X - RAYS
CHAPTER 44

1. λ = 0.1 nm

   a) Energy = $\frac{hc}{\lambda} = \frac{1242 \text{ ev.nm}}{0.1 \text{ nm}}$

   = 12420 ev = 12.42 Kev = 12.4 kev.

   b) Frequency = $\frac{C}{\lambda} = \frac{3\times10^8}{0.1\times10^{-9}} = \frac{3\times10^8}{10^{-10}} = 3\times10^{18}\text{Hz}$

   c) Momentum = E/C = $\frac{12.4\times10^3\times1.6\times10^{-19}}{3\times10^8}$ = 6.613 × 10^{-24} kg-m/s = 6.62 × 10^{-24} kg-m/s.

2. Distance = 3 km = 3 × 10^3 m
   C = 3 × 10^8 m/s

   t = $\frac{\text{Dist}}{\text{Speed}} = \frac{3\times10^3}{3\times10^8} = 10^{-5}$ sec.

   ⇒ 10 × 10^{-8} sec = 10 μs in both case.

3. V = 30 KV

   $\lambda = \frac{hc}{E} = \frac{hc}{eV} = \frac{1242 \text{ ev} - \text{nm}}{e\times30\times10^3}$ = 414 × 10^{-4} nm = 41.4 Pm.

4. λ = 0.10 nm = 10^{-10} m ;   h = 6.63 × 10^{-34} J-s
   C = 3 × 10^8 m/s ;   e = 1.6 × 10^{-19} C

   $\lambda_{min} = \frac{hc}{eV}$ or $V = \frac{hc}{e\lambda}$

   $= \frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times10^{-10}}$ = 12.43 × 10^3 V = 12.4 KV.

   Max. Energy = $\frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{10^{-10}}$ = 19.89 × 10^{-18} = 1.989 × 10^{-15} = 2 × 10^{-15} J.

5. λ = 80 pm, E = $\frac{hc}{\lambda} = \frac{1242}{80\times10^{-3}}$ = 15.525 × 10^3 eV = 15.5 KeV

6. We know $\lambda = \frac{hc}{V}$

   Now $\lambda = \frac{hc}{1.01V} = \frac{\lambda}{1.01}$

   $\lambda - \lambda' = \frac{0.01}{1.01}\lambda$.

   % change of wave length = $\frac{0.01\times\lambda}{1.01\times\lambda}\times100 = \frac{1}{1.01}$ = 0.9900 = 1%.

7. d = 1.5 m, λ = 30 pm = 30 × 10^{-3} nm

   E = $\frac{hc}{\lambda} = \frac{1242}{30\times10^{-3}}$ = 41.4 × 10^3 eV

   Electric field = $\frac{V}{d} = \frac{41.4\times10^3}{1.5}$ = 27.6 × 10^3 V/m = 27.6 KV/m.

8. Given $\lambda' = \lambda$ – 26 pm, V′ = 1.5 V

   Now, $\lambda = \frac{hc}{ev}$, $\lambda' = \frac{hc}{ev'}$

   or $\lambda V = \lambda' V'$

   ⇒ $\lambda V = (\lambda - 26\times10^{-12})\times1.5\text{ V}$

$\Rightarrow \lambda = 1.5\ \lambda - 1.5 \times 26 \times 10^{-12}$

$\Rightarrow \lambda = \dfrac{39 \times 10^{-12}}{0.5} = 78 \times 10^{-12}$ m

$V = \dfrac{hc}{e\lambda} = \dfrac{6.63 \times 3 \times 10^{-34} \times 10^{8}}{1.6 \times 10^{-19} \times 78 \times 10^{-12}} = 0.15937 \times 10^{5} = 15.93 \times 10^{3}$ V = 15.93 KV.

9. $V = 32$ KV $= 32 \times 10^{3}$ V
When accelerated through 32 KV
$E = 32 \times 10^{3}$ eV
$\lambda = \dfrac{hc}{E} = \dfrac{1242}{32 \times 10^{3}} = 38.8 \times 10^{-3}$ nm = 38.8 pm.

10. $\lambda = \dfrac{hc}{eV}$; V = 40 kV, f = 9.7×10^{18} Hz
or, $\dfrac{h}{c} = \dfrac{h}{eV}$; or, $\dfrac{i}{f} = \dfrac{h}{eV}$; or $h = \dfrac{eV}{f} V - s$
$= \dfrac{eV}{f} V - s = \dfrac{40 \times 10^{3}}{9.7 \times 10^{18}} = 4.12 \times 10^{-15}$ eV-s.

11. $V = 40$ KV $= 40 \times 10^{3}$ V
Energy = 40×10^{3} eV
Energy utilized = $\dfrac{70}{100} \times 40 \times 10^{3} = 28 \times 10^{3}$ eV
$\lambda = \dfrac{hc}{E} = \dfrac{1242 - \text{ev nm}}{28 \times 10^{3}\ \text{ev}} \Rightarrow 44.35 \times 10^{-3}$ nm = 44.35 pm.
For other wavelengths,
E = 70% (left over energy) = $\dfrac{70}{100} \times (40 - 28)10^{3} = 84 \times 10^{2}$.
$\lambda' = \dfrac{hc}{E} = \dfrac{1242}{8.4 \times 10^{3}} = 147.86 \times 10^{-3}$ nm = 147.86 pm = 148 pm.
For third wavelength,
$E = \dfrac{70}{100} = (12 - 8.4) \times 10^{3} = 7 \times 3.6 \times 10^{2} = 25.2 \times 10^{2}$
$\lambda' = \dfrac{hc}{E} = \dfrac{1242}{25.2 \times 10^{2}} = 49.2857 \times 10^{-2}$ nm = 493 pm.

12. $K_{\lambda} = 21.3 \times 10^{-12}$ pm, Now, $E_K - E_L = \dfrac{1242}{21.3 \times 10^{-3}} = 58.309$ kev
$E_L = 11.3$ kev, $E_K = 58.309 + 11.3 = 69.609$ kev
Now, Ve = 69.609 KeV, or V = 69.609 KV.

13. $\lambda = 0.36$ nm
$E = \dfrac{1242}{0.36} = 3450$ eV $(E_M - E_K)$
Energy needed to ionize an organ atom = 16 eV
Energy needed to knock out an electron from K-shell
= (3450 + 16) eV = 3466 eV = 3.466 KeV.

14. $\lambda_1 = 887$ pm
$v = \dfrac{C}{\lambda} = \dfrac{3 \times 10^{8}}{887 \times 10^{-12}} = 3.382 \times 10^{7} = 33.82 \times 10^{16} = 5.815 \times 10^{8}$
$\lambda_2 = 146$ pm
$v = \dfrac{3 \times 10^{8}}{146 \times 10^{-12}} = 0.02054 \times 10^{20} = 2.054 \times 10^{18} = 1.4331 \times 10^{9}$.

We know, $\sqrt{v} = a(z-b)$

$$\Rightarrow \frac{\sqrt{5.815 \times 10^8} = a(13-b)}{\sqrt{1.4331 \times 10^9} = a(30-b)}$$

$$\Rightarrow \frac{13-b}{30-b} = \frac{5.815 \times 10^{-1}}{1.4331} = 0.4057.$$

$\Rightarrow 30 \times 0.4057 - 0.4057\ b = 13 - b$

$\Rightarrow 12.171 - 0.4.57\ b + b = 13$

$$\Rightarrow b = \frac{0.829}{0.5943} = 1.39491$$

$$\Rightarrow a = \frac{5.815 \times 10^8}{11.33} = 0.51323 \times 10^8 = 5 \times 10^7.$$

For 'Fe',

$\sqrt{v} = 5 \times 10^7 (26 - 1.39) = 5 \times 24.61 \times 10^7 = 123.05 \times 10^7$

$c/\lambda = 15141.3 \times 10^{14}$

$$= \lambda = \frac{3 \times 10^8}{15141.3 \times 10^{14}} = 0.000198 \times 10^{-6}\ \text{m} = 198 \times 10^{-12} = 198\ \text{pm}.$$

15. $E = 3.69$ kev = 3690 eV

$$\lambda = \frac{hc}{E} = \frac{1242}{3690} = 0.33658\ \text{nm}$$

$\sqrt{c/\lambda} = a(z-b)$; $a = 5 \times 10^7\ \sqrt{\text{Hz}}$, $b = 1.37$ (from previous problem)

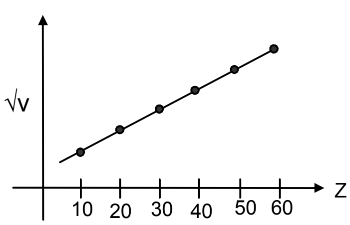
$$\sqrt{\frac{3 \times 10^8}{0.34 \times 10^{-9}}} = 5 \times 10^7 (Z - 1.37) \Rightarrow \sqrt{8.82 \times 10^{17}} = 5 \times 10^7 (Z - 1.37)$$

$\Rightarrow 9.39 \times 10^8 = 5 \times 10^7 (Z - 1.37) \Rightarrow 93.9 / 5 = Z - 1.37$

$\Rightarrow Z = 20.15 = 20$

$\therefore$ The element is calcium.

16. K_B radiation is when the e jumps from n = 3 to n = 1 (here n is principal quantum no)



$$\Delta E = h\nu = Rhc\,(z-h)^2 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \sqrt{v} = \sqrt{\frac{9RC}{8}}(z-h)$$

$\therefore \sqrt{v} \propto z$

Second method :

We can directly get value of v by `

hv = Energy

$$\Rightarrow v = \frac{\text{Energy(in kev)}}{h}$$

This we have to find out $\sqrt{v}$ and draw the same graph as above.

17. b = 1

For $\propto$ a (57)

$\sqrt{v} = a\,(Z - b)$

$\Rightarrow \sqrt{v} = a\,(57 - 1) = a \times 56$ …(1)

For Cu(29)

$\sqrt{1.88 \times 10^{78}} = a(29 - 1) = 28\,a$ …(2)

dividing (1) and (2)

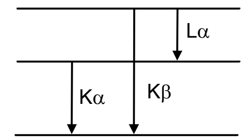
$$\sqrt{\frac{v}{1.88 \times 10^{18}}} = \frac{a \times 56}{a \times 28} = 2.$$

$\Rightarrow v = 1.88 \times 10^{18}(2)^2 = 4 \times 1.88 \times 10^{18} = 7.52 \times 10^{8}$ Hz.

18. $K_\alpha = E_K - E_L$,,,(1) $\lambda K_\alpha = 0.71$ A°

$K_\beta = E_K - E_M$,,,(2) $\lambda K_\beta = 0.63$ A°

$L_\alpha = E_L - E_M$,,,(3)



Subtracting (2) from (1)

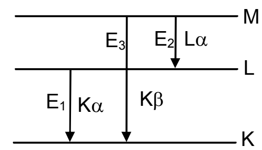
$K_\alpha - K_\beta = E_M - E_L = -L_\alpha$

or, $L_\alpha = K_\beta - K_\alpha = \dfrac{3 \times 10^8}{0.63 \times 10^{-10}} - \dfrac{3 \times 10^8}{0.71 \times 10^{-10}}$

$= 4.761 \times 10^{18} - 4.225 \times 10^{18} = 0.536 \times 10^{18}$ Hz.

Again $\lambda = \dfrac{3 \times 10^8}{0.536 \times 10^{18}} = 5.6 \times 10^{-10} = 5.6$ A°.

19. $E_1 = \dfrac{1242}{21.3 \times 10^{-3}} = 58.309 \times 10^3$ ev



$E_2 = \dfrac{1242}{141 \times 10^{-3}} = 8.8085 \times 10^3$ ev

$E_3 = E_1 + E_2 \Rightarrow (58.309 + 8.809)$ ev $= 67.118 \times 10^3$ ev

$\lambda = \dfrac{hc}{E_3} = \dfrac{1242}{67.118 \times 10^3} = 18.5 \times 10^{-3}$ nm = 18.5 pm.

20. $E_K = 25.31$ KeV, $E_L = 3.56$ KeV, $E_M = 0.530$ KeV

$K_\alpha = E_K - K_L = hv$

$\Rightarrow v = \dfrac{E_K - E_L}{h} = \dfrac{25.31 - 3.56}{4.14 \times 10^{-15}} \times 10^3 = 5.25 \times 10^{15}$ Hz

$K_\beta = E_K - K_M = hv$

$\Rightarrow v = \dfrac{E_K - E_M}{h} = \dfrac{25.31 - 0.53}{4.14 \times 10^{-15}} \times 10^3 = 5.985 \times 10^{18}$ Hz.

21. Let for, k series emission the potential required = v

$\therefore$ Energy of electrons = ev

This amount of energy ev = energy of L shell

The maximum potential difference that can be applied without emitting any electron is 11.3 ev.

22. V = 40 KV, i = 10 mA

1% of T_{KE} (Total Kinetic Energy) = X ray

i = ne or $n = \dfrac{10^{-2}}{1.6 \times 10^{-19}} = 0.625 \times 10^{17}$ no.of electrons.

KE of one electron = eV $= 1.6 \times 10^{-19} \times 40 \times 10^3 = 6.4 \times 10^{-15}$ J

$T_{KE} = 0.625 \times 6.4 \times 10^{17} \times 10^{-15} = 4 \times 10^2$ J.

a) Power emitted in X-ray $= 4 \times 10^2 \times (-1/100) = 4$w

b) Heat produced in target per second = 400 – 4 = 396 J.

23. Heat produced/sec = 200 w

$\Rightarrow \dfrac{neV}{t} = 200 \Rightarrow (ne/t)V = 200$

$\Rightarrow$ i = 200 /V = 10 mA.

24. Given : $v = (25 \times 10^{14}$ Hz$)(Z - 1)^2$

Or $C/\lambda = 25 \times 10^{14}(Z - 1)^2$

a) $\dfrac{3 \times 10^8}{78.9 \times 10^{-12} \times 25 \times 10^{14}} = (Z - 1)^2$

or, $(Z - 1)^2 = 0.001520 \times 10^6 = 1520$

$\Rightarrow$ Z – 1 = 38.98 or Z = 39.98 = 40. It is (Zr)

b) $\dfrac{3\times10^8}{146\times10^{-12}\times25\times10^{14}}=(Z-1)^2$

or, $(Z - 1)^2 = 0.0008219 \times 10^6$

$\Rightarrow$ Z – 1 = 28.669 or Z = 29.669 = 30. It is (Zn).

c) $\dfrac{3\times10^8}{158\times10^{-12}\times25\times10^{14}}=(Z-1)^2$

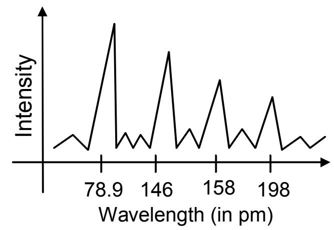
or, $(Z - 1)^2 = 0.0007594 \times 10^6$

$\Rightarrow$ Z – 1 = 27.5589 or Z = 28.5589 = 29. It is (Cu).

d) $\dfrac{3\times10^8}{198\times10^{-12}\times25\times10^{14}}=(Z-1)^2$

or, $(Z - 1)^2 = 0.000606 \times 10^6$

$\Rightarrow$ Z – 1 = 24.6182 or Z = 25.6182 = 26. It is (Fe).



25. Here energy of photon = E

E = 6.4 KeV = 6.4×10^3 ev

Momentum of Photon = E/C = $\dfrac{6.4\times10^3}{3\times10^8}$ = 3.41×10^{-24} m/sec.

According to collision theory of momentum of photon = momentum of atom

$\therefore$ Momentum of Atom = P = 3.41×10^{-24} m/sec

$\therefore$ Recoil K.E. of atom = P^2 / 2m

$\Rightarrow \dfrac{(3.41\times10^{-24})^2\,eV}{(2)(9.3\times10^{-26}\times1.6\times10^{-19})}=$ 3.9 eV [1 Joule = 1.6×10^{-19} ev]

26. $V_0 \rightarrow$ Stopping Potential, $\lambda \rightarrow$ Wavelength, $eV_0 = h\nu - h\nu_0$

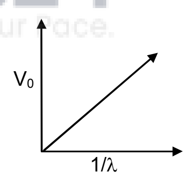
$eV_0 = hc/\lambda \Rightarrow V_0\lambda = hc/e$

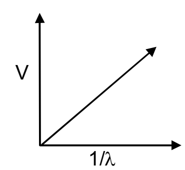
V $\rightarrow$ Potential difference across X-ray tube, $\lambda \rightarrow$ Cut of wavelength

λ = hc / eV or $V\lambda$ = hc / e

Slopes are same i.e. $V_0\lambda = V\lambda$

$\dfrac{hc}{e}=\dfrac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}}$ = 1.242×10^{-6} Vm





27. λ = 10 pm = 100×10^{-12} m

D = 40 cm = 40×10^{-2} m

β = 0.1 mm = 0.1×10^{-3} m

$\beta=\dfrac{\lambda D}{d}$

$\Rightarrow$ d = $\dfrac{\lambda D}{\beta}=\dfrac{100\times10^{-12}\times40\times10^{-2}}{10^{-3}\times0.1}$ = 4×10^{-7} m.

♠ ♠ ♠