EXERCISE 1.7

For any two sets A and B, prove that: A' – B' = B – A Solution:
To prove, A' – B' = B – A
Firstly we need to show
A' – B' ⊆ B – A
Let, x ∈ A' – B'
⇒ x ∈ A' and x ∉ B'
⇒ x ∉ A and x ∈ B (since, A ∩ A' = ϕ)
⇒ x ∈ B – A
It is true for all x ∈ A' – B'
∴ A' – B' = B – A
Hence Proved.

1. For any two sets A and B, prove the following:
(i) A ∩ (A' ∪ B) = A ∩ B
(ii) A – (A – B) = A ∩ B
(iii) A ∩ (A ∪ B') = ϕ
(iv) A – B = A Δ (A ∩ B)
Solution:
(i) A ∩ (A' ∪ B) = A ∩ B
Let us consider LHS A ∩ (A' ∪ B)
Expanding
(A ∩ A') ∪ (A ∩ B)
We know, (A ∩ A') =ϕ
⇒ ϕ ∪ (A∩ B)
⇒ (A ∩ B)
∴ LHS = RHS
Hence proved.

(ii) A – (A – B) = A ∩ B
For any sets A and B we have De-Morgan's law
(A ∪ B)' = A' ∩ B', (A ∩ B) ' = A' ∪ B'
Consider LHS
= A – (A–B)
= A ∩ (A–B)'
= A ∩ (A∩B')'

$= A \cap (A' \cup B')')$ (since, $(B')' = B$)
$= A \cap (A' \cup B)$
$= (A \cap A') \cup (A \cap B)$
$= \phi \cup (A \cap B)$ (since, $A \cap A' = \phi$)
$= (A \cap B)$ (since, $\phi \cup x = x$, for any set)
= RHS
∴ LHS=RHS
Hence proved.

(iii) $A \cap (A \cup B)' = \phi$
Let us consider LHS $A \cap (A \cup B)'$
$= A \cap (A \cup B)'$
$= A \cap (A' \cap B')$ (By De–Morgan's law)
$= (A \cap A') \cap B'$ (since, $A \cap A' = \phi$)
$= \phi \cap B'$
$= \phi$ (since, $\phi \cap B' = \phi$)
= RHS
∴ LHS=RHS
Hence proved.

(iv) $A - B = A \Delta (A \cap B)$
Let us consider RHS $A \Delta (A \cap B)$
$A \Delta (A \cap B)$ (since, $E \Delta F = (E–F) \cup (F–E)$)
$= (A - (A \cap B)) \cup (A \cap B - A)$ (since, $E - F = E \cap F'$)
$= (A \cap (A \cap B)') \cup (A \cap B \cap A')$
$= (A \cap (A' \cup B')) \cup (A \cap A' \cap B)$ (by using De-Morgan's law and associative law)
$= (A \cap A') \cup (A \cap B') \cup (\phi \cap B)$ (by using distributive law)
$= \phi \cup (A \cap B') \cup \phi$
$= A \cap B'$ (since, $A \cap B' = A–B$)
$= A - B$
= LHS
∴ LHS=RHS
Hence Proved

2. If A, B, C are three sets such that $A \subset B$, then prove that $C - B \subset C - A$.
Solution:
Given, ACB
To prove: $C - B \subset C - A$
Let us consider, $x \in C–B$

⇒ $x \in C$ and $x \notin B$
⇒ $x \in C$ and $x \notin A$
⇒ $x \in C – A$
Thus, $x \in C–B \Rightarrow x \in C – A$
This is true for all $x \in C–B$
∴ $C – B \subset C – A$
Hence proved.

3. For any two sets A and B, prove that
(i) (A ∪ B) – B = A – B
(ii) A – (A ∩ B) = A – B
(iii) A – (A – B) = A ∩ B
(iv) A ∪ (B – A) = A ∪ B
(v) (A – B) ∪ (A ∩ B) = A
Solution:
(i) $(A \cup B) – B = A – B$
Let us consider LHS $(A \cup B) – B$
$= (A–B) \cup (B–B)$
$= (A–B) \cup \phi$ (since, $B–B = \phi$)
$= A–B$ (since, $x \cup \phi = x$ for any set)
= RHS
Hence proved.

(ii) $A – (A \cap B) = A – B$
Let us consider LHS $A – (A \cap B)$
$= (A–A) \cap (A–B)$
$= \phi \cap (A – B)$ (since, $A-A = \phi$)
$= A – B$
= RHS
Hence proved.

(iii) $A – (A – B) = A \cap B$
Let us consider LHS $A – (A – B)$
Let, $x \in A – (A–B) = x \in A$ and $x \notin (A–B)$
$x \in A$ and $x \notin (A \cap B)$
$= x \in A \cap (A \cap B)$
$= x \in (A \cap B)$
$= (A \cap B)$
= RHS

Hence proved.

(iv) A ∪ (B – A) = A ∪ B
Let us consider LHS A ∪ (B – A)
Let, x ∈ A ∪ (B –A) ⇒ x ∈ A or x ∈ (B – A)
⇒ x ∈ A or x ∈ B and x ∉ A
⇒ x ∈ B
⇒ x ∈ (A ∪ B) (since, B ⊂ (A ∪ B))
This is true for all x ∈ A ∪ (B–A)
∴ A ∪ (B–A) ⊂ (A ∪ B)…… (1)

Conversely,
Let x ∈ (A ∪ B) ⇒ x ∈ A or x ∈ B
⇒ x ∈ A or x ∈ (B–A) (since, B ⊂ (A ∪ B))
⇒ x ∈ A ∪ (B–A)
∴ (A ∪ B) ⊂ A ∪ (B–A)…… (2)
From 1 and 2 we get,
A ∪ (B – A) = A ∪ B
Hence proved.

(v) (A – B) ∪ (A ∩ B) = A
Let us consider LHS (A – B) ∪ (A ∩ B)
Let, x ∈ A
Then either x ∈ (A–B) or x ∈ (A ∩ B)
⇒ x ∈ (A–B) ∪ (A ∩ B)
∴ A ⊂ (A – B) ∪ (A ∩ B)…. (1)

Conversely,
Let x ∈ (A–B) ∪ (A ∩ B)
⇒ x ∈ (A–B) or x ∈ (A ∩ B)
⇒ x ∈ A and x ∉ B or x ∈ B
⇒ x ∈ A
(A–B) ∪ (A ∩ B) ⊂ A… .... (2)
∴ From (1) and (2), We get
(A–B) ∪ (A ∩ B) = A
Hence proved.