Exercise 5(C)

Solution 1:

$$25a^2 - 9b^2 = (5a)^2 - (3b)^2$$
$$= (5a - 3b)(5a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (a - 2a - 3b)(a + 2a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (-a - 3b)(3a + 3b)$$
$$= -3(a + 3b)(a + b)$$

Solution 3:

$$a^2 - 81(b - c)^2 = (a)^2 - [9(b - c)]^2$$
$$= (a - (9b - 9c))(a + (9b - 9c)) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (a - 9b + 9c)(a + 9b - 9c)$$

Solution 4:

$$25(2a - b)^2 - 81b^2 = [5(2a - b)]^2 - (9b)^2$$
$$= [5(2a - b) - 9b][5(2a - b) + 9b]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [10a - 5b - 9b][10a - 5b + 9b]$$
$$= [10a - 14b][10a + 4b]$$
$$= 2 \times (5a - 7b) \times 2 \times (5a + 2b)$$
$$= 4(5a - 7b)(5a + 2b)$$

Solution 5:

$$50a^3 - 2a = 2a(25a^2 - 1)$$
$$= 2a[(5a)^2 - 1^2]$$
$$= 2a(5a + 1)(5a - 1) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

Solution 6:

$$4a^2b - 9b^3 = b(4a^2 - 9b^2)$$
$$= b[(2a)^2 - (3b)^2]$$
$$= b(2a - 3b)(2a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

Solution 7:

$$3a^5 - 108a^3 = 3a^3(a^2 - 36)$$
$$= 3a^3[(a)^2 - (6)^2]$$
$$= 3a^3(a - 6)(a + 6) \quad \because a^2 - b^2 = (a + b)(a - b)$$

Solution 8:

$$9(a-2)^2-16(a+2)^2=[3(a-2)]^2-[4(a+2)]^2$$
$$=[3(a-2)-4(a+2)][3(a-2)+4(a+2)]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[3a-6-4a-8][3a-6+4a+8]$$
$$=(-a-14)(7a+2)$$
$$=-(a+14)(7a+2)$$

Solution 9:

$$a^4-1=(a^2)^2-(1)^2$$
$$=(a^2+1)(a^2-1)\ [\because a^2-b^2=(a+b)(a-b)]$$
$$=(a^2+1)((a)^2-(1)^2)$$
$$=(a^2+1)(a+1)(a-1)$$

Solution 10:

$$a^3+2a^2-a-2=a^2(a+2)-1(a+2)$$
$$=(a^2-1)(a+2)$$
$$=(a+1)(a-1)(a+2)\quad[\because a^2-b^2=(a+b)(a-b)]$$

Solution 11:

$$(a+b)^3-a-b=(a+b)^3-(a+b)$$
$$=(a+b)\left[(a+b)^2-1\right]$$
$$=(a+b)\left[(a+b)^2-1^2\right]$$
$$=(a+b)((a+b)+1)((a+b)-1)$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=(a+b)(a+b+1)(a+b-1)$$

Solution 12:

$$a(a-1)-b(b-1)=a^2-a-b^2+b$$
$$=a^2-b^2-a+b$$
$$=(a+b)(a-b)-(a-b)$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=(a-b)[(a+b)-1]$$
$$=(a-b)[a+b-1]$$

Solution 13:

$$4a^2-\left(4b^2+4bc+c^2\right)=(2a)^2-(2b+c)^2$$
$$=[2a-(2b+c)][2a+(2b+c)]$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=[2a-2b-c][2a+2b+c]$$

Solution 14:

$$4a^2-49b^2+2a-7b=\left[(2a)^2-(7b)^2\right]+[2a-7b]$$
$$=[2a-7b][2a+7b]+[2a-7b]$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=[2a-7b][2a+7b+1]$$

Solution 15:

$$9a^2+3a-8b-64b^2=9a^2-64b^2+3a-8b$$
$$=(3a)^2-(8b)^2+3a-8b$$
$$=(3a-8b)(3a+8b)+(3a-8b)$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=(3a-8b)(3a+8b+1)$$

Solution 16:

$$4a^2-12a+9-49b^2=(2a)^2-12a+(3)^2-49b^2$$
$$=(2a-3)^2-49b^2$$
$$=(2a-3)^2-(7b)^2$$
$$=(2a-3-7b)(2a-3+7b)$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$

Solution 17:

$$4xy-x^2-4y^2+z^2=z^2-\left(x^2+4y^2-4xy\right)$$
$$=z^2-(x-2y)^2$$
$$=[z-(x-2y)][z+(x-2y)]$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=[z-x+2y][z+x-2y]$$

Solution 18:

$$a^2+b^2-c^2-d^2+2ab-2cd$$
$$=(a^2+b^2+2ab)-(c^2+d^2+2cd)$$
$$=(a+b)^2-(c+d)^2$$
$$=[(a+b)-(c+d)][(a+b)+(c+d)] \quad [\because a^2-b^2=(a+b)(a-b)]$$
$$=(a+b-c-d)(a+b+c+d)$$

Solution 19:

$$4x^2-12ax-y^2-z^2-2yz+9a^2$$
$$=4x^2+9a^2-12ax-y^2-z^2-2yz$$

$$=(2x)^2+(3a)^2-12ax-(y^2+z^2+2yz)$$
$$=(2x-3a)^2-(y+z)^2$$
$$=[(2x-3a)-(y+z)][(2x-3a)+(y+z)]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[2x-3a-y-z][2x-3a+y+z]$$

Solution 20:

$$(a^2-1)(b^2-1)+4ab=a^2b^2-a^2-b^2+1+4ab$$
$$=a^2b^2+1+2ab-a^2-b^2+2ab$$
$$=(a^2b^2+1+2ab)-(a^2+b^2-2ab)$$
$$=(ab+1)^2-(a-b)^2$$
$$=[(ab+1)-(a-b)][(ab+1)+(a-b)]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[ab+1-a+b][ab+1+a-b]$$

Solution 21:

$$x^4+x^2+1=x^4+2x^2+1-x^2$$
$$=(x^2)^2+2x^2+(1)^2-x^2$$
$$=(x^2+1)^2-(x)^2$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=(x^2+1-x)(x^2+1+x)$$

Solution 22:

$$\begin{aligned}(a^2+b^2-4c^2)^2-4a^2b^2 &= (a^2+b^2-4c^2)^2-(2ab)^2\\ &= (a^2+b^2-4c^2-2ab)(a^2+b^2-4c^2+2ab)\\ &\quad [\because a^2-b^2=(a+b)(a-b)]\\ &= (a^2+b^2-2ab-4c^2)(a^2+b^2+2ab-4c^2)\\ &= ((a-b)^2-(2c)^2)((a+b)^2-(2c)^2)\\ &= (a-b+2c)(a-b-2c)(a+b+2c)(a+b-2c)\end{aligned}$$

Solution 23:

$$\begin{aligned}&(x^2+4y^2-9z^2)^2-16x^2y^2\\ &= (x^2+4y^2-9z^2)^2-(4xy)^2\\ &= (x^2+4y^2-9z^2-4xy)(x^2+4y^2-9z^2+4xy)\\ &\quad [\because a^2-b^2=(a+b)(a-b)]\\ &= (x^2+4y^2-4xy-9z^2)(x^2+4y^2+4xy-9z^2)\\ &= [(x-2y)^2-(3z)^2][(x+2y)^2-(3z)^2]\\ &= [(x-2y)-3z][(x-2y)+3z][(x+2y)-3z][(x+2y)+3z]\\ &= [x-2y-3z][x-2y+3z][x+2y-3z][x+2y+3z]\end{aligned}$$

Solution 24:

$$\begin{aligned}&(a+b)^2-a^2+b^2\\ &= a^2+2ab+b^2-a^2+b^2\\ &= 2ab+2b^2\\ &= 2b(a+b)\end{aligned}$$

Solution 25:

$$\begin{aligned}&a^2-b^2-(a+b)^2\\ &= a^2-b^2-(a^2+2ab+b^2)\\ &= a^2-b^2-a^2-2ab-b^2\\ &= -2ab-2b^2\\ &= -2b(a+b)\end{aligned}$$

Solution 26:

$9a^2 - (a^2 - 4)^2$

$= (3a)^2 - (a^2 - 4)^2$

$= [3a - (a^2 - 4)][3a + (a^2 - 4)]$

$= [3a - a^2 - 4][3a + a^2 - 4]$

$= [-a^2 + 3a - 4][a^2 + 3a - 4]$

$= [-a^2 + 4a - a - 4][a^2 + 4a - a - 4]$

$= [a(-a + 4) + 1(-a + 4)][a(a + 4) - 1(a + 4)]$

$= [(a + 1)(4 - a)][(a + 4)(a - 1)]$

$= (a + 1)(4 - a)(a + 4)(a - 1)$

Solution 27:

$x^2 + \frac{1}{x^2} - 11$

$= x^2 + \frac{1}{x^2} - 2 - 9$

$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 9$

$= \left(x - \frac{1}{x}\right)^2 - (3)^2$

$= \left(x - \frac{1}{x} + 3\right)\left(x - \frac{1}{x} - 3\right)$

Solution 28:

$4x^2 + \frac{1}{4x^2} + 1$

$= 4x^2 + \frac{1}{4x^2} + 2 - 1$

$= 4x^2 + \frac{1}{4x^2} + 2 \times 2x \times \frac{1}{2x} - 1$

$= \left(2x + \frac{1}{2x}\right)^2 - (1)^2$

$= \left(2x + \frac{1}{2x} + 1\right)\left(2x + \frac{1}{2x} - 1\right)$

Solution29:

$4x^4 - x^2 - 12x - 36$

$= 4x^4 - (x^2 + 12x + 36)$

$= (2x^2)^2 - (x^2 + 2 \times x \times 6 + 6^2)$

$= (2x^2)^2 - (x + 6)^2$

$= (2x^2 + x + 6)(2x^2 - x - 6)$

$= (2x^2 + x + 6)(2x^2 - 4x + 3x - 6)$

$= (2x^2 + x + 6)[2x(x - 2) + 3(x - 2)]$

$= (2x^2 + x + 6)[(x - 2)(2x + 3)]$

$= (2x^2 + x + 6)(x - 2)(2x + 3)$

Solution 30:

$a^2(b + c) - (b + c)^3$

$= (b + c)[a^2 - (b + c)^2]$

$= (b + c)[(a + b + c)(a - b - c)]$

$= (b + c)(a + b + c)(a - b - c)$