EXERCISE 23.11

Prove that the following sets of three lines are concurrent:
(i) 15x – 18y + 1 = 0, 12x + 10y – 3 = 0 and 6x + 66y – 11 = 0
(ii) 3x – 5y – 11 = 0, 5x + 3y – 7 = 0 and x + 2y = 0
Solution:
(i) 15x – 18y + 1 = 0, 12x + 10y – 3 = 0 and 6x + 66y – 11 = 0
Given:
15x – 18y + 1 = 0 …… (i)
12x + 10y – 3 = 0 …… (ii)
6x + 66y – 11 = 0 …… (iii)
Now, consider the following determinant:

$$\begin{vmatrix} 15 & -18 & 1 \\ 12 & 10 & -3 \\ 6 & 66 & -11 \end{vmatrix} = 15(-110+198)+18(-132+18)+1(792-60)$$

=> 1320 – 2052 + 732 = 0
Hence proved, the given lines are concurrent.

(ii) 3x – 5y – 11 = 0, 5x + 3y – 7 = 0 and x + 2y = 0
Given:
3x – 5y – 11 = 0 …… (i)
5x + 3y – 7 = 0 …… (ii)
x + 2y = 0 …… (iii)
Now, consider the following determinant:

$$\begin{vmatrix} 3 & -5 & -11 \\ 5 & 3 & -7 \\ 1 & 2 & 0 \end{vmatrix} = 3 \times 14 + 5 \times 7 - 11 \times 7 = 0$$

Hence, the given lines are concurrent.

2. For what value of λ are the three lines 2x – 5y + 3 = 0, 5x – 9y + λ = 0 and x – 2y + 1 = 0 concurrent?
Solution:
Given:
2x − 5y + 3 = 0 … (1)
5x − 9y + λ = 0 … (2)
x − 2y + 1 = 0 … (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$\therefore \begin{vmatrix} 2 & -5 & 3 \\ 5 & -9 & \lambda \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$2(-9 + 2\lambda) + 5(5 - \lambda) + 3(-10 + 9) = 0$
$-18 + 4\lambda + 25 - 5\lambda - 3 = 0$
$\lambda = 4$
∴ The value of λ is 4.

3. Find the conditions that the straight lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ may meet in a point.
Solution:
Given:
$m_1x - y + c_1 = 0$ … (1)
$m_2x - y + c_2 = 0$ … (2)
$m_3x - y + c_3 = 0$ … (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$\therefore \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$$

$m_1(-c_3 + c_2) + 1(m_2c_3 - m_3c_2) + c_1(-m_2 + m_3) = 0$
$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$
∴ The required condition is $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

4. If the lines $p_1x + q_1y = 1$, $p_2x + q_2y = 1$ and $p_3x + q_3y = 1$ be concurrent, show that the points (p_1, q_1), (p_2, q_2) and (p_3, q_3) are collinear.
Solution:
Given:
$p_1x + q_1y = 1$
$p_2x + q_2y = 1$
$p_3x + q_3y = 1$
The given lines can be written as follows:
$p_1 x + q_1 y - 1 = 0$ … (1)
$p_2 x + q_2 y - 1 = 0$ … (2)
$p_3 x + q_3 y - 1 = 0$ … (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$\begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0$$

$$-\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

Hence proved, the given three points, (p_1, q_1), (p_2, q_2) and (p_3, q_3) are collinear.

5. Show that the straight lines $L_1 = (b + c)x + ay + 1 = 0$, $L_2 = (c + a)x + by + 1 = 0$ and $L_3 = (a + b)x + cy + 1 = 0$ are concurrent.

Solution:

Given:

$L_1 = (b + c)x + ay + 1 = 0$

$L_2 = (c + a)x + by + 1 = 0$

$L_3 = (a + b)x + cy + 1 = 0$

The given lines can be written as follows:

$(b + c)\, x + ay + 1 = 0$ … (1)

$(c + a)\, x + by + 1 = 0$ … (2)

$(a + b)\, x + cy + 1 = 0$ … (3)

Consider the following determinant.

$$\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix}$$

Let us apply the transformation $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = \begin{vmatrix} a+b+c & a & 1 \\ c+a+b & b & 1 \\ a+b+c & c & 1 \end{vmatrix}$$

$$\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = 0$$

Hence proved, the given lines are concurrent.