EXERCISE 31.6

Check the validity of the following statements:
(i) p: 100 is a multiple of 4 and 5.
(ii) q: 125 is a multiple of 5 and 7.
(iii) r: 60 is a multiple of 3 or 5.
Solution:
(i) p: 100 is a multiple of 4 and 5.
We know that 100 is a multiple of 4 as well as 5. So, the given statement is true.
Hence, the statement is true.

(ii) q: 125 is a multiple of 5 and 7
We know that 125 is a multiple of 5 and not a multiple of 7. So, the given statement is false.
Hence, the statement is false.

(iii) r: 60 is a multiple of 3 or 5.
We know that 60 is a multiple of 3 as well as 5. So, the given statement is true.
Hence, the statement is true.

2. Check whether the following statement is true or not:
(i) p: If x and y are odd integers, then x + y is an even integer.
(ii) q : if x, y are integer such that xy is even, then at least one of x and y is an even integer.
Solution:
(i) p: If x and y are odd integers, then x + y is an even integer.
Let us assume that 'p' and 'q' be the statements given by
p: x and y are odd integers.
q: x + y is an even integer
the given statement can be written as :
if p, then q.
Let p be true. Then, x and y are odd integers
$x = 2m+1$, $y = 2n+1$ for some integers m, n
$x + y = (2m+1) + (2n+1)$
$x + y = (2m+2n+2)$
$x + y = 2(m+n+1)$
x + y is an integer
q is true.
So, p is true and q is true.

Hence, "if p, then q "is a true statement."

(ii) q: if x, y are integer such that xy is even, then at least one of x and y is an even integer.
Let us assume that p and q be the statements given by
p: x and y are integers and xy is an even integer.
q: At least one of x and y is even.
Let p be true, and then xy is an even integer.
So,
$xy = 2(n + 1)$
Now,
Let $x = 2(k + 1)$
Since, x is an even integer, $xy = 2(k + 1)$. y is also an even integer.

Now take $x = 2(k + 1)$ and $y = 2(m + 1)$
$xy = 2(k + 1).2(m + 1) = 2.2(k + 1)(m + 1)$
So, it is also true.
Hence, the statement is true.

3. Show that the statement
p : "If x is a real number such that $x^3 + x = 0$, then x is 0" is true by
(i) Direct method
(ii) method of Contrapositive
(iii) method of contradiction
Solution:
(i) Direct Method:
Let us assume that 'q' and 'r' be the statements given by
q: x is a real number such that $x^3 + x=0$.
r: x is 0.
The given statement can be written as:
if q, then r.
Let q be true. Then, x is a real number such that $x^3 + x = 0$
x is a real number such that $x(x^2 + 1) = 0$
$x = 0$
r is true
Thus, q is true
Therefore, q is true and r is true.
Hence, p is true.

(ii) Method of Contrapositive:
Let r be false. Then,
R is not true
$x \neq 0, x \in R$
$x(x^2+1) \neq 0, x \in R$
q is not true
Thus, -r = -q
Hence, p : q and r is true

(iii) Method of Contradiction:
If possible, let p be false. Then,
P is not true
-p is true
-p (p => r) is true
q and –r is true
x is a real number such that $x^3+x = 0$and $x \neq 0$
$x = 0$ and $x \neq 0$
This is a contradiction.
Hence, p is true.

4. Show that the following statement is true by the method of the contrapositive p: "If x is an integer and x^2 is odd, then x is also odd."

Solution:

Let us assume that 'q' and 'r' be the statements given
q: x is an integer and x^2 is odd.
r: x is an odd integer.
The given statement can be written as:
p: if q, then r.
Let r be false. Then,
x is not an odd integer, then x is an even integer
x = (2n) for some integer n
$x^2 = 4n^2$
x^2 is an even integer
Thus, q is False
Therefore, r is false and q is false
Hence, p: " if q, then r" is a true statement.

5. Show that the following statement is true "The integer n is even if and only if n^2 is even"

Solution:
Let the statements,
p: Integer n is even
q: If n^2 is even
Let p be true. Then,
Let n = 2k
Squaring both the sides, we get,
$n^2 = 4k^2$
$n^2 = 2.2k^2$
n^2 is an even number.
So, q is true when p is true.
Hence, the given statement is true.

6. By giving a counter example, show that the following statement is not true.
p: "If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle."
Solution:
Let us consider a triangle ABC with all angles equal.
Then, each angle of the triangle is equal to 60.
So, ABC is not an obtuse angle triangle.
Hence, the statement "p: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle" is False.

7. Which of the following statements are true and which are false? In each case give a valid reason for saying so
(i) p: Each radius of a circle is a chord of the circle.
(ii) q: The centre of a circle bisect each chord of the circle.
(iii) r: Circle is a particular case of an ellipse.
(iv) s: If x and y are integers such that x > y, then – x < - y.
(v) t: √11 is a rational number.
Solution:
(i) p: Each radius of a circle is a chord of the circle.
The Radius of the circle is not it chord.
Hence, this statement is False.

(ii) q: The centre of a circle bisect each chord of the circle.
A chord does not have to pass through the center.
Hence, this statement is False.

(iii) r: Circle is a particular case of an ellipse.
A circle can be an ellipse in a particular case when the circle has equal axes.
Hence, this statement is true.

(iv) s: If x and y are integers such that $x > y$, then $-x < -y$.
For any two integers, if $x - y$ id positive then $-(x-y)$ is negative.
Hence, this statement is true.

(v) t: $\sqrt{11}$ is a rational number.
Square root of prime numbers is irrational numbers.
Hence, this statement is False.

8. Determine whether the argument used to check the validity of the following statement is correct:
p: "If x^2 is irrational, then x is rational."
The statement is true because the number $x^2 = \pi^2$ is irrational, therefore $x = \pi$ is irrational.
Solution:
Argument Used: $x^2 = \pi^2$ is irrational, therefore $x = \pi$ is irrational.
p: "If x^2 is irrational, then x is rational."

Let us take an irrational number given by $x = \sqrt{k}$, where k is a rational number.
Squaring both sides, we get,
$x^2 = k$
x^2 is a rational number and contradicts our statement.
Hence, the given argument is wrong.