

EXERCISE

SHORT ANSWER TYPE:

Evaluate:

$$1. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

Solution:

Given $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

The above equation can be written as

$$= \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{x - 3}$$

On simplifying and applying limits we get

$$\Rightarrow \lim_{x \rightarrow 3} (x + 3) = 6$$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$$

$$2. \lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$$

Solution:

Given $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$

The above equation can be written as

$$= \lim_{x \rightarrow \frac{1}{2}} \frac{(2x)^2 - 1}{2x - 1}$$

Using $a^2 - b^2$ formula and expanding we get

$$= \lim_{x \rightarrow \frac{1}{2}} \frac{(2x - 1)(2x + 1)}{2x - 1}$$



On simplifying and applying the limits we get

$$\Rightarrow \lim_{x \rightarrow \frac{1}{2}} (2x + 1) = 2$$

$$\Rightarrow \lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = 2$$

$$6. \lim_{x \rightarrow a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$$

Solution:

$$\text{Given } \lim_{x \rightarrow a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$$

Now by adding and subtracting 2 to denominator for further simplification we get

$$= \lim_{x \rightarrow a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(x+2) - (a+2)}$$

$$\text{Now we have } \lim_{x \rightarrow a} \frac{x^n - a^n}{x-a} = na^{n-1}$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(x+2) - (a+2)}$$

By using the above formula we get

$$= \frac{5}{2} (a+2)^{\frac{5}{2}-1}$$

Simplifying and applying the limits we get

$$= \frac{5}{2} (a+2)^{\frac{3}{2}}$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a} = \frac{5}{2} (a+2)^{\frac{3}{2}}$$

$$7. \lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1}$$

Now to rationalize the denominator by multiplying the given equation by its rationalizing factor we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1} = \lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1} \times \left(\frac{\sqrt{x} + 1}{\sqrt{x} + 1} \right)$$

On simplifying the above equation we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1} \times \left(\frac{\sqrt{x} + 1}{\sqrt{x} + 1} \right) = \lim_{x \rightarrow 1} \frac{x^4 \sqrt{x} + x^4 - x - \sqrt{x}}{x - 1}$$

Taking \sqrt{x} common we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\sqrt{x}(x^4 - x^4 - x - \sqrt{x})}{x - 1} = \lim_{x \rightarrow 1} \frac{\sqrt{x}(x^4 - 1) + x(x^3 - 1)}{x - 1}$$

Using $a^3 - b^3$ and $a^2 - b^2$ formula and expanding we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\sqrt{x}(x^4 - 1) + x(x^3 - 1)}{x - 1} = \lim_{x \rightarrow 1} \frac{\sqrt{x}(x - 1)(x + 1)(x^2 + 1) + x(x - 1)(x^2 + x + 1)}{x - 1}$$

On simplification we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\sqrt{x}(x - 1)(x + 1)(x^2 + 1) + x(x - 1)(x^2 + x + 1)}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(\sqrt{x}(x + 1)(x^2 + 1) + x(x^2 + x + 1))}{x - 1}$$

$$\Rightarrow \lim_{x \rightarrow 1} \left(\sqrt{x}(x + 1)(x^2 + 1) + x(x^2 + x + 1) \right) = 4 + 3 = 7$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1} = 7$$

$$8. \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x - 2} - \sqrt{x + 2}}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x - 2} - \sqrt{x + 2}}$$

Now to rationalize the denominator by multiplying the given equation by its rationalizing factor we get

$$\Rightarrow \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x-2} - \sqrt{x+2}} = \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x-2} - \sqrt{x+2}} \times \left(\frac{\sqrt{3x-2} + \sqrt{x+2}}{\sqrt{3x-2} + \sqrt{x+2}} \right)$$

Taking $(x - 2)(x + 2)$ as common we get

$$\Rightarrow \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{3x-2} + \sqrt{x+2})}{(3x-2) - (x+2)} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)(\sqrt{3x-2} + \sqrt{x+2})}{2x-4}$$

Taking common and simplifying we get

$$\Rightarrow \lim_{x \rightarrow 2} \frac{(x-2)(x+2)(\sqrt{3x-2} + \sqrt{x+2})}{2x-4} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)(\sqrt{3x-2} + \sqrt{x+2})}{2(x-2)}$$

Now by applying the limit we get

$$\Rightarrow \lim_{x \rightarrow 2} \frac{(x+2)(\sqrt{3x-2} + \sqrt{x+2})}{2} = 8$$

$$\Rightarrow \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x-2} - \sqrt{x+2}} = 8$$

10. $\lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

Solution:

This question can be easily solved using LH rule that is L. Hospital's rule which is given below

$$\Rightarrow \text{if } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Given $\lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

If we apply the limit we will get in determinant form

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2} = \frac{0}{0}$$

So now we have to apply L. Hospital's rule

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2} = \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(x^7 - 2x^5 + 1)}{\frac{d}{dx}(x^3 - 3x^2 + 2)}$$

Now by differentiating we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(x^7 - 2x^5 + 1)}{\frac{d}{dx}(x^3 - 3x^2 + 2)} = \lim_{x \rightarrow 1} \frac{7x^6 - 10x^4}{3x^2 - 6x}$$

Now by applying the limit we get

$$\Rightarrow \lim_{x \rightarrow 1} \frac{7x^6 - 10x^4}{3x^2 - 6x} = \frac{7-10}{3-6} = \frac{-3}{-3} = 1$$

$$11. \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$$

Now to rationalize the denominator by multiplying the given equation by its rationalizing factor we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} \times \left(\frac{\sqrt{1+x^3} + \sqrt{1-x^3}}{\sqrt{1+x^3} + \sqrt{1-x^3}} \right)$$

On simplifying the above equation we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} \times \left(\frac{\sqrt{1+x^3} + \sqrt{1-x^3}}{\sqrt{1+x^3} + \sqrt{1-x^3}} \right) = \lim_{x \rightarrow 0} \frac{(1+x^3) - (1-x^3)}{x^2 (\sqrt{1+x^3} + \sqrt{1-x^3})}$$

The above equation can be written as

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(1+x^3) - (1-x^3)}{x^2 (\sqrt{1+x^3} + \sqrt{1-x^3})} = \lim_{x \rightarrow 0} \frac{2x^3}{x^2 (\sqrt{1+x^3} + \sqrt{1-x^3})} = \lim_{x \rightarrow 0} \frac{2x}{(\sqrt{1+x^3} + \sqrt{1-x^3})}$$

Now by applying the limit we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2x}{(\sqrt{1+x^3} + \sqrt{1-x^3})} = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} = 0$$

$$14. \text{ Find 'n', if } \lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80, n \in \mathbb{N}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80$$

$$\text{We know that } \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

By using this formula we get

$$\Rightarrow n2^{n-1} = 80 = 5 \times 2^4 = 5 \times 2^{5-1}$$

$$\Rightarrow n = 5$$

$$16. \lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 4x}$$

Solution:

Given $\lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 4x}$

Multiply and divide both numerator and denominator by $4x^2/16x^2$ then we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 4x} = \lim_{x \rightarrow 0} \frac{(\sin^2 2x)/4x^2}{(\sin^2 4x)/16x^2} \times \frac{4x^2}{16x^2}$$

On simplifying

$$\Rightarrow \lim_{x \rightarrow 0} \left[\frac{\left(\frac{\sin 2x}{2x}\right)^2}{\left(\frac{\sin 4x}{4x}\right)^2} \right] \times \frac{4}{16}$$

Now as $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\Rightarrow \lim_{x \rightarrow 0} \left[\frac{\left(\frac{\sin 2x}{2x}\right)^2}{\left(\frac{\sin 4x}{4x}\right)^2} \right] \times \frac{4}{16} = \frac{4}{16}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 4x} = \frac{4}{16}$$

$$= \frac{1}{4}$$

17. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$

Solution:

Given $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$

Now by substituting the formula $\cos 2x = 1 - 2 \sin^2 x$ we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{1 - (1 - 2 \sin^2 x)}{x^2}$$

The above equation can be written as

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - (1 - 2 \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2}$$

By applying the limits we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = 2$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2$$

$$18. \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$$

We know that $\sin 2x = 2 \sin x \cos x$, using this formula we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x - 2 \sin x \cos x}{x^3}$$

Again by taking $2 \sin x$ common we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin x - 2 \sin x \cos x}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x (1 - \cos x)}{x^3}$$

Now we have $\cos x = 1 - 2 \sin^2(x/2)$

Using this identity above equation can be written as

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin x (1 - \cos x)}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x (1 - (1 - 2 \sin^2(\frac{x}{2})))}{x^3}$$

On simplifying we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin x (1 - (1 - 2 \sin^2(\frac{x}{2})))}{x^3} = \lim_{x \rightarrow 0} \frac{4 \sin x \sin^2(\frac{x}{2})}{x^3}$$

By splitting the above equation we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{4 \sin x \sin^2(\frac{x}{2})}{x^3} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{\sin^2(\frac{x}{2})}{(\frac{x}{2})^2}$$

The above equation becomes

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{\sin^2(\frac{x}{2})}{(\frac{x}{2})^2} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \left[\lim_{x \rightarrow 0} \frac{\sin(\frac{x}{2})}{(\frac{x}{2})} \right]^2$$

$$\text{Now as } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

By applying the limit we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x}{x} \left[\lim_{x \rightarrow 0} \frac{\sin(\frac{x}{2})}{(\frac{x}{2})} \right]^2 = 1 \cdot 1^2 = 1$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} = 1$$

$$19. \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$$

Here $\cos mx$ can be written as

$$\Rightarrow \cos mx = 1 - 2 \sin^2 \frac{mx}{2}$$

And similarly

$$\Rightarrow \cos nx = 1 - 2 \sin^2 \frac{nx}{2}$$

Using these two identities in given equation we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \lim_{x \rightarrow 0} \frac{1 - (1 - 2 \sin^2 \frac{mx}{2})}{1 - (1 - 2 \sin^2 \frac{nx}{2})}$$

On simplification we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - (1 - 2 \sin^2 \frac{mx}{2})}{1 - (1 - 2 \sin^2 \frac{nx}{2})} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{mx}{2}}{2 \sin^2 \frac{nx}{2}} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{mx}{2}}{\sin^2 \frac{nx}{2}}$$

Again by using trigonometric identity the above equation can be written as

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin^2 \frac{mx}{2}}{\sin^2 \frac{nx}{2}} = \lim_{x \rightarrow 0} \frac{\left(\frac{m}{2}\right)^2 \left[\frac{\sin^2 \frac{mx}{2}}{\left(\frac{m}{2}\right)^2}\right]}{\left(\frac{n}{2}\right)^2 \left[\frac{\sin^2 \frac{nx}{2}}{\left(\frac{n}{2}\right)^2}\right]}$$

Taking common and simplifying we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\left(\frac{m}{2}\right)^2 \left[\frac{\sin^2 \frac{mx}{2}}{\left(\frac{m}{2}\right)^2}\right]}{\left(\frac{n}{2}\right)^2 \left[\frac{\sin^2 \frac{nx}{2}}{\left(\frac{n}{2}\right)^2}\right]} = \frac{m^2}{n^2} \frac{\lim_{x \rightarrow 0} \left[\frac{\sin^2 \frac{mx}{2}}{\left(\frac{m}{2}\right)^2}\right]}{\lim_{x \rightarrow 0} \left[\frac{\sin^2 \frac{nx}{2}}{\left(\frac{n}{2}\right)^2}\right]}$$

$$\text{Now as } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\Rightarrow \frac{m^2}{n^2} \frac{\lim_{x \rightarrow 0} \left[\frac{\sin^2 \frac{mx}{2}}{\left(\frac{m}{2}\right)^2}\right]}{\lim_{x \rightarrow 0} \left[\frac{\sin^2 \frac{nx}{2}}{\left(\frac{n}{2}\right)^2}\right]} = \frac{m^2}{n^2} \frac{1}{1} = \frac{m^2}{n^2}$$

By applying the limits we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \frac{m^2}{n^2}$$

$$20. \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)}$$

Solution:

Given $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)}$

Now by using the formula

$$\cos 6x = 1 - 2 \sin^2 3x$$

Then the above equation becomes,

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - (1 - 2 \sin^2 3x)}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)}$$

Again using $\sin 3x$ formula the above equation can be written as

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - (1 - 2 \sin^2 3x)}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{2} |\sin 3x|}{\sqrt{2} \left(\frac{\pi}{3} - x \right)}$$

Now we have

$$\sin 3x = \sin(\pi - 3x) = \sin 3 \left(\frac{\pi}{3} - x \right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{2} |\sin 3x|}{\sqrt{2} \left(\frac{\pi}{3} - x \right)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin 3 \left(\frac{\pi}{3} - x \right)}{\left(\frac{\pi}{3} - x \right)}$$

On simplifying we get

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin 3 \left(\frac{\pi}{3} - x \right)}{\left(\frac{\pi}{3} - x \right)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{3 \sin 3 \left(\frac{\pi}{3} - x \right)}{3 \left(\frac{\pi}{3} - x \right)} = 3 \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin(\pi - 3x)}{(\pi - 3x)}$$

Now as $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

By substituting the limit we get

$$\Rightarrow 3 \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin(\pi - 3x)}{(\pi - 3x)} = 3 \cdot 1 = 3$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)} = 3$$

$$21. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

Solution:

$$\text{Given } \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

We have

$$\sin x - \cos x = \sqrt{2} \left(\frac{\sin x}{\sqrt{2}} - \frac{\cos x}{\sqrt{2}} \right) = \sqrt{2} \left(\sin x \cos \left(\frac{\pi}{4} \right) - \cos x \sin \left(\frac{\pi}{4} \right) \right)$$

By using this formula in given equation we get

$$\Rightarrow \sqrt{2} \left(\sin x \cos \left(\frac{\pi}{4} \right) - \cos x \sin \left(\frac{\pi}{4} \right) \right) = \sqrt{2} \sin \left(x - \frac{\pi}{4} \right)$$

On simplification we get

$$\Rightarrow \sin x - \cos x = \sqrt{2} \sin \left(x - \frac{\pi}{4} \right)$$

Now substituting these values in given equation we get

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin \left(x - \frac{\pi}{4} \right)}{x - \frac{\pi}{4}}$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin \left(x - \frac{\pi}{4} \right)}{x - \frac{\pi}{4}} = \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin \left(x - \frac{\pi}{4} \right)}{x - \frac{\pi}{4}}$$

$$\text{Now as } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

And by substituting the limits we get

$$\Rightarrow \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin \left(x - \frac{\pi}{4} \right)}{x - \frac{\pi}{4}} = \sqrt{2} \cdot 1 = \sqrt{2}$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}} = \sqrt{2}$$

$$22. \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$$

Solution:

$$\text{Given } \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$$

$$\text{Consider } \sqrt{3} \sin x - \cos x = 2 \left(\frac{\sqrt{3} \sin x}{2} - \frac{\cos x}{2} \right) = 2 \left(\sin x \cos \left(\frac{\pi}{6} \right) - \cos x \sin \left(\frac{\pi}{6} \right) \right)$$

On simplification the above equation can be written as

$$\Rightarrow 2 \left(\sin x \cos \left(\frac{\pi}{6} \right) - \cos x \sin \left(\frac{\pi}{6} \right) \right) = 2 \sin \left(x - \frac{\pi}{6} \right)$$

$$\Rightarrow \sqrt{3} \sin x - \cos x = 2 \sin \left(x - \frac{\pi}{6} \right)$$

Now by substituting these values in given equation we get

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin \left(x - \frac{\pi}{6} \right)}{x - \frac{\pi}{6}}$$

Taking constant term 2 outside

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin \left(x - \frac{\pi}{6} \right)}{x - \frac{\pi}{6}} = 2 \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin \left(x - \frac{\pi}{6} \right)}{x - \frac{\pi}{6}}$$

Now as $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Now by applying the limit we get

$$\Rightarrow 2 \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin \left(x - \frac{\pi}{6} \right)}{x - \frac{\pi}{6}} = 2 \cdot 1 = 2$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} = 2$$

23. $\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \tan 3x}$

Solution:

Given $\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \tan 3x}$

Multiply and divide the numerator of given equation by 2x

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \tan 3x} = \lim_{x \rightarrow 0} \frac{2x(\sin 2x)/2x + 3x}{2x + 3x(\tan 3x)/3x}$$

Now by splitting the limits we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2x(\sin 2x)/2x + 3x}{2x + 3x(\tan 3x)/3x} = \frac{\lim_{x \rightarrow 0} 2x \cdot \lim_{x \rightarrow 0} \left[\frac{\sin 2x}{2x} \right] + \lim_{x \rightarrow 0} 3x}{\lim_{x \rightarrow 0} 2x + \lim_{x \rightarrow 0} 3x \cdot \lim_{x \rightarrow 0} \left[\frac{\tan 3x}{3x} \right]}$$

Now as $\lim_{x \rightarrow 0} \left[\frac{\tan 3x}{3x} \right]$ and $\lim_{x \rightarrow 0} \left[\frac{\sin 2x}{2x} \right]$ both will be 1.

Substituting these in above equation and simplifying we get



$$\begin{aligned} & \lim_{x \rightarrow 0} 2x \cdot \lim_{x \rightarrow 0} \left[\frac{\sin 2x}{2x} \right] + \lim_{x \rightarrow 0} 3x = \lim_{x \rightarrow 0} 2x \cdot 1 + \lim_{x \rightarrow 0} 3x = \lim_{x \rightarrow 0} \frac{2x+3x}{2x+3x} = \lim_{x \rightarrow 0} 1 = 1 \\ \Rightarrow & \lim_{x \rightarrow 0} \frac{2x + \lim_{x \rightarrow 0} 3x \cdot \lim_{x \rightarrow 0} \left[\frac{\tan 3x}{3x} \right]}{2x + \lim_{x \rightarrow 0} 3x \cdot 1} = \lim_{x \rightarrow 0} \frac{2x+3x}{2x+3x} = \lim_{x \rightarrow 0} 1 = 1 \\ \Rightarrow & \lim_{x \rightarrow 0} \frac{\sin 2x+3x}{2x+\tan 3x} = 1 \end{aligned}$$

24. $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$

Solution:

Given $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$

Now we have to rationalize the denominator by multiplying the dividing by its rationalizing factor then we get

$$\Rightarrow \lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} = \lim_{x \rightarrow a} \left[\frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} \right]$$

On simplifying and splitting the denominator we get

$$\Rightarrow \lim_{x \rightarrow a} \left[\frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} \right] = \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a})$$

Now as $\sin x - \sin a = 2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right)$

Substituting this in above equation we get

$$\begin{aligned} \Rightarrow & \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a}) = \lim_{x \rightarrow a} \frac{2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right)}{x - a} \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a}) \\ \Rightarrow & \lim_{x \rightarrow a} \frac{2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right)}{x - a} \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a}) = 2\sqrt{a} \lim_{x \rightarrow a} \frac{\sin \left(\frac{x-a}{2} \right)}{\frac{x-a}{2}} \lim_{x \rightarrow a} \cos \left(\frac{x+a}{2} \right) \end{aligned}$$

Now as $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Applying the limits in above equation we get

$$\Rightarrow 2\sqrt{a} \lim_{x \rightarrow a} \frac{\sin \left(\frac{x-a}{2} \right)}{\frac{x-a}{2}} \lim_{x \rightarrow a} \cos \left(\frac{x+a}{2} \right) = 2\sqrt{a} \cdot 1 \cdot \cos a$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} = 2\sqrt{a} \cos a$$

25. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

Solution:

$$\text{Given } \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

We know that

$$\cot^2 x = \operatorname{cosec}^2 x - 1$$

By using this in given equation we get

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\operatorname{cosec}^2 x - 1) - 3}{\operatorname{cosec} x - 2} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

Again using $a^2 - b^2$ identity the above equation can be written as

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{\operatorname{cosec} x - 2}$$

On simplification and applying the limits we get

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{\operatorname{cosec} x - 2} = \lim_{x \rightarrow \frac{\pi}{6}} (\operatorname{cosec} x + 2) = 2 + 2 = 4$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} = 4$$

$$26. \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

Solution:

$$\text{Given } \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

Multiply and divide the given equation by $\sqrt{2} - \sqrt{1 + \cos x}$

Then we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} \times \left(\frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}} \right)$$

Now by splitting the limits we have

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} \times \left(\frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}} \right) = \lim_{x \rightarrow 0} \frac{2 - (1 + \cos x)}{\sin^2 x} \lim_{x \rightarrow 0} \left(\frac{1}{\sqrt{2} + \sqrt{1 + \cos x}} \right)$$

$$\text{Now } \sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x)$$

Substituting this in above equation

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 - (1 + \cos x)}{\sin^2 x} \lim_{x \rightarrow 0} \left(\frac{1}{\sqrt{2} + \sqrt{1 + \cos x}} \right) = \frac{1}{2\sqrt{2}} \lim_{x \rightarrow 0} \frac{(1 - \cos x)}{(1 - \cos x)(1 + \cos x)}$$

Now by applying the limits we get



$$\Rightarrow \frac{1}{2\sqrt{2}} \lim_{x \rightarrow 0} \frac{(1 - \cos x)}{(1 - \cos x)(1 + \cos x)} = \frac{1}{2\sqrt{2}} \lim_{x \rightarrow 0} \frac{1}{(1 + \cos x)} = \frac{1}{2\sqrt{2}} \cdot \frac{1}{2}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} = \frac{1}{4\sqrt{2}}$$

27. $\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$

Solution:

Given $\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$

Now by splitting the limits in above equation we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} - \lim_{x \rightarrow 0} \frac{2 \sin 3x}{x} + \lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

Taking constant term outside the limits we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x}{x} - \lim_{x \rightarrow 0} \frac{2 \sin 3x}{x} + \lim_{x \rightarrow 0} \frac{\sin 5x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} - 2(3) \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} + (5) \lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$$

Now as $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

By substituting and applying the limits we get

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x}{x} - 2(3) \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} + (5) \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 1 - 6 + 5 = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} = 0$$

Differentiate each of the functions with respect to x in Exercises 29 to 42.

29. $\frac{x^4 + x^3 + x^2 + 1}{x}$

Solution:

Let $y = \frac{x^4 + x^3 + x^2 + 1}{x}$

$$\Rightarrow y = \frac{x^4 + x^3 + x^2 + 1}{x}$$

Dividing by x we get

$$\Rightarrow y = x^3 + x^2 + x + \frac{1}{x}$$

Differentiating given equation with respect to x

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left(x^3 + x^2 + x + \frac{1}{x} \right)$$

On differentiation we get

$$\Rightarrow \frac{d}{dx} \left(x^3 + x^2 + x + \frac{1}{x} \right) = 3x^2 + 2x + 1 - \frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^4 + 2x^3 + x^2 - 1}{x^2}$$

$$dy/dx = d/dx(x^3 + x^2 + x + (1/x))$$

On differentiation we get,

$$d/dx(x^3 + x^2 + x + (1/x)) = 3x^2 + 2x + 1 - (1/x^2)$$

Hence, the required answer is $3x^2 + 2x + 1 - (1/x^2)$

30. $x + \frac{1}{x}^3$

Solution:

Let $y = \left(x + \frac{1}{x}\right)^3$

Now differentiating y with respect to x we get

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left(x + \frac{1}{x}\right)^3$$

Expanding the equation using $(a + b)^3$ formula then we get

$$= \frac{d}{dx} \left(x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}\right)$$

Splitting the differential we get

$$= \frac{d}{dx} (x^3) + \frac{d}{dx} \left(\frac{1}{x^3}\right) + \frac{d}{dx} (3x) + \frac{d}{dx} \left(\frac{3}{x}\right)$$

On differentiating we get

$$= 3x^2 - 3x^{-4} + 3 - 3x^{-2}$$

$$= 3x^2 - \frac{3}{x^4} + 3 - \frac{3}{x^2}$$

31. $(3x + 5)(1 + \tan x)$

Solution:

Given $(3x + 5)(1 + \tan x)$

Let $y = (3x + 5)(1 + \tan x)$

Applying product rule of differentiation that is

$$\Rightarrow \frac{d}{dx} (t.y) = y \cdot \frac{dt}{dx} + t \cdot \frac{dy}{dx}$$

$$\Rightarrow y = (3x + 5)(1 + \tan x)$$

$$\Rightarrow \frac{dy}{dx} = (1 + \tan x) \frac{d}{dx} (3x + 5) + (3x + 5) \frac{d}{dx} (1 + \tan x)$$

$$\Rightarrow \frac{dy}{dx} = 3(1 + \tan x) + (3x + 5) \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = 3x \sec^2 x + 5 \sec^2 x + 3 + 3 \tan x \text{ (by using product rule)}$$

Hence, the required answer is $3x \sec^2 x + 5 \sec^2 x + 3 \tan x + 3$

32. $(\sec x - 1)(\sec x + 1)$

Solution:

Given $(\sec x - 1)(\sec x + 1)$

Let $y = (\sec x - 1)(\sec x + 1)$

The above equation can be written as

$$\Rightarrow y = (\sec x - 1)(\sec x + 1) = \sec^2 x - 1 = \tan^2 x$$

$$\Rightarrow y = \tan^2 x$$

Now applying the chain rule we get

$$\Rightarrow \frac{dy}{dx} = \frac{d}{d(\tan x)} (\tan^2 x) \cdot \frac{d}{dx} (\tan x)$$

$$\Rightarrow \frac{dy}{dx} = 2 \tan x \sec^2 x$$

$$33. \frac{3x+4}{5x^2-7x+9}$$

Solution:

$$\text{Given } y = \frac{3x+4}{5x^2-7x+9}$$

Applying quotient rule of differentiation that is

$$\Rightarrow \frac{d}{dx} \left(\frac{t}{y} \right) = \frac{y \cdot \frac{dt}{dx} - t \cdot \frac{dy}{dx}}{y^2}$$

$$\Rightarrow y = \frac{3x+4}{5x^2-7x+9}$$

Applying the rule

$$\Rightarrow \frac{dy}{dx} = \frac{(5x^2 - 7x + 9) \frac{d}{dx} (3x + 4) - (3x + 4) \frac{d}{dx} (5x^2 - 7x + 9)}{(5x^2 - 7x + 9)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(5x^2 - 7x + 9) - (3x + 4)(10x - 7)}{(5x^2 - 7x + 9)^2}$$

On differentiation we get

$$\Rightarrow \frac{3(5x^2 - 7x + 9) - (3x + 4)(10x - 7)}{(5x^2 - 7x + 9)^2}$$

On differentiation we get,

$$= (15x^2 - 21x + 27 - 30x^2 + 21x - 40x + 28)/(5x^2 - 7x + 9)^2$$

$$= (-15x^2 - 40x + 55)/(5x^2 - 7x + 9)^2$$

$$= (55 - 40x - 15x^2)/(5x^2 - 7x + 9)^2$$

Hence, the required answer is,

$$(55 - 40x - 15x^2)/(5x^2 - 7x + 9)^2$$

34. $\frac{x^5 - \cos x}{\sin x}$

Solution:

Given $y = \frac{x^5 - \cos x}{\sin x}$

$$d/dx(x^5 - \cos x)/\sin x = [\sin x \cdot d/dx(x^5 - \cos x) - (x^5 - \cos x) \cdot d/dx(\sin x)]/\sin^2 x$$

By using quotient rule,

$$= [\sin x (5x^4 + \sin x) - (x^5 - \cos x)(\cos x)]/\sin^2 x$$

$$= [5x^4 \cdot \sin x + \sin^2 x - x^5 \cos x + \cos^2 x]/\sin^2 x$$

$$= [5x^4 \sin x - x^5 \cos x + (\sin^2 x + \cos^2 x)]/\sin^2 x$$

$$= [5x^4 \sin x - x^5 \cos x + 1]/\sin^2 x$$

Hence, the required answer is $[5x^4 \sin x - x^5 \cos x + 1]/\sin^2 x$

36. $(ax^2 + \cot x) (p + q \cos x)$

Solution:

Given $y = (ax^2 + \cot x)(p + q\cos x)$

Applying product rule of differentiation that

$$\Rightarrow \frac{d}{dx}(t.y) = y \cdot \frac{dt}{dx} + t \cdot \frac{dy}{dx}$$

$$\Rightarrow y = (ax^2 + \cot x)(p + q\cos x)$$

Now splitting the differentials,

$$\Rightarrow \frac{dy}{dx} = (p + q\cos x) \frac{d}{dx}(ax^2 + \cot x) + (ax^2 + \cot x) \frac{d}{dx}(p + q\cos x)$$

On differentiation we get

$$\Rightarrow \frac{dy}{dx} = (p + q\cos x)(2ax - \operatorname{cosec}^2 x) + (ax^2 + \cot x)(-q\sin x)$$

$$\Rightarrow \frac{dy}{dx} = (p + q\cos x)(2ax - \operatorname{cosec}^2 x) - q\sin x(ax^2 + \cot x)$$

37. $\frac{a + b\sin x}{c + d\cos x}$

Solution:

Given $y = \frac{a + b\sin x}{c + d\cos x}$

Applying division rule or quotient rule of differentiation that is

$$\Rightarrow \frac{d}{dx}\left(\frac{t}{y}\right) = \frac{y \cdot \frac{dt}{dx} - t \cdot \frac{dy}{dx}}{y^2}$$

$$\Rightarrow y = \frac{a + b\sin x}{c + d\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(c + d\cos x) \frac{d}{dx}(a + b\sin x) - (a + b\sin x) \frac{d}{dx}(c + d\cos x)}{(c + d\cos x)^2}$$

On differentiating we get

$$\Rightarrow \frac{dy}{dx} = \frac{(c + d\cos x)(b\cos x) - (a + b\sin x)(-d\sin x)}{(c + d\cos x)^2}$$

$$= [c b \cos x + b d \cos^2 x + a d \sin x + b d \sin^2 x] / (c + d \cos x)^2$$

$$= [c b \cos x + a d \sin x + b d (\cos^2 x + \sin^2 x)] / (c + d \cos x)^2$$

$$= [c b \cos x + a d \sin x + b d] / (c + d \cos x)^2$$

39. $(2x - 7)^2 (3x + 5)^3$

Solution:

$$\text{Given } y = (2x - 7)^2(3x + 5)^3$$

Applying product rule of differentiation that is

$$\Rightarrow \frac{d}{dx}(t.y) = y \cdot \frac{dt}{dx} + t \cdot \frac{dy}{dx}$$

$$\Rightarrow y = (2x - 7)^2(3x + 5)^3$$

$$\Rightarrow \frac{dy}{dx} = (3x + 5)^3 \frac{d}{dx}(2x - 7)^2 + (2x - 7)^2 \frac{d}{dx}(3x + 5)^3$$

On differentiating we get

$$\Rightarrow \frac{dy}{dx} = (2)(3x + 5)^3 2(2x - 7)^1 + (3)(2x - 7)^2 3(3x + 5)^2$$

$$\Rightarrow \frac{dy}{dx} = 4(3x + 5)^3 (2x - 7) + 9(2x - 7)^2 (3x + 5)^2$$

On simplification we get

$$\Rightarrow \frac{dy}{dx} = (2x - 7)(3x + 5)^2 [4(3x + 5) + 9(2x - 7)]$$

$$\Rightarrow \frac{dy}{dx} = (2x - 7)(3x + 5)^2 (30x - 43)$$

40. $x^2 \sin x + \cos 2x$

Solution:

Applying product rule of differentiation for given equation

That is

$$\Rightarrow \frac{d}{dx}(t.y) = y \cdot \frac{dt}{dx} + t \cdot \frac{dy}{dx}$$

$$\Rightarrow y = x^2 \sin x + \cos 2x$$

$$\Rightarrow \frac{dy}{dx} = \sin x \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos 2x)$$

On differentiating we get

$$\Rightarrow \frac{dy}{dx} = \sin x(2x) + x^2 \cos x + (-\sin 2x)(2)$$

$$\Rightarrow \frac{dy}{dx} = 2x \sin x + x^2 \cos x - 2 \sin 2x$$



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