Chapter 5: Arithmetic Progressions

EXERCISE 5.3:

1. Find the sum of the following APs:
   (i) 2, 7, 12, . . ., to 10 terms.
   (ii) –37, –33, –29, . . ., to 12 terms.
   (iii) 0.6, 1.7, 2.8, . . ., to 100 terms.
   (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots,$ to 11 terms.

Solution:(i) Here, a = 2, d = 5 and n = 10

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5]$

= 5(4 + 45)

= 5×49

= 245

Thus, sum of the 10 terms of given AP (S_n)=245

(ii) Here, a = - 37, d = 4 and n = 12

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$S_{12} = \frac{12}{2}[2 \times (-37) + (11)4]$

= 6(-74 + 44)

= $6 \times (-30)$

=-180

Thus, sum of the 12 terms of given AP (S_n)= -180

(iii) Here, a = 0.6, d = 1.1 and n = 100

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{100} = \frac{100}{2}[2 \times (0.6) + (99)1.1]$$

$= 50(1.2 + 108.9)$

$= 50 \times (110.1)$

$=5505$

Thus, sum of the 100 terms of given AP (S_n)= -5505

(iv) Here,

a= $\frac{1}{15}$, n = 11,

d = $\frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[2 \times (\frac{1}{15}) + (10)\frac{1}{60}\right]$$

$= \frac{11}{2}(\frac{2}{15} + \frac{1}{6})$

$= \frac{11}{2} \times (\frac{4+5}{30})$

$= \frac{33}{20}$

Thus, sum of the 100 terms of given AP (S_n)=$\frac{33}{20}$

2. Find the sums given below :

   (i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

   (ii) $34 + 32 + 30 + \ldots. + 10$

   (iii) $-5 + (-8) + (-11) + \ldots. + (-230)$

Solution: (i) Here, a = 7, d = 3.5 and last term = 84

Number of terms can be calculated as follows;

a_n =a + (n-1)d

Or, 84 = 7 + (n-1)3.5

Or, (n-1)3.5 = 84-7

Or, n - 1 = 77/3.5 = 22

Or, n = 23

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{23} = \frac{23}{2}[2 \times (7) + (22)3.5]$$

$$= \frac{23}{2}(14 + 77)$$

$$= \frac{23}{2} \times (91)$$

$$= 1046\frac{1}{2}$$

(ii) Here, a = 34, d = - 2 and last term = 10

Number of terms can be calculated as follows:

an = a + (n – 1)d

Or, 10 = 34 + (n – 1)(- 2)

Or, 10 = 34 – (n – 1)(2)

Or, (n – 1)2 = 34 – 10 = 24

Or, n – 1 = 12

Or, n = 13

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}[2 \times (34) + (12)(-2)]$$

$$= \frac{13}{2}[68 + (-24)]$$

$$= \frac{13}{2} \times (44)$$

$$= 286$$

Thus, sum of the given AP (S_n)=286

(iii) Here, a = - 5, d = - 3 and last term = - 230

Number of terms can be calculated as follows:

$a_n = a + (n – 1)d$

Or, - 230 = - 5 + (n – 1)(- 3)

Or, - 230 = - 5 – (n – 1)3

Or, (n – 1)3 = - 5 + 230 = 225

Or, n – 1 = 75

Or, n = 76

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{23} = \frac{76}{2}[2 \times (-5) + (-3)75]$$

$$= \frac{76}{2}(-10 -225)$$

$$= 38 \times (-235)$$

$$=-8930$$

3. In an AP:

(i) given $a = 5, d = 3, a_n = 50$, find n and S_n.

(ii) given $a = 7, a_{13} = 35$, find d and S_{13}.

(iii) given $a = 7, a_{13} = 35$, find a and S_{12}.

(iv) given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

(v) given $d = 5, S_9 = 75$, find a and a_9.

(vi) given $a = 2, d = 8, S_n = 90$, find n and a_n.

(vii) given $a_n = 8, a_n = 62, S_n = 210$, find n and d.

(viii) given $a_n = 4, d = 2, S_n = -14$, find n and a.

(ix) given $a = 3, n = 8, S = 192$, find d.

(x) given $l = 28$, $S = 144$, and there are total 9 terms. Find a.

Solution: (i) Number of terms can be calculated as follows:

$a_n = a + (n – 1)d$

Or, 50 = 5 + (n – 1)3

Or, (n – 1)3 = 50 – 5 = 45

Or, n – 1 = 15

Or, n = 16

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2 \times (5) + (15)3]$$

$$= 8\ (10 + 45)$$

$$= 8 \times (55)$$

$$=440$$

(ii) Common difference can be calculated as follows:

$a_n = a + (n – 1)d$

Or, 35 = 7 + 12d

Or, 12d = 35 – 7 = 28

Or, d = 7/3

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}\left[2 \times (7) + (12)\frac{7}{3}\right]$$

$$= \frac{13}{2}(14 + 28)$$

$$= \frac{13}{2} \times (42)$$

$$=273$$

(iii) First term can be calculated as follows:

$a_n = a + (n – 1)d$

Or, 37 = a + 11 x 3

Or, a = 37 – 33 = 4

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$S_{12} = \frac{12}{2}[2 \times (4) + (11)3]$

= 6(8 + 33)

= 6 * 41

= 246

(iv) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$S_{10} = \frac{10}{2}[2 \times (a) + (9)d]$

125 = 5(2a + 9d)

25 = 2a * 9d (i)

According to the question; the 3rd term is 15, which means;

a + 2d = 15 (ii)

Now,

Subtracting equation (ii) from equation (i), we get;

2a + 9d – a – 2d = 25 – 15

Or, a + 7d = 10 (iii)

Subtracting (ii) from equation (iii), we get;

a + 7d – a – 2d = 10 – 15

Or, 5d = - 5

Or, d = - 1

Now,

Substituting the value of d in equation (2), we get;

a + 2(- 1) = 15

Or, a – 2 = 15

Or, a = 17

10th term can be calculated as follows;

a_{10} = a + 9d

= 17 – 9 = 8

Thus, d = - 1 and 10th term = 8

(v) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

75= $\frac{9}{2}[2 \times (a) + (8)5]$

75= $\frac{9}{2}$ (2a + 40)

$\frac{50}{3}$ = 2a * 40

$$a = -\frac{35}{3}$$

Now,

9^{th} term can be calculated as follows:

a_9 = a + 8d

= -$\frac{35}{3} + 40$

= $\frac{85}{3}$

(vi) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

90= $\frac{N}{2}[2 \times (2) + (N-1)8]$

90= $\frac{N}{2}$ (4 + 8N -8)

90 = N(2 +4N -4)

$4N^2$ -2N -90 =0

$2N^2$ -N -45 =0

(2N +9)(N -5)

Hence, n = - 9/2 and n = 5

Rejecting the negative value,

We have n = 5

Now, 5^{th}term will be:

a_5 = a + 4d

= 2 + 4 x 8

= 2 + 32 = 34

(vii) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$210 = \frac{n}{2}[a + a + (n - 1)d]$
420 = n (8 + 62)
420 = n *70
n = 6

Now, for calculating d:

a_6 = a + 5d

Or, 62 = 8 + 5d

Or, 5d = 62 – 8 = 54

Or, d = 54/5

(viii) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

-14 = $\frac{n}{2}[a + a + (n - 1)d]$
-28 = n (a + 4)
$n = \frac{-28}{a+4}$ (i)

We know;

a_n = a + (n – 1)d

Or, 4 = a + (n – 1)2

Or, 4 = a + 2n – 2

Or, a + 2n = 6

Or, 2n = 6 – a

Or, n = (6 – a)/2 (ii)

Using (i) and (ii):

$$\frac{-28}{a+4} = \frac{6-a}{2}$$

-56 = (6 –a)(a +4)

24 + 2a – a^2 = -56

a^2 -2a -80= 0

(a + 8)(a – 10)= 0

Therefore, a = - 8 and a = 10

As a is smaller than 10 and d has positive value, hence we'll take a = - 8

Now, we can find the number of terms as follows:

a_n = a + (n – 1)d

4 = - 8 + (n – 1)2

(n – 1)2 = 4 + 8 = 12

n – 1 = 6Or, n = 7

Hence, n = 7 and a = - 8

(ix) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

192 = $\frac{8}{2}[2*3+(7)d]$

192 = 4 (6 + 7d)

7d = 42

d = 6

(x) Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

144 = $\frac{9}{2}$(a + a_n)

288 = n (a + 28)

9a + 252 = 288

9a = 288 -252

9a = 36

a = 4

4. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

Solution:Given that, a = 9, d = 8 and S_n = 636

Now

We know:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$636 = \frac{N}{2}[2 \times (9) + (N-1)8]$

$636 = \frac{N}{2}(18 + 8N - 8)$

636 = N(9 +4N -4)

$4N^2 + 5N - 636 = 0$

$2N^2 - N - 45 = 0$

(4N +53)(N – 12)

Now, n = - 53/4 and n = 12

Taking the integral value and rejecting the fractional value, we have n = 12

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of termsand the common difference.

Solution: We can calculate the sum of N number of terms as :

$$S = \frac{N}{2}[2a + (n-1)d]$$

$400 = \frac{n}{2}[2 * 5 + (n-1)d]$

800 = n (5 + 45)

$50n = 800$

n = 16

Now, common difference can be calculated as follows:

a_n = a + (n – 1)d

45 = 5 + 15d

15d = 40

d = 40/15 = 8/3

Hence, n = 16 and d = 8/3

6. The first and the last terms of an AP are 17 and 350 respectively. If the common differenceis 9, how many terms are there and what is their sum?

Solution:We have; a = 17, a_n = 350 and d = 9

We know;

a_n = a + (n – 1)d

350 = 17 + (n – 1)9

(n – 1)9 = 350 – 17

n – 1 = 333/9 = 37

n = 38

Now, sum can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{38}{2}[17 + 350]$$

= 19*367

= 6973

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Solution: We have; n = 22, d = 7 and a_{22} = 149

We know;

a_n = a + (n – 1)d

Or, a = 149 – 147 = 2

The sum can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{22}{2}[2 + 149]$$

= 11*151

= 1661

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18respectively.

Solution: We have; $a_2 = 14$, $a_3 = 18$ and n = 51

We know,

$a_3 – a_2 = 18 – 14 = 4$

Therefore, d = 4

$a_2 – a = 4$

14 – a = 4

a = 10

Now, sum can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{51}{2}[2 * 10 + 50 * 4]$$

= 51*(10 + 100)

= 51*110

= 5610

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum offirst n terms.

Solution: According to the question:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$49 = \frac{7}{2}[2 * a + 6d]$$

49 = 7*(a + 3d)

7= a + 3d

We can observe that it is 4^{th} term,

Similarly,

$S_{17} = \frac{17}{2}[2a + (16)d]$

289 = 17(a + 8d)

17 = a + 8d

We can observe that it is 9th term,

This is the 9th term

Subtracting 4th term from 9th term, we get;

a + 8d – a – 3d = 17 – 7

Or, 5d = 10

Or, d = 2

Using the value of d in equation for fourth term, we can find a as follows:

a + 3d =7

Or, a + 6 = 7

Or, a = 1

Using the values of a and d; we can find the sum of first n terms as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$= \frac{N}{2}[2(1) + (n-1)2]$$
$$= N(1 + N - 1)$$

$= N^2$

10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below :
(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Solution: (i)Let us take different values for a, i.e. 1, 2, 3 and so on

a = 3 + 4 = 7

a_2 = 3 + 4 x 2 = 11

a_3 = 3 + 4 x 3 = 15

$a_4 = 3 + 4 \times 4 = 19$

Here; each subsequent member of the series is increasing by 4 and hence it is an AP.

(ii) Let us take values for a, i.e. 1, 2, 3 and so on

$a = 9 – 5 = 4$

$a_2 = 9 – 5 \times 2 = - 1$

$a_3 = 9 – 5 \times 3 = - 6$

$a_4 = 9 – 5 \times 4 = - 11$

Here; each subsequent member of the series is decreasing by 5 and hence it is an AP.

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S1)? Whatis the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th andthe nth terms.

Solution:We can find the first term as follows:

$S_1 = 4 \times 1 - 1^2 = 3$

Now; sum of first two terms can be calculated as follows:

$S_2 = 4 \times 2 - 2^2$

$= 8 - 4 = 4$

Hence; second term = 4 – 3 = 1

And d = 1 – 3 = - 2

So, 3rd term can be calculated as follows:

$a_3 = a + (n – 1)d$

$= 3 + 2(- 2) = 3 – 4 = - 1$

Similarly, 10th term can be calculated as follows:

$a_{10} = a + 9d$

$= 3 + 9(- 2)$

$= 3 – 18$

= - 15

12. Find the sum of the first 40 positive integers divisible by 6.

Solution:The smallest positive integer which is divisible by 6 is 6 itself and its 40th multiple will be 6 x 40 = 240

So, we have a = 6, d = 6, n = 40 and 40th term = 240.

Sum of first 40 positive integers divisible by 6 can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$= \frac{40}{2}[2*6 + (39)6]$$

= 20(12 + 234)

= 20*246

= 4920

13. Find the sum of the first 15 multiples of 8.

Solution:Given: a = 8, d = 8 and n = 15

Sum of first 15 multiples of 8 can be calculated as:

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$= \frac{15}{2}[2*8 + (14)8]$$

= 15(8 + 56)

= 15*64

= 960

14. Find the sum of the odd numbers between 0 and 50

Solution:There are 25 odd numbers between 0 and 50

And, we know that

Sum of first n odd numbers = n^2.

= 25^2

= 625

15. A contract on construction job specifies a penalty for delay of completion beyond acertain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for thepreceding day. How much money the contractor has to pay as penalty, if he has delayedthe work by 30 days?

Solution:Given: a = 200, d = 50 and n = 30

We can find the penalty by using the sum of 30 terms:

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$= \frac{30}{2}[2 * 200 + (29)50]$$

= 15(400 + 1450)

= 15*1850

= 27750

Hence, he has to pay an amount of Rs. 27750 as penalty

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for theiroverall academic performance. If each prize is Rs 20 less than its preceding prize, find thevalue of each of the prizes.

Solution:Here; d = 20 n = 7 and S_n = 700

Now,

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$700 = \frac{n}{2}[2 * a + (6)20]$$

700 = 7(a + 60)

100 = a + 60

a= 40

Hence, the prizes in ascending order are:

Rs. 40, Rs. 60, Rs. 80, Rs. 100, Rs. 120, Rs. 140 and Rs. 160

17. In a school, students thought of planting trees in and around the school to reduce airpollution. It was decided that the number of trees, that each section of each class willplant, will be the same as the class, in which they are studying, e.g., a section of Class Iwill plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There arethree sections of each class. How many trees will be planted by the students?

Solution:Since each section of class 1 will plant 1 tree, so 3 trees will be planted by 3 sections of class 1.

Similarly,

No. of trees planted by 3 sections of class 2 = 6

No. of trees planted by 3 sections of class 3 = 9

No. of trees planted by 3 sections of class 4 = 12

We have; a = 3, d = 3 and n = 12

We can find the total number of trees as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$= \frac{12}{2}[2 * 3 + (11)3]$$

= 6(6 + 33)

= 6*39

= 234

18. A spiral is made up of successive semicircles, with centres alternately at A and B,starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown inFig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)

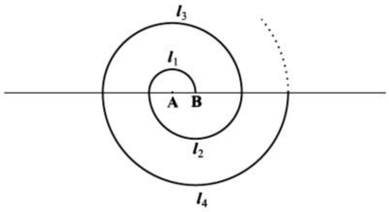

Fig. 5.4

[Hint : Length of successive semicircles is l_1, l_2, l_3, l_4,....with centres at A, B, A, B, . . .,respectively.]

Solution:Circumference of 1st semicircle = πr = 0.5π

Circumference of 2nd semicircle = πr = 1π = π

Circumference of 3rd semicircle =πr = 1.5π

It is clear that a = 0.5 π, d = 0.5π and n = 13

Hence; length of spiral can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{13}{2}[2 * 0.5\pi + (12)0.5\pi]$$

$= \frac{13}{2}(7\pi)$

$= \frac{13}{2} \times 7 \times \frac{22}{7}$

= 143 cm

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row,18 in the row next to it and so on (see Fig. 5.5). In how may rows are the 200 logs placedand how many logs are in the top row?


Fig. 5.5

Solution:We have; a = 20, d = - 1 and S_n = 200

We know;

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$200 = \frac{N}{2}[2 \times (20) + (N-1)(-1)]$$

400= N (40 - N +1)

N^2 -41N +400 =0

(N -16)(N –25)

Thus, n = 16 and n = 25

If number of rows is 25 then;

a_{25} = 20 + 24 x (- 1)

= 20 – 24 = - 4

Since; negative value for number of logs is not possible hence; number of rows = 16

a_{16} = 20 + 15 x (- 1)

= 20 – 15 = 5

Thus, number of rows = 16 and number of logs in top rows = 5

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato,and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in theline (see Fig. 5.6).

Fig. 5.6

A competitor starts from the bucket, picks up the nearest potato, runs back with it, dropsit in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, andshe continues in the same way until all the potatoes are in the bucket. What is the totaldistance the competitor has to run?

[Hint : To pick up the first potato and the second potato, the total distance (in metres)run by a competitor is $2 \times 5 + 2 \times (5+3)$]

Solution:Distance covered in picking and dropping 1st potato = 2 x 5 = 10 m

Distance covered in picking and dropping 2nd potato = 2 (5+3) = 16 m

Distance covered in picking and dropping 3rd potato = 2(5 + 3 + 3) = 22 m

Therefore, a = 10, d = 6 and n = 10

Total distance can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2 * 10 + (9)6]$$

= 5(20 + 54)

= 5*74

= 370 m