}=\frac{AD}{AC}=\frac{24}{26}=\frac{12}{13}$$
$$\cos y=\frac{\text{base}}{\text{hypotenuse}}=\frac{BD}{AB}=\frac{32}{40}=\frac{4}{5}$$

Therefore

$$\frac{6}{\cos x}-\frac{5}{\cos y}+8\tan y$$
$$=\frac{6}{\frac{12}{13}}-\frac{5}{\frac{4}{5}}+8\left(\frac{3}{4}\right)$$
$$=\frac{13}{2}-\frac{25}{4}+6$$
$$=\frac{26-25+24}{4}$$
$$=\frac{25}{4}$$
$$=6\frac{1}{4}$$