13. Method of Integration

Exercise 13A

1. Question

Evaluate the following integrals:

$$\int (2x+9)^5 dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x + 9 = t ⇒ 2 dx = dt

$$\int t^5 (\frac{dt}{2}) = \frac{1}{2}\int t^5 dt = \frac{1}{2}\frac{t^6}{6} + c = \frac{t^6}{12} + c$$

$$= \frac{(2x+9)^6}{12} + c$$

2. Question

Evaluate the following integrals:

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 7 - 3x = t ⇒ -3 dx = dt

$$\int t^4 (\frac{dt}{-3}) = \frac{1}{-3}\int t^4 dt = \frac{1}{-3}\frac{t^5}{5} + c = -\frac{t^5}{15} + c$$

$$= -\frac{(7-3x)^5}{15} + c$$

3. Question

Evaluate the following integrals:

$$\int \sqrt{3x-5}\, dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3x - 5 = t ⇒ 3 dx = dt

$$\int t^{0.5} (\frac{dt}{3}) = \frac{1}{3}\int t^{0.5} dt = \frac{1}{3} \times \frac{t^{1.5}}{1.5} + c = \frac{2}{1} \times \frac{t^{1.5}}{9} + c$$

$$= \frac{2(3x-5)^5}{9} + c$$

4. Question

Evaluate the following integrals:

$$\int \frac{1}{\sqrt{4x+3}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 4x + 3 = t ⇒ 4 dx = dt

$$\int t^{-0.5}(\frac{dt}{4}) = \frac{1}{4}\int t^{-0.5}dt = \frac{1}{4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{4} \times \frac{t^{0.5}}{1} + c$$

$$= \frac{\sqrt{4x+3}}{2} + c$$

5. Question

Evaluate the following integrals:

$$\int \frac{1}{\sqrt{3-4x}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3 - 4x = t ⇒ -4 dx = dt

$$\int t^{-0.5}(\frac{dt}{-4}) = \frac{1}{-4}\int t^{-0.5}dt = \frac{1}{-4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{-4} \times \frac{t^{0.5}}{1} + c$$

$$= -\frac{\sqrt{3-4x}}{2} + c$$

6. Question

Evaluate the following integrals:

$$\int \frac{1}{(2x-3)^{3/2}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x - 3 = t ⇒ 2 dx = dt

$$\int t^{-\frac{3}{2}}(\frac{dt}{2}) = \frac{1}{2}\int t^{-\frac{3}{2}}dt = \frac{1}{2} \times \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{2} \times \frac{t^{-0.5}}{1} + c$$

$$= -\frac{1}{\sqrt{2x-3}} + c$$

7. Question

Evaluate the following integrals:

$\int e^{(2x-1)} dx$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 2x – 1 = t ⇒ 2 dx = dt

$$\int e^t (\frac{dt}{2}) = \frac{1}{2}\int e^t dt = \frac{1}{2} \times e^t + c = \frac{e^{2x-1}}{2} + c$$

$$= \frac{e^{(2x-1)}}{2} + c$$

8. Question

Evaluate the following integrals:

$\int e^{(1-3x)} dx$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 1 - 3x = t ⇒ -3 dx = dt

$$\int e^t (\frac{dt}{-3}) = \frac{1}{-3}\int e^t dt = \frac{1}{-3} \times e^t + c = \frac{e^{1-3x}}{-3} + c$$

$$= -\frac{e^{(1-3x)}}{3} + c$$

9. Question

Evaluate the following integrals:

$\int 3^{(2-3x)} dx$

Answer

Formula = $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put 2 - 3x = t ⇒ -3 dx = dt

$$\int 3^t (\frac{dt}{-3}) = \frac{1}{-3}\int 3^t dt = \frac{1}{-3} \times (\frac{3^t}{\log 3}) + c = \frac{3^t}{-3\log 3} + c$$

$$= -\frac{3^{(2-3x)}}{3\log 3} + c$$

10. Question

Evaluate the following integrals:

$\int \sin 3x \, dx$

Answer

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put 3x = t ⇒ 3 dx = dt

$$\int \sin t\,(\frac{dt}{3}) = \frac{1}{3}\int \sin t\,dt = \frac{1}{3}\times(-\cos t) + c = \frac{-\cos 3x}{3} + c$$

$$= -\frac{\cos 3x}{3} + c$$

11. Question

Evaluate the following integrals:

$$\int \cos(5+6x)\,dx$$

Answer

Formula = $\int \cos x\,dx = \sin x + c$

Therefore ,

Put 5 + 6x = t ⇒ 6 dx = dt

$$\int \cos t\,(\frac{dt}{6}) = \frac{1}{6}\int \cos t\,dt = \frac{1}{6}\times(\sin t) + c = \frac{\sin 5 + 6x}{6} + c$$

$$= \frac{\sin(5+6x)}{6} + c$$

12. Question

Evaluate the following integrals:

$$\int \sin x\sqrt{1+\cos 2x}\,dx$$

Answer

Formula $\int \cos x\,dx = \sin x + c$

$1 + \cos 2x = 2cos^2 x$

Therefore ,

$$\int \sin x\sqrt{1+\cos 2x}\,dx = \int \sin x\sqrt{2}\cos x + c$$

$$\int \sqrt{2}\sin x\cos x\ \ dx$$

Put sin x =t ⇒ cos x dx = dt

$$\int \sqrt{2}\sin x\cos x\ \ dx = \int \sqrt{2}t\,dt = \sqrt{2}\ \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{\sqrt{2}} + c$$

13. Question

Evaluate the following integrals:

$$\int \mathrm{cosec}^2(2x+5)\,dx$$

Answer

Formula $\int cosec^2 x\,dx = -\cot x + c$

Therefore ,

Put 2x + 5 =t ⇒ 2 dx = dt

$$\int cosec^2 t\,\frac{dt}{2} = -\frac{1}{2}\cot t + c = -\frac{1}{2}\cot(2x+5) + c$$

$$= -\frac{1}{2}\cot(2x+5) + c$$

14. Question

Evaluate the following integrals:

$$\int \sin x \cos x \, dx$$

Answer

Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{2} + c$$

15. Question

Evaluate the following integrals:

$$\int \sin^3 x \cos x \, dx$$

Answer

Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t^3\,dt = \frac{t^4}{4} + c$$

$$= \frac{(\sin x)^4}{4} + c$$

16. Question

Evaluate the following integrals:

$$\int \left(\sqrt{\cos x}\right) \sin x \, dx$$

Answer

Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put cos x =t ⇒ -sin x dx = dt

$$\int t^{0.5}(-1)\,dt = -\frac{t^{1.5}}{1.5} + c$$

$$= -\frac{2(\cos x)^{\frac{3}{2}}}{3} + c$$

17. Question

Evaluate the following integrals:

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

Answer

Formula $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$ $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Therefore ,

Put $\sin^{-1} x$ =t ⇒ $\frac{1}{\sqrt{1-x^2}}$ dx = dt

$$\int t^1 dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

18. Question

Evaluate the following integrals:

$$\int \frac{\sin\left(2\tan^{-1} x\right)}{\left(1+x^2\right)} dx.$$

Answer

Formula $\int \sin t \, dx = -\cos t + c$ $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

Therefore ,

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\int \sin 2t \, dt = \frac{-\cos 2t}{2} + c$$

$$= -\frac{\cos(2\tan^{-1} x)}{2} + c$$

19. Question

Evaluate the following integrals:

$$\int \frac{\cos(\log x)}{x} dx$$

Answer

Formula $\int \cos t \, dx = \sin t + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\int \cos t\, dt = \sin t + c$

$= \sin(\log x) + c$

20. Question

Evaluate the following integrals:

$$\int \frac{\operatorname{cosec}^2(\log x)}{x} dx$$

Answer

Formula $\int cosec^2 x\, dx = -\cot x + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\int cosec^2 t \frac{dt}{1} = -\cot t + c = -\cot(\log x) + c$

$= -\cot(\log x) + c$

21. Question

Evaluate the following integrals:

$$\int \frac{1}{x \log x} dx$$

Answer

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\int \frac{dt}{t} = \log t + c = \log(\log x) + c$

$= \log(\log x) + c$

22. Question

Evaluate the following integrals:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = \int \frac{x+1}{x} \times \frac{(x+\log x)^2}{1} dx$$
$$= \int (1+\frac{1}{x}) \times \frac{(x+\log x)^2}{1} dx$$

Therefore ,

Put $x + \log x = t \Rightarrow (1 + \frac{1}{x}) dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(x + \log x)^3}{3} + c$$

23. Question

Evaluate the following integrals:

$$\int \frac{(\log x)^2}{x} dx$$

Answer

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$$

$$= \frac{(\log x)^3}{3} + c$$

24. Question

Evaluate the following integrals:

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

Answer

Formula $\int \cos t \, dx = \sin t + c$ $\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Therefore ,

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\int \cos t \; 2dt = 2 \sin t + c$$

$$= 2 \sin(\sqrt{x}) + c$$

25. Question

Evaluate the following integrals:

$$\int e^{\tan x} \sec^2 x \, dx$$

Answer

Formula = $\int e^x dx = e^x + c$ $\frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

Put tan x = t $\Rightarrow sec^2 x$ dx = dt

$$\int e^t dt = e^t + c$$

$= e^{\tan x} + c$

26. Question

Evaluate the following integrals:

$$\int e^{\cos^2 x} \sin 2x \, dx$$

Answer

Formula = $\int e^x dx = e^x + c$ $\frac{d(\cos^2 x)}{dx} = 2\cos x(-\sin x) = -\sin 2x$

Therefore ,

Put $\cos^2 x$ = t ⇒ $-\sin 2x$ dx = dt

$$\int -e^t dt = -e^t + c$$

$$= -e^{\cos^2 x} + c$$

27. Question

Evaluate the following integrals:

$$\int \sin(ax+b)\cos(ax+b)dx$$

Answer

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put ax+b = t ⇒ adx = dt

$$\int \sin t \cos t \frac{dt}{a} = \frac{1}{a}\int \sin t \cos t \, dt$$

Put sin t = z � cos t dt = dz

$$\frac{1}{a}\int z dz = \frac{1}{a} \times \frac{z^2}{2} + c$$

$$= \frac{(\sin ax + b)^2}{2a} + c$$

28. Question

Evaluate the following integrals:

$$\int \cos^3 x \, dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

$\cos 3x = 3\cos x - 4\cos^3 x$

Therefore ,

$$\int \left(\frac{3\cos x}{4} - \frac{\cos 3x}{4}\right)dx = \frac{3\sin x}{4} - \frac{\sin 3x}{4 \times 3} + c$$

$$= \frac{3\sin x}{4} - \frac{\sin 3x}{12} + c$$

29. Question

Evaluate the following integrals:

$$\int \frac{1}{x^2} e^{-1/x} dx$$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $-\frac{1}{x}$ = t ⇒ $\frac{1}{x^2} dx = dt$

$$\int e^t (dt) = \int e^t dt = e^t + c = e^{-\frac{1}{x}} + c$$

$$= e^{-\frac{1}{x}} + c$$

30. Question

Evaluate the following integrals:

$$\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put $-\frac{1}{x}$ = t ⇒ $\frac{1}{x^2} dx = dt$

$$\int \cos t \, (dt) = \int \cos t \, dt = \sin t + c = \sin(-\frac{1}{x}) + c$$

$$= -\sin\frac{1}{x} + c$$

31. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(e^x + e^{-x}\right)}$$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

$$\int \frac{e^x}{1 + e^{2x}} dx$$

Put e^x = t ⇒ $e^x dx = dt$

$$\int \frac{1}{1 + t^2} (dt) = \int \frac{1}{1 + t^2} dt = \tan^{-1} t + c$$

$$= \tan^{-1}(e^x) + c$$

32. Question

Evaluate the following integrals:

$$\int \frac{e^{2x}}{\left(e^{2x}-2\right)} dx$$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $e^{2x} - 2$ = t ⇒ $2e^{2x} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(e^{2x} - 2) + c$$

33. Question

Evaluate the following integrals:

$$\int \cot x \log(\sin x) dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x\, dx = dt$

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log \sin x)^2}{2} + c$$

34. Question

Evaluate the following integrals:

$$\int \frac{\cot x}{\log(\sin x)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x\, dx = dt$

$$\int \frac{1}{t} dt = \log t + c$$

$$= \log(\log \sin x) + c$$

35. Question

Evaluate the following integrals:

$$\int 2x \sin\left(x^2 + 1\right) dx$$

Answer

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put $x^2 + 1 = t \Rightarrow 2x \, dx = dt$

$$\int \sin t \; dt = -\cos t + c$$

$= -\cos(x^2 + 1) + c$

36. Question

Evaluate the following integrals:

$$\int \sec x \log(\sec x + \tan x) dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sec x + tan x)= t

$$\frac{1}{\sec x + \tan x} \times (\sec x \tan x + sec^2 x) \; dx = dt$$

$$\frac{1}{\sec x + \tan x} \times \sec x \; (\sec x + \tan x) \; dx = dt$$

Sec x dx = dt

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log(\sec x + \tan x))^2}{2} + c$$

37. Question

Evaluate the following integrals:

$$\int \frac{\tan \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\tan \sqrt{x}$= t

$$sec^2 \sqrt{x} \times (\frac{1}{2\sqrt{x}}) \; dx = dt$$

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\tan \sqrt{x})^2}{2} + c$$

38. Question

Evaluate the following integrals:

$$\int \frac{x \tan^{-1} x^2}{\left(1+x^4\right)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\tan^{-1} x^2$ = t ⇒ $\frac{1}{1+(x^2)^2} \times 2x \times dx = dt$ � $\frac{2x}{1+x^4} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

39. Question

Evaluate the following integrals:

$$\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^2$ = t ⇒ $\frac{1}{\sqrt{1-(x^2)^2}} \times 2x \times dx = dt$ � $\frac{2x}{\sqrt{1-x^4}} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\sin^{-1} x^2)^2}{4} + c$$

40. Question

Evaluate the following integrals:

$$\int \frac{1}{\left(\sqrt{1-x^2}\right)\sin^{-1} x} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^1$ = t ⇒ $\frac{1}{\sqrt{1-(x^2)^1}} \times dx = dt$ � $\frac{1}{\sqrt{1-x^2}} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{1}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log \sin^{-1} x + c$$

41. Question

Evaluate the following integrals:

$$\int \frac{\sqrt{(2+\log x)}}{x} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put 2 + log x = t$\Rightarrow \frac{1}{x} \times dx = dt$

$$\int \sqrt{t}\left(\frac{dt}{1}\right) = \int \sqrt{t} dt = \frac{2t^{1.5}}{3} + c$$

$$= \frac{2(2+\log x)^{\frac{3}{2}}}{3} + c$$

42. Question

Evaluate the following integrals:

$$\int \frac{\sec^2 x}{(1+\tan x)} dx$$

Answer

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put 1 + tan x = t$\Rightarrow sec^2 x \times dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(1+\tan x) + c$$

43. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(1+\cos x)} dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 1 + cos x = t$\Rightarrow -\sin x \times dx = dt$

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$$= -\log(1+\cos x) + c$$

44. Question

Evaluate the following integrals:

$$\int\left(\frac{1+\tan x}{1-\tan x}\right)dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

$$\int\left(\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}\right)dx = \int\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)dx$$

Put cos x - sin x = t ⇒ (- cos x - sin x) dx = dt

$$\int\left(\frac{-dt}{t}\right) = -\int\frac{1}{t}dt = -\log t + c$$

$= -\log(\cos x - \sin x) + c$

45. Question

Evaluate the following integrals:

i. $\int\frac{(1+\tan x)}{(x+\log\sec x)}dx$

ii. $\int\frac{(1-\sin 2x)}{\left(x+\cos^2 x\right)}dx$

Answer

(i)

Formula = $\int\frac{1}{x}dx = \log x + c$

Therefore ,

Put x + log (sec x) = t ⇒ $1+\frac{1}{\sec x}\times\sec x\ \tan x\,dx = dt$

$(1+\tan x)dx = dt$

$$\int\left(\frac{dt}{t}\right) = \int\frac{1}{t}dt = \log t + c$$

$= \log(x+\log(\sec x)) + c$

(ii)

Formula = $\int\frac{1}{x}dx = \log x + c$

Therefore ,

Put $x + cos^2 x$ = t ⇒ $1 + 2\cos x \times(-\sin x)dx = dt$

$(1-\sin 2x)dx = dt$

$$\int\left(\frac{dt}{t}\right) = \int\frac{1}{t}dt = \log t + c$$

$= \log(x + cos^2 x) + c$

46. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{\left(a^2 + b^2 \sin^2 x\right)} dx$$

Answer

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 + b^2 sin^2 x = t$ � $b^2 \times 2 \sin x \times \cos x\, dx = dt$

$(b^2 \sin 2x) dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2}\right) = \frac{1}{b^2} \int \frac{1}{t} dt = \frac{1}{b^2} \log t + c$$

$$= \frac{1}{b^2} \log|a^2 + b^2 sin^2 x| + c$$

47. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)} dx$$

Answer

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 cos^2 x + b^2 sin^2 x = t$

$(a^2 \times 2 \cos x \times (-\sin x) + b^2 \times 2 \sin x \times \cos x) dx = dt$

$(b^2 - a^2) \sin 2x \ dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2 - a^2}\right) = \frac{1}{b^2 - a^2} \int \frac{1}{t} dt = \frac{1}{b^2 - a^2} \log t + c$$

$$= \frac{1}{b^2 - a^2} \log|a^2 cos^2 x + b^2 sin^2 x| + c$$

48. Question

Evaluate the following integrals:

$$\int \left(\frac{2\cos x - 3\sin x}{3\cos x + 2\sin x}\right) dx$$

Answer

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put 3cos x + 2sin x = t ⇒ (2cos x - 3sin x) dx = dt

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(3\cos x + 2\sin x) + c$$

49. Question

Evaluate the following integrals:

$$\int \frac{4x}{\left(2x^2+3\right)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $2x^2 + 3 = t \Rightarrow (4x)\, dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(2x^2 + 3) + c$$

50. Question

Evaluate the following integrals:

$$\int \frac{(x+1)}{\left(x^2+2x-3\right)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $x^2+2x+3 = t \Rightarrow (2x+2)\, dx = dt$ � $2(x+1)dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(x^2 + 2x + 3) + c$$

51. Question

Evaluate the following integrals:

$$\int \frac{(4x-5)}{\left(2x^2-5x+1\right)} dx$$

Answer

To find: Value of $\int \frac{4x-5}{(2x^2-5x+1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{4x-5}{(2x^2-5x+1)} dx$ ... (i)

Let $2x^2 - 5x + 1 = t$

$$\Rightarrow \frac{d(2x^2-5x+1)}{dx} = \frac{dt}{dx}$$

$\Rightarrow 4x - 5 = \frac{dt}{dx}$

$\Rightarrow (4x - 5)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$2x^2 - 5x + 1 = t$]

$I = \log|t| + c$

$I = \log|2x^2 - 5x + 1| + c$

Ans) $\log|2x^2 - 5x + 1| + c$

52. Question

Evaluate the following integrals:

$$\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$$

Answer

To find: Value of $\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$ ... (i)

Let $3x^3 - 2x^2 + 5x + 1 = t$

$\Rightarrow \frac{d(3x^3 - 2x^2 + 5x + 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow 9x^2 - 4x + 5 = \frac{dt}{dx}$

$\Rightarrow (9x^2 - 4x + 5)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$3x^3 - 2x^2 + 5x + 1 = t$]

$I = \log|t| + c$

$I = \log|3x^3 - 2x^2 + 5x + 1| + c$

Ans) $\log|3x^3 - 2x^2 + 5x + 1| + c$

53. Question

Evaluate the following integrals:

$$\int \frac{\sec x \ \text{cosec}\, x}{\log(\tan x)} dx$$

Answer

To find: Value of $\int \frac{\sec x \ \text{cosec} x}{\log(\tan x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sec x \ \mathrm{cosec} x}{\log(\tan x)} dx$ ... (i)

Let $\log(\tan x) = t$

$\Rightarrow \frac{d(\log(\tan x))}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(\log(\tan x))}{d\tan x}\frac{d\tan x}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{\tan x}\sec^2 x = \frac{dt}{dx}$

$\Rightarrow \sec x \ \mathrm{cosec} x = \frac{dt}{dx}$

$\Rightarrow (\sec x \ \mathrm{cosec} x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$\log(\tan x) = t$]

$I = \log|t| + c$

$I = \log|\log(\tan x)| + c$

Ans) $\log|\log(\tan x)| + c$

54. Question

Evaluate the following integrals:

$$\int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$$

Answer

To find: Value of $\int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$ ... (i)

Let $x + \sin x = t$

$\Rightarrow \frac{d(x + \sin x)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin x)}{dx} = \frac{dt}{dx}$

$\Rightarrow (1 + \cos x) = \frac{dt}{dx}$

$\Rightarrow (1 + \cos x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t^3}$ [$x + \sin x = t$]

$\Rightarrow I = -\frac{1}{2t^2} + c$

$I = -\frac{1}{2(x + \sin x)^2} + c$

Ans) $-\frac{1}{2(x + \sin x)^2} + c$

55. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(1+\cos x)^2} dx$$

Answer

To find: Value of $\int \frac{\sin x}{(1+\cos x)^2} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ ... (i)

Let $1 + \cos x = t$

$\Rightarrow \frac{d(1 + \cos x)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = \frac{dt}{dx}$

$\Rightarrow (0 - \sin x) = \frac{dt}{dx}$

$\Rightarrow (-\sin x)dx = dt$

Putting this value in equation (i)

$I = \int -\frac{dt}{t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{t} + c$

$I = \frac{1}{1 + \cos x} + c$

Ans) $\frac{1}{1 + \cos x} + c$

56. Question

Evaluate the following integrals:

$$\int \frac{(2x+3)}{\sqrt{x^2+3x-2}} dx$$

Answer

To find: Value of $\int \frac{(2x+3)}{\sqrt{x^2+3x-2}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ ... (i)

Let $x^2 + 3x - 2 = t$

$\Rightarrow (2x+3) = \frac{dt}{dx}$

$\Rightarrow (2x+3)\ dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{t}}\ [\ x^2 + 3x - 2 = t\ ]$

$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

$I = 2t^{\frac{1}{2}} + c$

$I = 2\sqrt{x^2 + 3x - 2} + c$

Ans) $2\sqrt{x^2 + 3x - 2} + c$

57. Question

Evaluate the following integrals:

$$\int \frac{(2x-1)}{\sqrt{x^2-x-1}} dx$$

Answer

To find: Value of $\int \frac{(2x-1)}{\sqrt{x^2-x-1}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ ... (i)

Let $x^2 - x - 1 = t$

$\Rightarrow \frac{d(x^2 - x - 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x - 1) = \frac{dt}{dx}$

$\Rightarrow (2x - 1)\ dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^{\frac{1}{2}}} \ [x^2 - x - 1 = t]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow I = \frac{2\sqrt{t}}{1} + c$$

$$I = \frac{2\sqrt{x^2 - x - 1}}{1} + c$$

Ans) $2\sqrt{x^2 - x - 1} + c$

58. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(\sqrt{x+a} + \sqrt{x+b}\right)}$$

Answer

To find: Value of $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{\left(\sqrt{x+a}\right)^2 - \left(\sqrt{x+b}\right)^2} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b} dx$$

$$I = \frac{1}{a-b}\left[\int \sqrt{x+a}\, dx - \int \sqrt{x+b}\, dx\right]$$

$$I = \frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}} dx - \int (x+b)^{\frac{1}{2}} dx\right]$$

$$I = \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$

59. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(\sqrt{1-3x}-\sqrt{5-3x}\right)}$$

Answer

To find: Value of $\int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}} \times \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\sqrt{1-3x}+\sqrt{5-3x}}$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\left(\sqrt{1-3x}\right)^2-\left(\sqrt{5-3x}\right)^2}dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{(1-3x)-(5-3x)}dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{1-3x-5+3x}dx$$

$$I = -\frac{1}{4}\left[\int \sqrt{1-3x}\,dx + \int \sqrt{5-3x}\,dx\right]$$

$$I = -\frac{1}{4}\left[\int (1-3x)^{\frac{1}{2}}\,dx + \int (5-3x)^{\frac{1}{2}}dx\right]$$

$$I = -\frac{1}{4}\left[\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} + \frac{(5-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)}\right]$$

$$I = -\frac{2}{-9\times 4}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

$$I = \frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$

60. Question

Evaluate the following integrals:

$$\int \frac{x^2}{\left(1+x^6\right)}dx$$

Answer

To find: Value of $\int \frac{x^2}{(1+x^6)}dx$

Formula used: $\int \frac{1}{1+x^2}dx = \tan^{-1} x$

We have, $I = \int \frac{x^2}{(1+x^6)} dx$ ... (i)

$$I = \int \frac{x^2}{1+(x^3)^2} dx$$

Let $x^3 = t$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow (x^2)dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \frac{1}{3}\int \frac{dt}{1+t^2} \; [\, 1 + \cos x = t \,]$$

$$\Rightarrow I = \frac{1}{3}\tan^{-1}(t) + c$$

$$I = \frac{1}{3}\tan^{-1}(x^3) + c$$

Ans) $\frac{1}{3}\tan^{-1}(x^3) + c$

61. Question

Evaluate the following integrals:

$$\int \frac{x^3}{\left(1+x^8\right)} dx$$

Answer

To find: Value of $\int \frac{x^3}{(1+x^8)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^3}{(1+x^8)} dx$ ... (i)

$$I = \int \frac{x^3}{1+(x^4)^2} dx$$

Let $x^4 = t$

$$\Rightarrow \frac{d(x^4)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (4x^3) = \frac{dt}{dx}$$

$$\Rightarrow (x^3)dx = \frac{dt}{4}$$

Putting this value in equation (i)

$I = \frac{1}{4}\int \frac{dt}{1+t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{4}\tan^{-1}(t) + c$

$I = \frac{1}{4}\tan^{-1}(x^4) + c$

Ans) $\frac{1}{4}\tan^{-1}(x^4) + c$

62. Question

Evaluate the following integrals:

$$\int \frac{x}{\left(1+x^4\right)} dx$$

Answer

To find: Value of $\int \frac{x}{(1+x^4)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x}{(1+x^4)} dx$ ... (i)

$I = \int \frac{x}{1+(x^2)^2} dx$

Let $x^2 = t$

$\Rightarrow \frac{d(x^2)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x) = \frac{dt}{dx}$

$\Rightarrow (x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{1+t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{2}\tan^{-1}(t) + c$

$I = \frac{1}{2}\tan^{-1}(x^2) + c$

Ans) $\frac{1}{2}\tan^{-1}(x^2) + c$

63. Question

Evaluate the following integrals:

$$\int \frac{x^5}{\sqrt{1+x^3}} dx$$

Answer

To find: Value of $\int \frac{x^5}{\sqrt{1+x^3}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ ... (i)

Let $1 + x^3 = t$

$\Rightarrow x^3 = t - 1$

$\Rightarrow \frac{d(x^3)}{dx} = \frac{d(t-1)}{dx}$

$\Rightarrow (3x^2) = \frac{dt}{dx}$

$\Rightarrow x^2\, dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \frac{x^3 x^2}{\sqrt{1 + x^3}} dx$

$I = \int \frac{(t-1)}{t^{\frac{1}{2}}} \frac{dt}{3}$ [$1 + x^3 = t$]

$\Rightarrow I = \frac{1}{3} \int \frac{t}{t^{\frac{1}{2}}} dt - \frac{1}{3} \int \frac{1}{t^{\frac{1}{2}}} dt$

$\Rightarrow I = \frac{1}{3}\left[\int t^{\frac{1}{2}} dt - \int t^{-\frac{1}{2}} dt\right]$

$\Rightarrow I = \frac{1}{3}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]$

$\Rightarrow I = \frac{2}{3}\left[\frac{(1+x^3)^{\frac{3}{2}}}{3} - \frac{(1+x^3)^{\frac{1}{2}}}{1}\right]$

$\Rightarrow I = \frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$

Ans) $\frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$

64. Question

Evaluate the following integrals:

$\int \frac{x}{\sqrt{1+x}} dx$

Answer

To find: Value of $\int \frac{x}{\sqrt{1+x}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x}{\sqrt{1+x}} dx$ ... (i)

Let $1 + x = t$

$\Rightarrow x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int \frac{t-1}{\sqrt{t}} dx \, [1 + x = t]$$

$$\Rightarrow I = \int \sqrt{t} dt - \int \frac{1}{\sqrt{t}} dt$$

$$\Rightarrow I = \left[\int t^{\frac{1}{2}} dt - \int t^{-\frac{1}{2}} dt\right]$$

$$\Rightarrow I = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right] + c$$

$$\Rightarrow I = 2\left[\frac{(1+x)^{\frac{3}{2}}}{3} - \frac{(1+x)^{\frac{1}{2}}}{1}\right] + c$$

$$\Rightarrow I = \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$$

Ans) $\frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

65. Question

Evaluate the following integrals:

$$\int \frac{1}{x\sqrt{x^4 - 1}} dx$$

Answer

To find: Value of $\int \frac{1}{x\sqrt{x^4-1}} dx$

Formula used: $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}} dx$ ... (i)

Multiplying numerator and denominator with x

$$I = \int \frac{x}{x^2\sqrt{(x^2)^2 - 1}} dx$$

Let $x^2 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2 - 1}}$ [$x^2 = t$]

$\Rightarrow I = \frac{1}{2}\sec^{-1}t + c$

$\Rightarrow I = \frac{1}{2}\sec^{-1}(x^2) + c$

Ans) $\frac{1}{2}\sec^{-1}(x^2) + c$

66. Question

Evaluate the following integrals:

$\int x\sqrt{-1}dx$

Answer

To find: Value of $\int x\sqrt{x-1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x-1}\,dx$ ... (i)

Let $x - 1 = t$

$x = t + 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int (t+1)\sqrt{t}\,dt$ [$x = t + 1$]

$\Rightarrow I = \int t\sqrt{t}dx + \int \sqrt{t}\,dx$

$\Rightarrow I = \int t^{\frac{3}{2}}dx + \int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$

Ans) $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$

67. Question

Evaluate the following integrals:

$\int(1-x)\sqrt{1+x}\ dx$

Answer

To find: Value of $\int (1 - x)\sqrt{1+x}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (1 - x)\sqrt{1+x}\,dx$ ... (i)

Let $1 + x = t$

$x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \{1 - (t - 1)\}\sqrt{t}\,dt\,[\,x = t - 1\,]$

$\Rightarrow I = \int \{1 - t + 1\}\sqrt{t}\,dt$

$\Rightarrow I = \int \{2 - t\}\sqrt{t}\,dt$

$\Rightarrow I = \int 2\sqrt{t}\,dt - \int t\sqrt{t}\,dt$

$\Rightarrow I = 2\int t^{\frac{1}{2}}dx - \int t^{\frac{3}{2}}\,dx$

$\Rightarrow I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$

$\Rightarrow I = \frac{4}{3}(1+x)^{\frac{3}{2}} - \frac{2}{5}(1+x)^{\frac{5}{2}} + c$

Ans) $\frac{4}{3}(1+x)^{\frac{3}{2}} - \frac{2}{5}(1+x)^{\frac{5}{2}} + c$

68. Question

Evaluate the following integrals:

$\int x\sqrt{x^2-1}dx$

Answer

To find: Value of $\int x\sqrt{x^2 - 1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x^2 - 1}\,dx$ ... (i)

Let $x^2 - 1 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \frac{1}{2}\sqrt{t}\,dt\,[\,x = x^2 - 1\,]$

$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\,t^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\,(x^2 - 1)^{\frac{3}{2}} + c$

Ans) $\frac{1}{3}\,(x^2 - 1)^{\frac{3}{2}} + c$

69. Question

Evaluate the following integrals:

$\int x\sqrt{3x-2}\,dx$

Answer

To find: Value of $\int x\sqrt{3x - 2}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{3x - 2}\,dx$ ... (i)

Let $3x - 2 = t$

$\Rightarrow 3x = t + 2$

$\Rightarrow x = \frac{t+2}{3}$

$\Rightarrow 3 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \left(\frac{t+2}{3}\right)\sqrt{t}\,\frac{dt}{3}\,[\,t = 3x - 2\,]$

$\Rightarrow I = \frac{1}{9}\left[\int t^{\frac{3}{2}}\,dx + 2\int t^{\frac{1}{2}}\,dx\right]$

$\Rightarrow I = \frac{1}{9}\left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$