EXERCISE 8.5

Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and remainder:

(i) $3x^2 + 4x + 5$, $x - 2$

(ii) $10x^2 – 7x + 8$, $5x - 3$

(iii) $5y^3 – 6y^2 + 6y – 1$, $5y – 1$

(iv) $x^4 – x^3 + 5x$, $x - 1$

(v) $y^4 + y^2$, $y^2 – 2$

Solution:

(i) $3x^2 + 4x + 5$, $x - 2$

By using long division method

$$\begin{array}{r|lll} & 3x & +10 & \\ \hline x-2 & 3x^2 & +4x & +5 \\ - & 3x^2 & -6x & \\ \hline & & 10x & +5 \\ & - & 10x & -20 \\ \hline & & & 25 \end{array}$$

∴ the Quotient is $3x + 10$ and the Remainder is 25.

(ii) $10x^2 – 7x + 8$, $5x - 3$

By using long division method

$$\begin{array}{r|lll} & 2x & -\frac{1}{5} & \\ \hline 5x-3 & 10x^2 & -7x & +8 \\ - & 10x^2 & -6x & \\ \hline & & -x & +8 \\ & - & -x & +\frac{3}{5} \\ \hline & & & \frac{37}{5} \end{array}$$

∴ the Quotient is $2x – 1/5$ and the Remainder is $37/5$.

(iii) $5y^3 - 6y^2 + 6y - 1$, $5y - 1$
By using long division method

$$
\begin{array}{r|l}
 & y^2 \quad -y \quad +1 \\
\hline
5y - 1 & 5y^3 \quad -6y^2 \quad +6y \quad -1 \\
 & - \\
 & 5y^3 \quad -y^2 \\
\hline
 & \qquad -5y^2 \quad +6y \quad -1 \\
 & \qquad - \\
 & \qquad -5y^2 \quad +y \\
\hline
 & \qquad\qquad 5y \quad -1 \\
 & \qquad\qquad - \\
 & \qquad\qquad 5y \quad -1 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

∴ the Quotient is $y^2 - y + 1$ and the Remainder is 0.

(iv) $x^4 - x^3 + 5x$, $x - 1$
By using long division method

$$
\begin{array}{r|l}
 & x^3 \quad +5 \\
\hline
x - 1 & x^4 \quad -x^3 \quad +0x^2 \quad +5x \quad +0 \\
 & - \\
 & x^4 \quad -x^3 \\
\hline
 & \qquad 0 \quad +0x^2 \quad +5x \quad +0 \\
 & \qquad - \\
 & \qquad 5x^3 \quad -5x^2 \\
\hline
 & \qquad -5x^3 \quad +5x^2 \quad +5x \quad +0
\end{array}
$$

∴ the Quotient is $x^3 + 5$ and the Remainder is 5.

(v) $y^4 + y^2$, $y^2 - 2$
By using long division method

$$
\begin{array}{r|l}
 & y^2 \quad +3 \\
\hline
y^2 - 2 & y^4 \quad +0y^3 \quad +y^2 \quad +0y \quad +0 \\
 & - \\
 & y^4 \quad +0y^3 \quad -2y^2 \\
\hline
 & \qquad\qquad 3y^2 \quad +0y \quad +0 \\
 & \qquad\qquad - \\
 & \qquad\qquad 3y^2 \quad +0y \quad -6 \\
\hline
 & \qquad\qquad\qquad\qquad 6
\end{array}
$$

∴ the Quotient is $y^2 + 3$ and the Remainder is 6.

2. Find Whether or not the first polynomial is a factor of the second:
(i) $x + 1$, $2x^2 + 5x + 4$
(ii) $y - 2$, $3y^3 + 5y^2 + 5y + 2$
(iii) $4x^2 - 5$, $4x^4 + 7x^2 + 15$
(iv) $4 - z$, $3z^2 - 13z + 4$
(v) $2a - 3$, $10a^2 - 9a - 5$
(vi) $4y + 1$, $8y^2 - 2y + 1$
Solution:
(i) $x + 1$, $2x^2 + 5x + 4$
Let us perform long division method,

$$
\begin{array}{r|l}
 & 2x \quad +3 \\
\hline
x + 1 & 2x^2 \quad +5x \quad +4 \\
 & - \\
 & 2x^2 \quad +2x \\
\hline
 & \qquad\quad 3x \quad +4 \\
 & \qquad\quad - \\
 & \qquad\quad 3x \quad +3 \\
\hline
 & \qquad\qquad\quad 1
\end{array}
$$

Since remainder is 1 therefore the first polynomial is not a factor of the second polynomial.

(ii) $y - 2$, $3y^3 + 5y^2 + 5y + 2$
Let us perform long division method,

$$
\begin{array}{r|llll}
 & 3y^2 & +11y & +27 & \\
\hline
y-2 & 3y^3 & +5y^2 & +5y & +2 \\
- & 3y^3 & -6y^2 & & \\
\hline
 & & 11y^2 & +5y & +2 \\
 & - & 11y^2 & -22y & \\
\hline
 & & & 27y & +2 \\
 & & - & 27y & -54 \\
\hline
 & & & & 56
\end{array}
$$

Since remainder is 56 therefore the first polynomial is not a factor of the second polynomial.

(iii) $4x^2 - 5$, $4x^4 + 7x^2 + 15$

Let us perform long division method,

$$
\begin{array}{r|lllll}
 & x^2 & +3 & & & \\
\hline
4x^2-5 & 4x^4 & +0x^3 & +7x^2 & +0x & +15 \\
- & 4x^4 & +0x^3 & -5x^2 & & \\
\hline
 & & & 12x^2 & +0x & +15 \\
 & & - & 12x^2 & +0x & -15 \\
\hline
 & & & & & 30
\end{array}
$$

Since remainder is 30 therefore the first polynomial is not a factor of the second polynomial.

(iv) $4 - z$, $3z^2 - 13z + 4$

Let us perform long division method,

$$\begin{array}{r|l} & -3z \quad +1 \\ \hline -z+4 & 3z^2 \quad -13z \quad +4 \\ & -\ (3z^2 \quad -12z) \\ \hline & \qquad -z \quad +4 \\ & \qquad -\ (-z \quad +4) \\ \hline & \qquad\qquad 0 \end{array}$$

Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a - 3$, $10a^2 - 9a - 5$
Let us perform long division method,

$$\begin{array}{r|l} & 5a \quad +3 \\ \hline 2a-3 & 10a^2 \quad -9a \quad -5 \\ & -\ (10a^2 \quad -15a) \\ \hline & \qquad 6a \quad -5 \\ & \qquad -\ (6a \quad -9) \\ \hline & \qquad\qquad 4 \end{array}$$

Since remainder is 4 therefore the first polynomial is not a factor of the second polynomial.

(vi) $4y + 1$, $8y^2 - 2y + 1$
Let us perform long division method,

$$\begin{array}{r|l} & 2y \quad -1 \\ \hline 4y+1 & 8y^2 \quad -2y \quad +1 \\ & - \\ & 8y^2 \quad +2y \\ \hline & \quad -4y \quad +1 \\ & \quad - \\ & \quad -4y \quad -1 \\ \hline & \qquad\qquad 2 \end{array}$$

Since remainder is 2 therefore the first polynomial is not a factor of the second polynomial.