Hence the total number of words in which vowels come together are 120×2 = 240 words.

(v) All consonants come together

There are 4 consonants V,W,L,S. consider this a group.

Therefore, a permutation of 3 groups is 3! = 6 ways.

The group of consonants also can be arranged in 4! = 24 ways.

Hence, the total number of words in which consonants come together is 6×24 = 144 words.

Q. 24. How many numbers divisible by 5 and lying between 3000 and 4000 can be formed by using the digits 3, 4, 5, 6, 7, 8 when no digit is repeated in any such number?

Answer : For a number to be divisible by 5, the last digit should either be 5 or 0.

In this case, 5 is only possible.

For a four digit number to be between 3000 to 4000, in this case, should start with 3.

Therefore, the other 2 digits can be arranged by 4 numbers in P(4,2)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 2 places is

P(4,2) = $\frac{4!}{(4-2)!}$

= $\frac{4!}{2!}$ = $\frac{24}{2}$ = 12.

Therefore, there are 12 numbers present between 3000 to 4000 formed by using numbers 3,1,5,6,7,8.

Q. 25. In an examination, there are 8 candidates out of which 3 candidates have to appear in mathematics and the rest in different subjects. In how many ways can they are seated in a row if candidates appearing in mathematics are not to sit together?

Answer : Candidates in mathematics are not sitting together = total ways – the

Students are appearing for mathematic sit together.

The total number of arrangements of 8 students is 8! = 40320

When students giving mathematics exam sit together, then consider

Them as a group.

Therefore, 6 groups can be arranged in P(6,6) ways.

The group of 3 can also be arranged in 3! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, total arrangement are

$P(6,6) \times 3! = \frac{6!}{(6-6)!} \times 3!$

$= \frac{6!}{0!} \times 3! = \frac{720}{1} \times 6 = 4320.$

The total number of possibilities when all the students giving

Mathematics exam sits together is 4320 ways.

Therefore, number of ways in which candidates appearing

Mathematics exam is 40320 – 4320 = 36000.

Q. 26. In how many ways can 5 children be arranged in a line such that

(i) two of them, Rajan and Tanvy, are always together?
(ii) two of them, Rajan and Tanvy, are never together,

Answer : (i) two of them, Rajan and Tanvy, are always together

Consider Rajan and Tanvy as a group which can be arranged in 2! = 2 ways.

The 3 children with this 1 group can be arranged in 4! = 24 ways.

The total number of possibilities in which they both come together is 2×24 = 48 ways.

(ii) Two of them, Rajan and Tanvy, are never together

Two of them are never together = total number of possible ways of sitting – total number of ways in which they sit together.

A total number of possible way of arrangement of 5 students is 5! = 120 ways.

Therefore, the total number of arrangement when they both don't sit together is = 120 – 48 = 72.

Q. 27. when a group photograph is taken, all the seven teachers should be in the first row, and all the twenty students should be in the second row. If the tow corners of the second row are reserved for the two tallest students, interchangeable only between them, and if the middle seat of the front row is reserved for the principal, how many arrangements are possible?

Answer : For the first row:

There are 7 teachers in which the position of principal is fixed.

Therefore, the teachers can be arranged in p(7,7) = 5040.

For the second row:

The tallest students are at the ends and can be arranged in 2! = 2 ways.

Rest 18 students can be arranged in P(18,18) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 18 different objects in 18 places is

$P(18,18) = \frac{18!}{(18-18)!}$

$= \frac{18!}{0!} = \frac{18!}{1} = 18!$

Therefore, a total number of arrangements of the second row is 2 × 18!

Total arrangements = 2 × 18! × 5040 = 10080 × 18!

The total number of arrangements is 10080 × 18!

Q. 28. Find the number of ways in which m boys and n girls may be arranged in a row so that no two of the girls are together; it is given that m > n.

Answer : In this question, n girls are to be seated alternatively between m boys.

There are m+1 spaces in which girls can be arranged.

1	2	3	4	5	....	m-1	m	m+1
Girl	boy	girl	boy	girl		girl	boy	girl

The number of ways of arranging n girls is $P(m+1,n) = \frac{(m+1)!}{(m-n+1)!}$ ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of n different objects in m+1 places is

$P(m+1,n) = \frac{(m+1)!}{(m+1-n)!}$

$= \frac{(m+1)!}{(m-n+1)!}$

The arrangement of m boys can be done in P(m, m) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n, r) = n!/(n-r)!

Therefore, a permutation of m different objects in m places is

P(m, m) = $\frac{m!}{(m-m)!}$ = $\frac{m!}{0!}$ = m!

Therefore the total number of arrangements is $\frac{(m+1)!}{(m-n+1)!} \times m!$.

Exercise 8E

Q. 1. Find the total number of permutations of the letters of each of the words given below:

(i) APPLE (ii) ARRANGE
(iii) COMMERCE (iv) INSTITUTE
(v) ENGINEERING
(vi) INTERMEDIATE

Answer : To find: number of permutations of the letters of each word

Number of permutations of n distinct letters is n!

Number of permutations of n letters where r letters are of one kind, s letters of another kind, t letters of a third kind and so on = $\frac{n!}{r!s!t!\ldots}$

(i) Here n = 5

P is repeated twice

So the number of permutations = $\frac{5!}{2!} = 5 \times 4 \times 3 = 60$

(ii) Here n = 7

A is repeated twice, and R is repeated twice

So, the number of permutations = $\frac{7!}{2!.2!} = \frac{7\times6\times5\times4\times3}{2} = 1260$

(iii) Here n = 8

M and E are repeated twice

So, the number of permutations = $\frac{8!}{2!2!} = \frac{8\times7\times6\times5\times4\times3\times2}{4} = 10080$

(iv) Here n = 9

I is repeated twice, T is repeated thrice

So, the number of permutations = $\frac{9!}{2!3!} = 30240$

(v) Here n = 11

E, N is repeated thrice, I,G are repeated twice

So the number of permutations = $\frac{11!}{3!3!2!2!} = 277200$

(vi) Here n = 12

I and T are repeated twice, E is repeated thrice

So, the number of permutations = $\frac{12!}{2!2!3!} = 19958400$

Q. 2. In how many ways can the letters of the expression $x^2y^2z^4$ be arranged when written without using exponents?

Answer : To find: number of ways the letters can be arranged

The following table shows the possible arrangements

Power Alphabet	2	2	4
Case 1	x	y	Z
Case 2	x	z	Y
Case 3	y	z	X
Case 4	y	x	Z
Case 5	z	x	Y
Case 6	z	y	X

However, we see that case 1 = $x^2y^2z^4$ is the same as case 4 = $y^2x^2z^4$

Similarly (case2,case 5), (case 3,case 6) are the same

So there are only 3 distinct cases

Hence the letters can be arranged in 3 distinct ways

Q. 3. There are 3 blue balls, 4 red balls and 5 green balls. In how many ways can they are arranged in a row?

Answer : To find: no of ways in which the balls can be arranged in a row where some balls are of the same kind

Total number of balls = 3+4+5 = 12

3 are of 1 kind, 4 are of another kind, 5 are of the third kind

Number of ways = $\frac{12!}{3!4!5!} = 27720$

They can be arranged in 27720 ways

Q. 4. A child has three plastic toys bearing the digits 3, 3, 5 respectively. How many 3 - digit numbers can he make using them?

Answer : To find: number of 3 digit numbers he can make

If all were distinct, he could have made 3! = 6 numbers

But 2 number are the same

So the number of possibilities = $\frac{3!}{2!} = \frac{6}{2} = 3$

He can make 3 three - digit numbers using them

Q. 5. How many different signals can be transmitted by arranging 2 red, 3 yellow and 2 green flags on a pole, if all the seven flags are used to transmit a signal?

Answer : To find: Number of distinct signals possible

Total number of fags = 7

2 are of 1 kind, 3 are of another kind, and 2 are of the 3^{rd} kind

⇒ Number of distinct signals = $\frac{7!}{2!3!2!} = 210$

Hence 210 different signals can be made

Q. 6. How many words can be formed by arranging the letters of the word ‘ARRANGEMENT’, so that the vowels remain together?

Answer : To find: number of words where vowels are together

Vowels in the above word are: A,A,E,E

Consonants in the above word: R,R,N,G,M,N,T

Let us denote the all the vowels by a single letter say Z

⇒ The word now has the letters, R,R,N,G,M,N,T,Z

R and N are repeated twice

Number of permutations = $\frac{8!}{2!2!}$

Now Z is comprised of 4 letters which can be permuted amongst themselves

A and E are repeated twice

⇒ Number of permutations of Z = $\frac{4!}{2!2!}$

⇒ Total number of permutations = $\frac{8!\times 4!}{2!^4} = 60480$

The number of words that can be formed is 60480

Q. 7. How many words can be formed by arranging the letters of the word 'INDIA', so that the vowels are never together?

Answer : To find: Number of words that can be formed so that vowels are never together

Number of words such that vowels are never the together = Total number of words - Number of words where vowels are together

Total number of words = $\frac{5!}{2!} = 60$

To find a number of words where vowels are together

Let the vowels I, I, A be represented by a single letter Z

⇒the new word is NDZ

A number of permutations = 3! = 6

Z is composed of 3 letters which can be permuted amongst themselves.

Number of permutations of Z = $\frac{3!}{2!} = 3$

Number of words where vowels are together = 6×3 = 18

⇒ Number of words where vowels are not together = 60 - 18 = 42

There are 42 words where vowels are not together

Q. 8. Find the number of arrangements of the letters of the word 'ALGEBRA' without altering the relative positions of the vowels and the consonants.

Answer : To find: number of arrangements without changing the relative position

The following table shows where the vowels and consonants can be placed

Consonants can be placed in the blank places

vowel				vowel			vowel

There are 3 spaces for vowels

There are 3 vowels out of which 2 are alike

Vowels can be placed in $\frac{3!}{2!} =$ 3 ways

There are 4 consonants, and they can be placed in $4!$ = 24 ways

⇒ Total number of arrangements = 24×3 = 72 ways

72 arrangements can be made

Q. 9. How many words can be formed from the letters of the word 'SERIES', which start with S and end with S?

Answer : To find: number of words which start and end with S

S	....	....	....	....	S

There are 4 places to fill up with 4 letters out of which 2 are of the same kind

⇒Number of words = $\frac{4!}{2!} = 12$

12 words are possible

Q. 10. In how many ways can the letters of the word 'PARALLEL' be arranged so that all L's do not come together?

Answer : To find: number of words where L do not come together

Let the three L's be treated as a single letter say Z

Number of words with L not the together = Total number of words - Words with L's together

The new word is PARAEZ

Total number of words = $\frac{8!}{2!3!} = 3360$

Words with L together = $6! = 720$

⇒ Words with L, not together = 3360 – 720 = 2640

There are 2640 words where L do not come together

Q. 11. How many different words can be formed with the letters of the word 'CAPTAIN'? In how many of these C and T are never together?

Answer : To find: number of words such that C and T are never together

Number of words where C and T are never the together = Total numbers of words - Number of words where C and T are together

Total number of words = $\frac{7!}{2!} = 2520$

Let C and T be denoted by a single letter Z

⇒New word is APAINZ

This can be permuted in $\frac{6!}{2!}$ = 360 ways

Z can be permuted among itself in 2 ways

⇒ Number of words where C and T are together = 360 × 2 = 720

⇒ Number of words where C and T are never together = 2520 - 720 = 1800

There are 1800 words where C and T are never together

Q. 12. In how many ways can the letters of the word 'ASSASSINATION' be arranged so that all S's are together?

Answer : To find: number of ways letters can be arranged such that all S's are together

Let all S's be represented by a single letter Z

New word is AAINATIONZ

Number of arrangements = $\frac{10!}{3!2!2!} = 151200$

Letters can be arranged in 151200 ways

Q. 13. (i)How many arrangements can be made by using all the letters of the word 'MATHEMATICS'?

(ii) How many of them begin with C?
(iii) How many of them begin with T?

Answer : (i) There are 11 letters of which 2 are of 1 kind, 2 are of another kind, 2 are of the 3rd kind

Total number of arrangements = $\frac{11!}{2!2!2!} = 4989600$

(ii)

C	....	....	....	....	....	...	....	....	...	...

There are 10 spaces to be filled by 10 letters of which 2 are of 3 different kinds

Number of arrangements = $\frac{10!}{2!2!2!} =$ 453600

(iii)

T	...	....	....	....	....	....	....	...	....	...

There are 10 spaces to be filled by 10 letters of which 2 are of 2 different kinds

Number of arrangements = $\frac{10!}{2!2!} =$ 907200

Q. 14. In how many ways can the letters of the word 'INTERMEDIATE' be arranged so that:

(i) The vowels always occupy even places?
(ii) The relative orders of vowels and consonants do not change?

Answer : (i)

	Even		Even		Even		Even		Even		even

There are 6 even places and 6 vowels out of which 2 are of 1 kind, 3 are of the 2^{nd} kind

The vowels can be arranged in $\frac{6!}{2!3!} =$ 60

There are 6 consonants out of which 2 is of one kind

Number of permutations = $\frac{6!}{2!} =$ 360

⇒ Total number of words = $360 \times 60 = 21600$

(ii)

Vowel			Vowel			Vowel		Vowel	Vowel		vowel

There are 6 vowels to arrange in $\frac{6!}{2!3!}$

There are 6 consonants which can be arranged in $\frac{6!}{2!}$

⇒ Total number of ways = $\frac{6!}{2!3!} \times \frac{6!}{2!} =$ 21600

Q. 15. (i)Find the number of different words by using all the letters of the word, 'INSTITUTION'.

In how many of them
(ii) are the three T' s together
(iii) are the first two letters the two N' s?

Answer : (i) There are 11 letters of which 3 are of 1 kind, 2 are of the 2nd kind ,3 are of the 3rd kind

Number of arrangements = $\frac{11!}{3!2!3!} =$ 554400

(ii) Let all the three T's be denoted by a single letter Z

New word is INSIUIONZ

Number of permutations = $\frac{9!}{3!2!} =$ 30240

(iii)

N	N	…	…	…	…	…	…	…	…	…

There are 9 places to be filled by 9 letters of which 3 are of 2 different kinds

Number of permutations = $\frac{9!}{3!3!} = 10080$

Q. 16. How many five - digit numbers can be formed with the digits 5, 4, 3, 5, 3?

Answer : To find: Number of 5 - digit numbers that can be formed

2 numbers are of 1 kind, and 2 are of another kind

Total number of permutations = $\frac{5!}{2!2!} = 30$

30 number can be formed

Q. 17. How many numbers can be formed with the digits 2, 3, 4, 5, 4, 3, 2 so that the odd digits occupy the odd places?

Answer : The table shows the places where the odd digits can be placed

Odd		odd		odd		Odd

There are 4 places

And 3 odd digits out of which 2 are of the same kind

Choose any 3 places out of the four places in ${}^4C_3$ ways = 4 ways

In each way, the 3 digits can be placed in $\frac{3!}{2!}$ ways = 3 ways

⇒ Total number of ways in which odd digits occupy odd places = 4×3 = 12

Now there are 4 remaining digits out of which 2 are same of 1 kind, and 2 are same as another kind

⇒ They can be arranged in the remaining places in $\frac{4!}{2!2!}$ = 6 ways

⇒ Total number of numbers where odd digit occupies odd places = 12 × 6 = 72

There are 72 such numbers

Q. 18. How many 7 - digit numbers can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Answer : To find: number of 7 digit

0 cannot be in the first place because that would make a 6 digit number

Total number of 7 - digit numbers = Total number of number possible - Number of numbers with 0 at the first place

Total number of numbers possible = $\frac{7!}{3!2!} = 420$

Number of numbers with 0 at first place = $\frac{6!}{3!2!} = 60$

⇒ Number of 7 - digit numbers = 420 - 60 = 360

360 seven - digit numbers are possible

Q. 19. How many 6 - digit numbers can be formed by using the digits 4, 5, 0, 3, 4, 5?

Answer : To find: number of 6 digit

0 cannot be in the first place because that would make a 5 - digit number

Total number of 6 - digit numbers = Total number of number possible - Number of numbers with 0 at the first place

Total number of numbers possible = $\frac{6!}{2!2!} = 180$

Number of numbers with 0 at first place = $\frac{5!}{2!2!} = 30$

⇒ Number of 6 - digit numbers = 180 - 30 = 150

150 six - digit numbers are possible

Q. 20. The letters of the word 'INDIA' are arranged as in a dictionary. What are the 1^{st}, 13^{th}, 49^{th} and 60^{th} words?

Answer : Alphabetical arrangement of letters: A,D,I,N

⇒ 1^{st} word: ADIIN

To find other words:

Case 1: words starting with A

Number of words = $\frac{4!}{2!} = 12$

⇒ 13^{th} word starts with D and is DAIIN

Case 2: words starting with D

Number of words = $\frac{4!}{2!} = 12$

Case 3: Words starting with I

Number of words = $4! = 24$

⇒ $(12+12+24+1)^{th}$ = 49^{th} word starts with N and is NAIID

Case 4: Words starting with N

Number of words = $\frac{4!}{2!} = 12$

⇒ $(48+12)^{th}$ word is the last word which starts with N

⇒ 60^{th} word = NDIIA

1st word: ADIIN

13^{th} word: DAIIN

9^{th} word: NAIID

60th word: NDIIA

Exercise 8F

Q. 1. A child has 6 pockets. In how many ways, he can put 5 marbles in his pocket?

Answer : The first marble can be put into the pockets in 6 ways,

i.e. Choose 1 Pocket From 6 by ${}^6C_1=6$

Similarly second, third, Fourth, fifth & Sixth marble. Thus, the number of ways in which the child can put the marbles is 6^5

Q. 2. In how many ways can 5 bananas be distributed among 3 boys, there being no restriction to the number of bananas each boy may get?

Answer : As there is 5 banana, So suppose it as B_1, B_2, B_3, B_4, B_5 And Let the Boy be A_1, A_2, A_3

So B_1 can Be distributed to 3 Boys (A_1, A_2, A_3) by 3 ways,

Similarly, B_2, B_3, B_4, B_5 Can be distributed to 3 Boys by 3^4

So total number of ways is 3^5

Q. 3. In how many ways can 3 letters can be posted in 2 letterboxes?

Answer : Let Suppose Letterbox be B_1, B_2 and letters are L_1, L_2, L_3

So L_1 can be posted in any 2 letterboxes (B_1, B_2) by 2 ways

Similarly, L_2 can be posted in any 2 letterbox (B_1, B_2) by 2 ways

Similarly, L_3 can be posted in any 2 letterbox (B_1, B_2) by 2 ways

So total number of ways is $2^3 = 8$

Q. 4. How many 3-digit numbers are there when a digit may be repeated any numbers of time?

Answer : Let Suppose 3 digit number as 3 boxes as shown below. First Box is at 100thplace, the Second box is at 10th place, and the Third box be at 1st place.

1^{st}	2^{nd}	3^{rd}

To make a 3 digit number,

1st box can be filled with nine numbers(1, 2, 3, 4, 5, 6, 7, 8, 9) if we include 0 in 1st box then it become 2 digit number(i.e 010 is 2 digit number not 3 digit)

2nd box can be filled with ten numbers(1, 2, 3, 4, 5, 6, 7, 8, 9, 0) as repetition is allowed.

Similarly 3rd box can be filled with ten numbers(1, 2, 3, 4, 5, 6, 7, 8, 9, 0)

Total number of ways is $9 \times 10 \times 10 = 900$

Q. 5. How many 4-digit numbers can be formed with the digits 0, 2, 3, 4, 5 when a digit may be repeated any numbers of time in any arrangement?

Answer : Let Suppose 4 digit number as 4 boxes as shown below. First Box is at 1000thplace, the Second box is at 100th place, the Third box is at 10th place, and Fourth box is at 1st place.

1^{st}	2^{nd}	3^{rd}	4^{th}

The 1st box can be filled with four numbers(2, 3, 4, 5) if we include 0 in the 1stbox then it becomes 3 digit number(i.e. 0234 is 3 digit number, not 4 digits)

The 2nd box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.

Similarly, the 3rd box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.

Similarly, the 4th box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.

Total number of ways is $4 \times 5 \times 5 \times 5 = 500$

Q. 6. In how many ways can 4 prizes be given to 3 boys when a boy is eligible for all prizes?

Answer : Let suppose 4 prizes be P_1, P_2, P_3, P_4 and 3 boys be B_1, B_2, B_3

Now P_1 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways,

Similarly, P_2 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways,

Similarly, P_3 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways,

And P_4 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways

So total number of ways is $3 \times 3 \times 3 \times 3 = 81$

Q. 7. There are 4 candidates for the post of a chairman, and one is to be elected by votes of 5 men. In how many ways can the vote be given?

Answer : Let suppose 4 candidates be C_1, C_2, C_3, C_4 and 5 men be M_1, M_2, M_3, M_4, M_5

Now M_1 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

Similarly, M_2 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

Similarly, M_3 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

Similarly, M_4 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

And M_5 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

So total numbers of ways are $4 \times 4 \times 4 \times 4 \times 4 = 1024$

Exercise 8G

Q. 1. In how many ways can 6 persons be arranged in

(i) a line, (ii) a circle?

Answer : (i) Let choose 1 person from 6 by ${}^6C_1 = 6$ and arranged it in line