Exercise 4(B)

Solution 1:

(i)

$$(a-b)^3 = a^3 - 3ab(a-b) - b^3$$

$$\begin{aligned}(3a-2b)^3 &= (3a)^3 - 3\times 3a\times 2b(3a-2b) - (2b)^3\\ &= 27a^3 - 18ab(3a-2b) - 8b^3\\ &= 27a^3 - 54a^2b + 36ab^2 - 8b^3\end{aligned}$$

(ii)

$$(a+b)^3 = a^3 + 3ab(a+b) + b^3$$

$$\begin{aligned}(5a+3b)^3 &= (5a)^3 + 3\times 5a\times 3b(5a+3b) + (3b)^3\\ &= 125a^3 + 45ab(5a+3b) + 27b^3\\ &= 125a^3 + 225a^2b + 135ab^2 + 27b^3\end{aligned}$$

(iii)

$$(a+b)^3 = a^3 + 3ab(a+b) + b^3$$

$$\left(2a+\frac{1}{2a}\right)^3 = (2a)^3 + 3\times 2a\times\frac{1}{2a}\times\left(2a+\frac{1}{2a}\right)+\left(\frac{1}{2a}\right)^3$$

$$= 8a^3 + 3\left(2a+\frac{1}{2a}\right)+\frac{1}{8a^3}$$

$$\left(2a+\frac{1}{2a}\right)^3 = 8a^3 + 6a + \frac{3}{2a} + \frac{1}{8a^3}$$

(iv)

$$(a-b)^3 = a^3 - 3ab(a-b) - b^3$$

$$\left(3a-\frac{1}{a}\right)^3 = (3a)^3 - 3\times 3a\times\frac{1}{a}\left(3a-\frac{1}{a}\right)-\left(\frac{1}{a}\right)^3$$

$$= 27a^3 - 9\left(3a-\frac{1}{a}\right)-\frac{1}{a^3}$$

$$= 27a^3 - 27a + \frac{9}{a} - \frac{1}{a^3}$$

Solution 2:

(i)

$$a^2 + \frac{1}{a^2} = 47$$

$$\left(a+\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 47 + 2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 49$$

$$\Rightarrow a + \frac{1}{a} = \pm\sqrt{49}$$

$$\Rightarrow a + \frac{1}{a} = \pm 7 \ldots.(1)$$

(ii)

$$\left(a+\frac{1}{a}\right)^3 = a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$$

$$\Rightarrow a^3+\frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$$

$$\Rightarrow a^3+\frac{1}{a^3} = (\pm 7)^3-3(\pm 7)\ \text{[from (1)]}$$

$$\Rightarrow a^3+\frac{1}{a^3} = \pm 322$$

Solution 3:

(i)

$$a^2+\frac{1}{a^2}=18$$

$$\left(a-\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}-2$$

$$\Rightarrow \left(a-\frac{1}{a}\right)^2 = 18-2$$

$$\Rightarrow \left(a-\frac{1}{a}\right)^2 = 16$$

$$\Rightarrow a-\frac{1}{a} = \pm\sqrt{16}$$

$$\Rightarrow a-\frac{1}{a} = \pm 4 \ldots (1)$$

(ii)

$$\left(a-\frac{1}{a}\right)^3 = a^3-\frac{1}{a^3}-3\left(a-\frac{1}{a}\right)$$

$$\Rightarrow a^3-\frac{1}{a^3} = \left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$$

$$\Rightarrow a^3-\frac{1}{a^3} = (\pm 4)^3+3(\pm 4)\ \text{[from (1)]}$$

$$\Rightarrow a^3-\frac{1}{a^3} = \pm 76$$

Solution 4:

Given that $a+\frac{1}{a}=p....(1)$

$\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$

$\Rightarrow a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$

$\Rightarrow a^3+\frac{1}{a^3}=(p)^3-3(p)$ [from (1)]

$\Rightarrow a^3+\frac{1}{a^3}=p\left(p^2-3\right)$

Solution 5:

Given that a+2b=5;

We need to find a^3+8b^3+30ab :

Now consider the cube of a+2b:

$(a+2b)^3=a^3+(2b)^3+3\times a\times 2b\times(a+2b)$

$=a^3+8b^3+6ab\times(a+2b)$

$5^3=a^3+8b^3+6ab\times(5)$ [$\because a+2b=5$]

$125=a^3+8b^3+30ab$

Thus the value of a^3+8b^3+30ab is 125.

Solution 6:

Given that $\left(a+\frac{1}{a}\right)^2 = 3$

$\Rightarrow a+\frac{1}{a} = \pm\sqrt{3}....(1)$

We need to find $a^3+\frac{1}{a^3}$:

Consider the identity,

$$\left(a+\frac{1}{a}\right)^3 = a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$$

$$\Rightarrow a^3+\frac{1}{a^3} = \left(\pm\sqrt{3}\right)^3 - 3\left(\pm\sqrt{3}\right) \text{ [from (1)]}$$

$$\Rightarrow a^3+\frac{1}{a^3} = \pm3\sqrt{3} - 3\left(\pm\sqrt{3}\right)$$

$$\Rightarrow a^3+\frac{1}{a^3} = 0$$

Solution 7:

Given that a+2b+c=0;

$\Rightarrow$ a+2b=– c....(1)

Now consider the expansion of $(a+2b)^3$:

$$(a+2b)^3 = (-c)^3$$

$$a^3+(2b)^3+3\times a\times 2b\times(a+2b) = -c^3$$

$$\Rightarrow \quad a^3+8b^3+3\times a\times 2b\times(-c) = -c^3 \quad \text{[from (1)]}$$

$$\Rightarrow \quad a^3+8b^3-6abc = -c^3$$

$$\Rightarrow \quad a^3+8b^3+c^3 = 6abc$$

Hence proved.

Solution 8:

Property is if a + b + c = 0 then $a^3 + b^3 + c^3 = 3abc$

(i) a = 13, b = -8 and c = -5

$13^3 + (-8)^3 + (-5)^3 = 3(13)(-8)(-5) = 1560$

(ii) a = 7, b = 3, c = -10

$7^3 + 3^3 + (-10)^3 = 3(7)(3)(-10) = -630$

(iii)a = 9, b = -5, c = -4

$9^3 - 5^3 - 4^3 = 9^3 + (-5)^3 + (-4)^3 = 3(9)(-5)(-4) = 540$

(iv) a = 38, b = -26, c = -12

$38^3 + (-26)^3 + (-12)^3 = 3(38)(-26)(-12) = 35568$

Solution 9:

(i)

$$a - \frac{1}{a} = 3$$

$$\left(a - \frac{1}{a}\right)^2 = 9$$

$$a^2 + \frac{1}{a^2} = 9 + 2 = 11$$

(ii)

$$a - \frac{1}{a} = 3$$

$$\left(a - \frac{1}{a}\right)^3 = 27$$

$$a^3 + \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right) = 27$$

$$a^3 + \frac{1}{a^3} = 27 + 9 = 36$$

Solution 10:

(i)

$$\left(a-\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}-2$$

$$\Rightarrow a^2+\frac{1}{a^2} = \left(a-\frac{1}{a}\right)^2+2$$

$$\Rightarrow a^2+\frac{1}{a^2} = (4)^2+2 \quad [\because a-\frac{1}{a}=4]$$

$$\Rightarrow a^2+\frac{1}{a^2} = 18.....(1)$$

(ii)

We know that

$$a^4+\frac{1}{a^4} = \left(a^2+\frac{1}{a^2}\right)^2-2$$

$$= (18)^2-2 \quad [\text{from } (1)]$$

$$= 324-2$$

$$\Rightarrow a^4+\frac{1}{a^4} = 322$$

(iii)

$$\left(a-\frac{1}{a}\right)^3 = a^3-\frac{1}{a^3}-3\left(a-\frac{1}{a}\right)$$

$$\Rightarrow a^3-\frac{1}{a^3} = \left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$$

$$\Rightarrow a^3-\frac{1}{a^3} = (4)^3+3(4) \quad [\because a-\frac{1}{a}=4]$$

$$\Rightarrow a^3-\frac{1}{a^3} = 64+12$$

$$\Rightarrow a^3-\frac{1}{a^3} = 76$$

Solution 11:

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

$$\Rightarrow x^2+\frac{1}{x^2} = (2)^2 - 2 \quad \left[\because x-\frac{1}{x}=2\right]$$

$$\Rightarrow x^2+\frac{1}{x^2} = 2 \ldots\ldots(1)$$

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow x^3+\frac{1}{x^3} = (2)^3 - 3(2) \quad \left[\because x+\frac{1}{x}=2\right]$$

$$\Rightarrow x^3+\frac{1}{x^3} = 8-6$$

$$\Rightarrow x^3+\frac{1}{x^3} = 2 \ldots.(2)$$

We know that

$$x^4+\frac{1}{x^4} = \left(x^2+\frac{1}{x^2}\right)^2 - 2$$

$$= (2)^2 - 2 \quad [\text{from } (1)]$$

$$= 4-2$$

$$\Rightarrow x^4+\frac{1}{x^4} = 2 \ldots\ldots(3)$$

Thus from equations (1), (2) and (3), we have

$$x^2+\frac{1}{x^2} = x^3+\frac{1}{x^3} = x^4+\frac{1}{x^4}$$

Solution 12:

Given that $2x - 3y = 10, xy = 16$

:. $(2x - 3y)^3 = (10)^3$

Þ $8x^3 - 27y^3 - 3(2x)(3y)(2x - 3y) = 1000$ Þ $8x^3 - 27y^3 - 18xy(2x - 3y) = 1000$

Þ $8x^3 - 27y^3 - 18(16)(10) = 1000$

Þ $8x^3 - 27y^3 - 2880 = 1000$

Þ $8x^3 - 27y^3 = 1000 + 2880$

Þ$8x^3 - 27y^3 = 3880$

Solution 13:

(i)

$(3x + 5y + 2z)(3x - 5y + 2z)$

$= \{(3x + 2z) + (5y)\}\{(3x + 2z) - (5y)\}$

$= (3x + 2z)^2 - (5y)^2$

{since $(a + b)(a - b) = a^2 - b^2$}

$= 9x^2 + 4z^2 + 2 \times 3x \times 2z - 25y^2$

$= 9x^2 + 4z^2 + 12xz - 25y^2$

$= 9x^2 + 4z^2 - 25y^2 + 12xz$

(ii)

$(3x - 5y - 2z)(3x - 5y + 2z)$

$= \{(3x - 5y) - (2z)\}\{(3x - 5y) + (2z)\}$

$= (3x - 5y)^2 - (2z)^2$ {since $(a + b)(a - b) = a^2 - b^2$}

$= 9x^2 + 25y^2 - 2 \times 3x \times 5y - 4z^2$

$= 9x^2 + 25y^2 - 30xy - 4z^2$

$= 9x^2 + 25y^2 - 4z^2 - 30xy$

Solution 14:

Given sum of two numbers is 9 and their product is 20.

Let the numbers be a and b.

$a + b = 9$

$ab = 20$

Squaring on both sides gives

$(a+b)^2 = 9^2$

$a^2 + b^2 + 2ab = 81$

$a^2 + b^2 + 40 = 81$

So sum of squares is 81 - 40 = 41

Cubing on both sides gives

$(a + b)^3 = 9^3$

$a^3 + b^3 + 3ab(a + b) = 729$

$a^3 + b^3 + 60(9) = 729$

$a^3 + b^3 = 729 - 540 = 189$

So the sum of cubes is 189.

Cubing on both sides gives

$(x - y)^3 = 5^3$

$x^3 - y^3 - 3xy(x - y) = 125$

$x^3 - y^3 - 72(5) = 125$

$x^3 - y^3 = 125 + 360 = 485$

So, difference of their cubes is 485.

Cubing both sides, we get

$(x + y)^3 = 11^3$

$x^3 + y^3 + 3xy(x + y) = 1331$

$x^3 + y^3 = 1331 - 72(11) = 1331 - 792 = 539$

So, sum of their cubes is 539.