NCERT Solutions for Class 9 Maths Chapter 8 Quadrilaterals Ex 8.1

Question 1.

The angles of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

Solution.

Let the common ratio between the angles be x. Therefore, the angles will be 3x, 5x, 9x, and 13x respectively

As the sum of all interior angles of a quadrilateral is 360^0,

$3x + 5x + 9x + 13x = 360^0$

$30x = 360^0$ x

$= 12^0$

Hence, the angles are

$3x = 3 \times 12 = 36^0$ $5x = 5 \times 12 = 60^0$

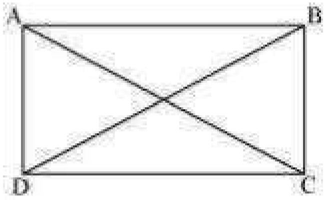
$9x = 9 \times 12 = 108^0$ $13x = 13 \times 12 = 156^0$

Question 2.

If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution.

Let ABCD be a parallelogram. To show that ABCD is a rectangle, We have to prove that: One of its interior angles is 90^0



In ΔABC and ΔDCB,

AB = DC (Opposite sides of a parallelogram are equal)

BC = BC (Common)

AC = DB (Given)

ΔABC ≅ ΔDCB (By SSS Congruence rule)

∠ABC = ∠DCB

It is known that the sum of the measures of angles on the same side of transversal is 180^0

∠ABC + ∠DCB = 180^0 (AB || CD)

∠ABC + ∠ABC = 180^0

∠ABC = 180^0

2 ∠ABC = 90^0

Since ABCD is a parallelogram and one of its interior angles is 90^0, ABCD is a rectangle

Question 3.

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

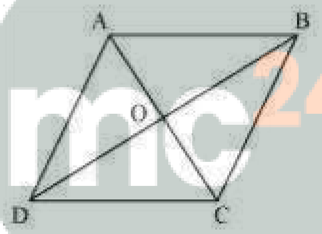
Solution.

Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right

Angle i.e., OA = OC, OB = OD, and

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^0$. To prove

ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.



In ΔAOD and ΔCOD,

OA = OC (Diagonals bisect each other)

$\angle AOD = \angle COD$ (Given)

OD = OD (Common)

$\Delta AOD \cong \Delta COD$ (By SAS congruence rule)

AD = CD (1)

Similarly,

AD = AB and CD = BC (2)

From equations (1) and (2),

AB = BC = CD = AD

Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that

ABCD is a rhombus

Question 4.

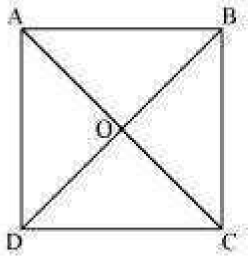
Show that the diagonals of a square
are equal and bisect each other at right angles.

Solution.

Let ABCD be a square. Let the diagonals AC and BD intersect each other at a point O.

To prove that the diagonals of a square are equal and bisect each other at right angles,

We have to prove: AC = BD, OA = OC, OB = OD and $\angle AOB = 90^0$



In ΔABC and ΔDCB,

AB = DC (Sides of a square are equal to each other)

∠ABC = ∠DCB (All interior angles are of 90)

BC = CB (Common side)

ΔABC ≅ ΔDCB (By SAS congruency)

AC = DB (By CPCT)

Hence, the diagonals of a square are equal in length.

In ΔAOB and ΔCOD,

∠AOB =∠ COD (Vertically opposite angles)

∠ABO = ∠CDO (Alternate interior angles)

AB = CD (Sides of a square are always equal)

ΔAOB ≅ ΔCOD (By AAS congruence rule)

AO = CO and

OB = OD (By CPCT)

Hence, the diagonals of a square bisect each other

In ΔAOB and ΔCOB,

As we had proved that diagonals bisect each other, therefore,

AO = CO

AB = CB (Sides of a square are equal)

BO = BO (Common)

$\Delta AOB \cong \Delta COB$ (By SSS congruency)

$\angle AOB = \angle COB$ (By CPCT)

However,

$\angle AOB + \angle COB = 180^0$ (Linear pair)

$2\angle AOB = 180^0$

$\angle AOB = 90^0$

Hence, the diagonals of a square bisect each other at right angles

Question 5.

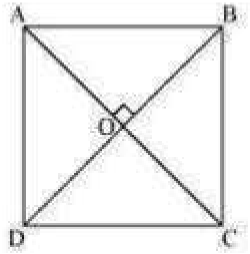
Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution.

Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O.

It is given that the diagonals of ABCD are equal and bisect each other at right angles. Therefore, AC = BD, OA = OC, OB = OD, and

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^0$



To prove ABCD is a square, we have to prove that ABCD is a parallelogram,

AB = BC = CD = AD, and one of its interior angles is 90^0

In ΔAOB and ΔCOD,

AO = CO (Diagonals bisect each other)

OB = OD (Diagonals bisect each other)

∠AOB = ∠COD (Vertically opposite angles)

ΔAOB ≅ ΔCOD (SAS congruence rule)

AB = CD (By CPCT) (1)

And,

∠OAB = ∠OCD (By CPCT)

However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel

AB || CD (2)

From equations (1) and (2), we obtain ABCD is a parallelogram

In ΔAOD and ΔCOD,

AO = CO (Diagonals bisect each other)

∠AOD = ∠COD (Given that each is 90^0)

OD = OD (Common)

ΔAOD ≅ ΔCOD (SAS congruence rule)

AD = DC (3)

However,

AD = BC and

AB = CD (Opposite sides of parallelogram ABCD)

AB = BC = CD = DA

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In ΔADC and ΔBCD,

AD = BC (Already proved)

AC = BD (Given)

DC = CD (Common)

ΔADC ≅ ΔBCD (SSS Congruence rule)

∠ADC = ∠BCD (By CPCT)

However,

$\angle ADC + \angle BCD = 180^0$ (Co-interior angles)

$\angle ADC + \angle ADC = 180^0$

$2\ \angle ADC = 180^0$

$\angle ADC = 90^0$ One of the interior angles of quadrilateral ABCD is a right angle.

Thus, we have obtained that ABCD is a parallelogram, AB = BC = CD = AD and one of its interior angles is 90^0

Therefore, ABCD is a square

Question 6.

Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig. 8.19). Show that

(i) It bisects $\angle$ C also,

(ii) ABCD is a rhombus.

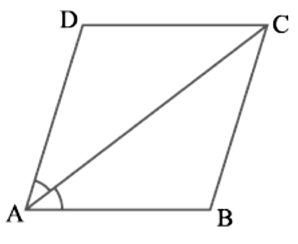


Fig. 8.19

Solution.

(i) ABCD is a parallelogram.

$\angle DAC = \angle BCA$ (Alternate interior angles) ... (1)

And,

∠BAC = ∠DCA (Alternate interior angles) ... (2) However, it is given that AC bisects A

∠DAC = ∠BAC ... (3)

From equations (1), (2), and (3), we obtain

∠DAC = ∠BCA = ∠BAC = ∠DCA ... (4)

∠DCA = ∠BCA

Hence, AC bisects C

From equation (4), we obtain

∠DAC = ∠DCA

DA = DC (Side opposite to equal angles are equal)

However,

DA = BC and AB = CD (Opposite sides of a parallelogram)

AB = BC = CD = DA

Hence, ABCD is a rhombus

Question 7.

ABCD is a rhombus. Show that diagonal AC bisects ∠ A as well as ∠ C and diagonal BD bisects ∠ B as well as ∠ D

Solution.

Let us join AC.

In ΔABC,

BC = AB (Sides of a rhombus are equal to each other)

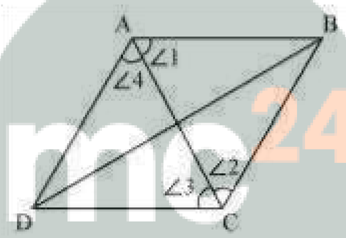
∠1 = ∠2 (Angles opposite to equal sides of a triangle are equal)

However,

∠1 = ∠3 (Alternate interior angles for parallel lines AB and CD)

∠2 = ∠3

Therefore, AC bisects C



Also,

∠2 = ∠4 (Alternate interior angles for || lines BC and DA)

∠1 = ∠4

Therefore,

AC bisects A

Similarly, it can be proved that BD bisects B and D as well

Question 8.

ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C

Show that:

(i) ABCD is a square

(ii) Diagonal BD bisects ∠B as well as ∠D

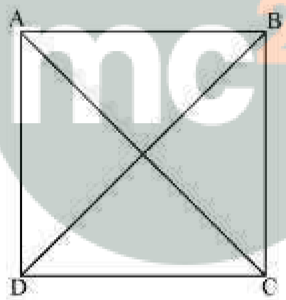
Solution.

It is given that ABCD is a rectangle

∠A = ∠C

$\frac{1}{2}$ ∠A = $\frac{1}{2}$ ∠C

∠DAC =∠ DCA (AC bisects A and C)



CD = DA (Sides opposite to equal angles are also equal)

However,

DA = BC and AB = CD (Opposite sides of a rectangle are equal)

AB = BC = CD = DA

ABCD is a rectangle and all of its sides are equal.

Hence, ABCD is a square

(ii) Let us join BD

In ΔBCD,

BC = CD (Sides of a square are equal to each other)

∠CDB = ∠CBD (Angles opposite to equal sides are equal)

However,

∠CDB = ∠ABD (Alternate interior angles for AB || CD)

∠CBD = ∠ABD

BD bisects B

Also,

∠CBD = ∠ADB (Alternate interior angles for BC || AD)

∠CDB = ∠ABD

BD bisects D

Question 9.

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.20). Show that:

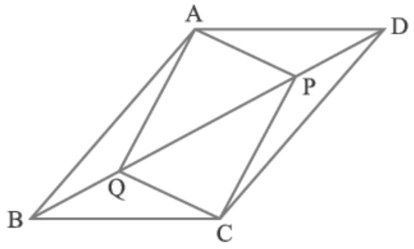


Fig. 8.20

(i) Δ APD ≅Δ CQB

(ii) AP = CQ

(iii) Δ AQB ≅Δ CPD

(iv) AQ = CP

(v) APCQ is a parallelogram

Solution.

(i) In ΔAPD and ΔCQB,

∠ADP = ∠CBQ (Alternate interior angles for BC || AD)

AD = CB (Opposite sides of parallelogram ABCD)

DP = BQ (Given)

ΔAPD ≅ ΔCQB (Using SAS congruence rule)

(ii) As we had observed that,

ΔAPD ≅ ΔCQB

AP = CQ (CPCT)

(iii) In ΔAQB and ΔCPD,

$\angle ABQ = \angle CDP$ (Alternate interior angles for AB || CD)

AB = CD (Opposite sides of parallelogram ABCD)

BQ = DP (Given)

$\Delta AQB \cong \Delta CPD$ (Using SAS congruence rule)

(iv) As we had observed that,

$\Delta AQB \cong \Delta CPD$,

AQ = CP (CPCT)

(v) From the result obtained in (ii) and (iv),

AQ = CP and AP = CQ

Since,

Opposite sides in quadrilateral APCQ are equal to each other,

APCQ is a parallelogram

Question 10.

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that

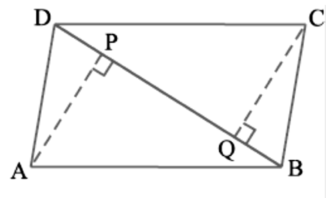


Fig. 8.21

(i) Δ APB ≅Δ CQD

(ii) AP = CQ

Solution.

(i) In ΔAPB and ΔCQD,

∠APB = ∠CQD (Each 90°)

AB = CD (Opposite sides of parallelogram ABCD)

∠ABP= ∠CDQ (Alternate interior angles for AB || CD)

ΔAPB ≅ ΔCQD (By AAS congruency)

(ii) By using the above result

ΔAPB ≅ ΔCQD, we obtain

AP = CQ (By CPCT)

Question 11.

In Δ ABC and Δ DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22). Show that

(i) Quadrilateral ABED is a parallelogram

(ii) Quadrilateral BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) Δ ABC ≅Δ DEF.

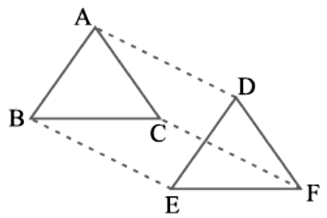


Fig. 8.22

Solution.

(i) Given that: AB = DE and

AB || DE

If two opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.

Therefore, quadrilateral ABED is a parallelogram

(ii) Again,

BC = EF and BC || EF

Therefore, quadrilateral BCEF is a parallelogram

(iii) As we had observed that ABED and BEFC are parallelograms

Therefore,

AD = BE and AD || BE

(Opposite sides of a parallelogram are equal and parallel)

And,

BE = CF and BE || CF

(Opposite sides of a parallelogram are equal and parallel)

AD = CF and AD || CF

(iv) As we had observed that one pair of opposite sides (AD and CF) of quadrilateral

ACFD are equal and parallel to each other, therefore, it is a parallelogram

(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other

AC || DF and AC = DF

(vi) ΔABC and ΔDEF,

AB = DE (Given)

BC = EF (Given)

AC = DF (ACFD is a parallelogram)

ΔABC ≅ ΔDEF (By SSS congruence rule)

Question 12.

ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.23). Show that

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) Diagonal AC = diagonal BD

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

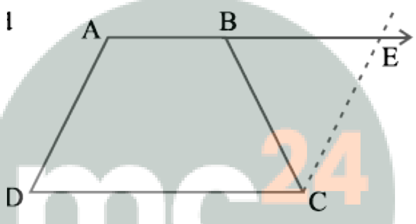


Fig. 8.23

Solution.

Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting AE at point E

It is clear that AECD is a parallelogram

(i) AD = CE (Opposite sides of parallelogram AECD)

However,

AD = BC (Given)

Therefore,

BC = CE

$\angle CEB = \angle CBE$ (Angle opposite to equal sides are also equal)

Consider parallel lines AD and CE. AE is the transversal line for them

$\angle A + \angle CEB = 180^0$ (Angles on the same side of transversal)

$\angle A + \angle CBE = 180^0$ (Using the relation CEB = CBE) (1)

However,

$\angle B + \angle CBE = 180^0$ (Linear pair angles) (2)

From equations (1) and (2), we obtain

$\angle A = \angle B$

(ii) AB || CD

$\angle A + \angle D = 180^0$ (Angles on the same side of the transversal)

Also,

$\angle C + \angle B = 180^0$ (Angles on the same side of the transversal)

$\angle A + \angle D = \angle C + \angle B$

However,

$\angle A = \angle B$ [Using the result obtained in (i)

$\angle C = \angle D$

(iii) In ΔABC and ΔBAD,

$AB = BA$ (Common side)

$BC = AD$ (Given)

$\angle B = \angle A$ (Proved before)

$\Delta ABC \cong \Delta BAD$ (SAS congruence rule)

(iv) We had observed that,

$\Delta ABC \cong \Delta BAD$

$AC = BD$ (By CPCT)