Chapter 20. Area and Perimeter of Plane Figures

Exercise 20(A)

Solution 1:

Since the sides of the triangle are 18cm,24cm and 30cm respectively.

$$s = \frac{18 + 24 + 30}{2}$$
$$= 36$$

Hence area of the triangle is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36(36-18)(36-24)(36-30)}$$
$$= \sqrt{36 \times 18 \times 12 \times 6}$$
$$= \sqrt{46656}$$
$$= 216 \text{sqcm}$$

Again

$$A = \frac{1}{2} \text{base} \times \text{altitude}$$

Hence

$$216 = \frac{1}{2} \times 30 \times h$$
$$h = 14.4 cm$$

Solution 2:

Let the sides of the triangle are

a=3x

b=4x

c=5x

Given that the perimeter is 144 cm.

hence

$$3x + 4x + 5x = 144$$
$$\Rightarrow 12x = 144$$
$$\Rightarrow x = \frac{144}{12}$$
$$\Rightarrow x = 12$$
$$s = \frac{a+b+c}{2} = \frac{12x}{2} = 6x = 72$$

The sides are a=36 cm, b=48 cm and c=60 cm

Area of the triangle is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{72(72-36)(72-48)(72-60)}$$
$$= \sqrt{72 \times 36 \times 24 \times 12}$$
$$= \sqrt{746496}$$
$$= 864\ cm^2$$

Solution 3:

(i)

Area of the triangle is given by

$$A = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 4 \times 4$$

$$= 8 \text{ sq.cm}$$

(ii)

Again area of the triangle

$$A = \frac{1}{2} \times BC \times h$$

$$8 = \frac{1}{2} \times \left(\sqrt{4^2 + 4^2}\right) \times h$$

$$h = 2.83 cm$$

Solution 4:

Area of an equilateral triangle is given by

$$\frac{\sqrt{3}}{4} \times (side)^2 = A$$

$$\frac{\sqrt{3}}{4} \times (side)^2 = 36\sqrt{3}$$

$$(side)^2 = 144$$

$$side = 12\ cm$$

Hence

$$\text{perimeter} = 3 \times (\text{its side})$$

$$= 3 \times 12$$

$$= 36\ cm$$

Solution 5:

Since the perimeter of the isosceles triangle is 36cm and base is 16cm.

hence the length of each of equal sides are $\dfrac{36-16}{2} = 10cm$

Here

It is given that

a = equa lsides = $10cm$

b = base = $16cm$

Let 'h' be the altitude of the isosceles triangle.

Since the altitude from the vertex bisects the base perpendicularly, we can apply Pythagoras Theorem.

Thus we have,

$$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{4a^2 - b^2}$$

We know that

$$\text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\text{Area of the triangle} = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 16 \times \sqrt{400 - 256}$$

$$= 48\text{sq.cm}$$

Solution 6:

It is given that

$Area = 192\ sq.cm$

$base = 24\ cm$

Knowing the length of equal side, a, and base, b, of an isosceles triangle, the area can be calculated using the formula,

$$A = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

Let 'a' be the length of an equal side.

$$\text{Area} = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$192 = \frac{1}{4} \times 24 \times \sqrt{4a^2 - 576}$$

$$192 = 6\sqrt{4a^2 - 576}$$

$$\sqrt{4a^2 - 576} = 32$$

$$4a^2 - 576 = 1024$$

$$4a^2 = 1600$$

$$a = 20cm$$

Hence perimeter= $20 + 20 + 24 = 64cm$

Solution 7:

From ΔABC,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{16^2 - 8^2}$$
$$= \sqrt{192}$$

Area of ΔABC

$$\Delta ABC = \frac{1}{2} \times 8 \times \sqrt{192}$$
$$= 4\sqrt{192}$$

Area of ΔBCD

$$\Delta BCD = \frac{\sqrt{3}}{4} \times 8^2$$
$$= 16\sqrt{3}$$

Now

$$\Delta ABD = \Delta ABC - \Delta BDC$$
$$= 4\sqrt{192} - 16\sqrt{3}$$
$$= 32\sqrt{3} - 16\sqrt{3}$$
$$= 16\sqrt{3} \text{sq.cm}$$

Solution 8:

Given , AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and $\angle$DBC = 90°

$$BC = \sqrt{DC^2 - BD^2}$$
$$= \sqrt{13^2 - 12^2}$$
$$= 5cm$$

Hence perimeter=8+10+13+5=36cm

Area of ΔABD

$$\Delta ABD = \sqrt{15(15-8)(15-10)(15-12)}$$
$$= \sqrt{15 \times 7 \times 5 \times 3}$$
$$= 15\sqrt{7}$$
$$= 39.7$$

Area of ΔDBC

$$\Delta BDC = \frac{1}{2} \times 12 \times 5$$
$$= 30$$

Now

Solution 9:

$$\text{Area of } ABCD = \text{area of } \Delta ABD + \text{area of } \Delta BDC$$
$$= 39.7 + 30$$
$$= 69.7 \text{sq. cm}$$

$$\text{Area of the rectangular field} = \frac{49572}{36.72} = 135000$$

Let the height of the triangle be x

$$135000 = \frac{1}{2} \times x \times 3x$$

$$\Rightarrow x^2 = 90000$$

$$\Rightarrow x = 300$$

Height $= 300$ m

Base $= 900$ m

Solution 10:

(i)

Given that the sides of a triangle are in the ratio 5:3:4.

Also, given that the perimeter of the triangle is 180.

Thus, we have, $5x + 4x + 3x = 180$

$\Rightarrow 12x = 180$

$\Rightarrow x = \frac{180}{12}$

$\Rightarrow x = 15$

Thus, the sides are 5×15, 3×15 and 4×15.

That is the sides are 75 m, 45 m and 60 m.

Since the sides are in the ratio, 5:3:4, it is a Pythagorean triplet.

Therefore, the triangle is a right angled triangle.

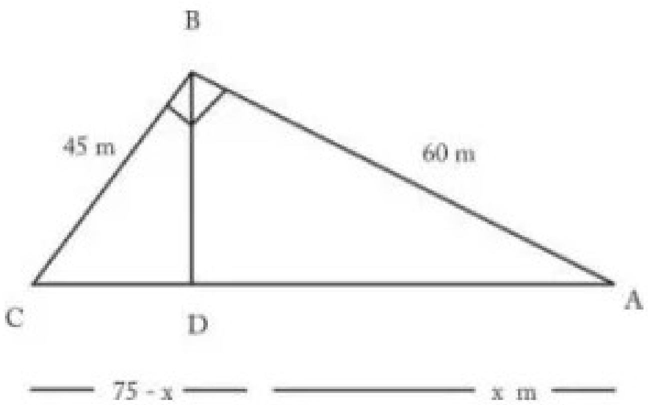
Area of a right angled triangle $= \frac{1}{2} \times \text{base} \times \text{altitude}$

$\Rightarrow = \frac{1}{2} \times 45 \times 60$

$\Rightarrow = 45 \times 30 = 1350\ m^2$

(ii)

Consider the following figure.



In the above figure,

The largest side is AC = 75 m.

The altitude corresponding to AC is BD.

We need to find the value of BD.

Now consider the triangles ΔBCD and ΔBAD.

We have,

$\angle B = \angle B$ [common]

$BD = BD$ [common]

$\angle D = \angle D = 90^\circ$

Thus, by Angle-Side-Angle criterion of congruence, we have ΔBCD ∼ ΔABD.

Similar triangles have similar proportionality.

Thus, we have,

$$\frac{CD}{BD} = \frac{BD}{AD}$$

$$\Rightarrow BD^2 = AD \times CD \ldots(1)$$

From subpart(i), the sides of the triangle are

AC = 75 m, AB = 60 m and BC = 45 m

Let AD = x m $\Rightarrow$ CD= $(75 - x)$ m

Thus applying Pythgoras Theroem,

from right triangle ΔBCD, we have

$$45^2 = (75 - x)^2 + BD^2$$

$$\Rightarrow BD^2 = 45^2 - (75 - x)^2$$

$$\Rightarrow BD^2 = 2025 - (5625 + x^2 - 150x)$$

$$\Rightarrow BD^2 = 2025 - 5625 - x^2 + 150x$$

$$\Rightarrow BD^2 = -3600 - x^2 + 150x \ldots(2)$$

Now applying Pythgoras Theroem,

from right triangle ΔABD, we have

$$60^2 = x^2 + BD^2$$

$$\Rightarrow BD^2 = 60^2 - x^2$$

$$\Rightarrow BD^2 = 3600 - x^2 \ldots(3)$$

From equations (2) and (3), we have,

$$-3600 - x^2 + 150x = 3600 - x^2$$

$$\Rightarrow 150x = 3600 + 3600$$

$$\Rightarrow 150x = 7200$$

$\Rightarrow x = \frac{7200}{150}$

$\Rightarrow x = 48$

Thus, AD = 48 and CD = 75 – 48 = 27

Substituting the values AD=48 m

and CD=27 m in equation (1), we have

$BD^2 = 48 \times 27$

$\Rightarrow BD^2 = 1296$

$\Rightarrow BD = 36$ m

The altitude of the triangle corresponding to

its largest side is BD = 36 m

(iii)

The area of the triangular field

from subpart(i) is 1350 m^2

The cost of levelling the field is

Rs.10 per square metre.

Thus, the total cost of

levelling the field = 1350×10 = Rs.13,500

Solution 11:

Let the height of the triangle be x cm.

Equal sides are (x+4) cm.

According to Pythagoras theorem,

$$(x+4)^2 = x^2 + 12^2$$

$$8x = 128$$

$$x = 16cm$$

Hence perimeter= $20 + 20 + 24 = 64cm$

Area of the isosceles triangle is given by

Here a=20cm

b=24cm

hence

$$Area = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 24 \times \sqrt{1024}$$

$$= 192 sq.cm$$

Solution 12:

Each side of the triangle is $\frac{60}{3} = 20cm$

Hence the area of the equilateral triangle is given by

$$A = \frac{\sqrt{3}}{4} \times 20^2$$
$$= 100\sqrt{3}$$
$$= 173.2\,sq.cm$$

The height h of the triangle is given by

$$\frac{1}{2} \times 20 \times h = 173.2$$
$$h = 17.32cm$$

Solution 13:

The area of the triangle is given as 150sq.cm

$$\frac{1}{2} \times x \times (x+5) = 150$$
$$x^2 + 5x - 300 = 0$$
$$(x+20)(x-15) = 0$$
$$x = 15$$

Hence AB=15cm,AC=20cm and

$$BC = \sqrt{15^2 + 20^2}$$
$$= 25cm$$

Solution 14:

Let the two sides be x cm and (x-3) cm.

Now

$$\frac{1}{2}\times x\times(x-3) = 54$$

$$x^2 - 3x - 108 = 0$$

$$(x-12)(x+9) = 0$$

$$x = 12cm$$

Hence the sides are 12cm, 9cm and $\sqrt{12^2 + 9^2} = 15cm$

The required perimeter is 12+9+15=36cm.

Solution 15:

$$\text{Area of } \Delta ABC = \frac{1}{4}\times 36\times\sqrt{4\times 30^2 - 36^2}$$

$$= \frac{1}{4}\times 36\times\sqrt{2304}$$

$$= \frac{1}{4}\times 36\times 48$$

$$= 432$$

Since AB=AC and $\angle BOC = 90^\circ$

$$\angle BOD = \angle COD = 45^\circ$$

hence $\angle OBD = 45^\circ$ and $OD = BD = 18cm$

Now

$$\text{Area of } \Delta BOC = \frac{1}{2}\times 36\times 18$$

$$= 324$$

$$\text{Area of } ABOC = \text{Area of } \Delta ABC - \text{Area of } \Delta BOC$$

$$= 432 - 324$$

$$= 108\,\text{sq.cm}$$