EXERCISE 7.4

Find all the angles of an equilateral triangle. Solution:

In equilateral triangle,
All the sides are equal.
Therefore, all angles are also equal
Let the angles of an equilateral triangle = x
According to angle sum property,
We know that the sum of the interior angles is equal to 180º.
$x+x+x=180^\circ$
$3x=180$
$x=60^\circ$
Therefore, all the angles of an equilateral triangle are 60°

1. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.
[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].

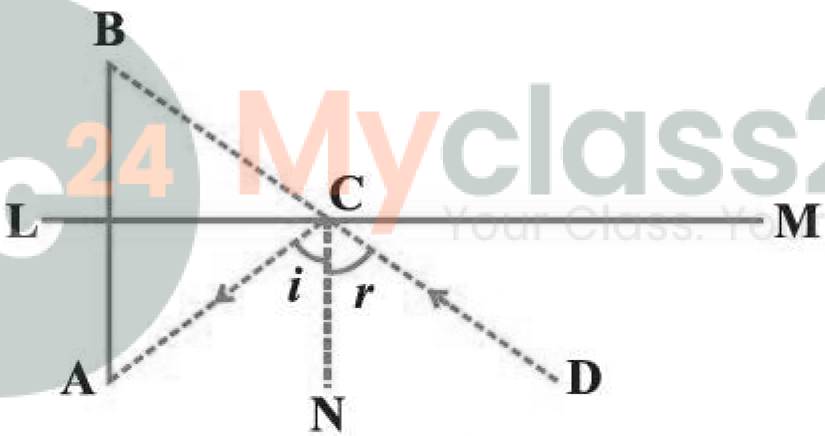
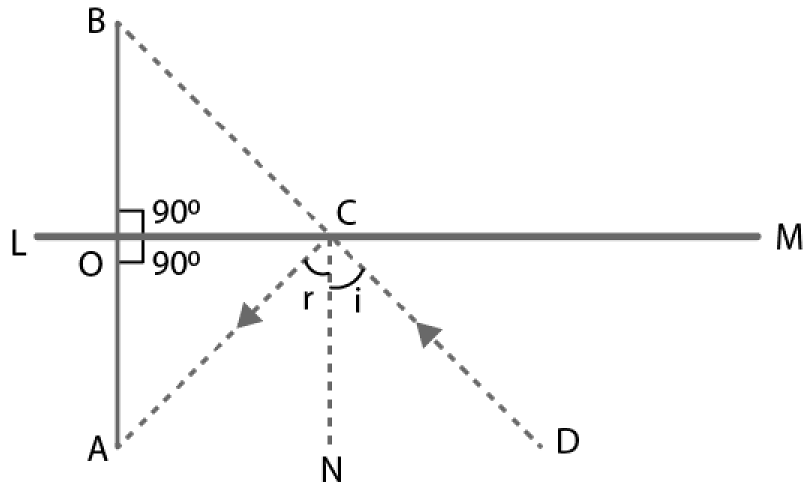


Fig. 7.12

Solution:

Let AB intersect LM at O. We have to prove that AO = BO.
Now, ∠i = ∠r …(1)
[∵Angle of incidence = Angle of reflection]
∠B = ∠i [Corresponding angles] …(2)
And ∠A = ∠r [Alternate interior angles] …(3)



From (1), (2) and (3), we get
∠B = ∠A
⇒ ∠BCO = ∠ACO
In ΔBOC and ΔAOC we have
∠1 = ∠2 [Each = 90º]
OC = OC [Common side]
And ∠BCO = ∠ACO [Proved above]
ΔBOC ≅ ΔAOC [ASA congruence rule]
Hence, AO = BO [CPCT]

2. ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD ⊥ BC (Fig. 7.13). To prove that ∠BAD = ∠CAD, a student proceeded as follows:
In Δ ABD and Δ ACD,
AB = AC (Given)
∠B = ∠C (because AB = AC)
And ∠ADB = ∠ADC
Therefore, Δ ABD Δ Δ ACD (AAS)
So, ∠BAD = ∠CAD (CPCT)

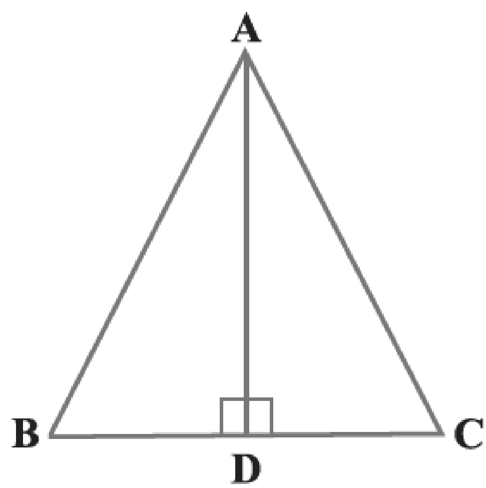
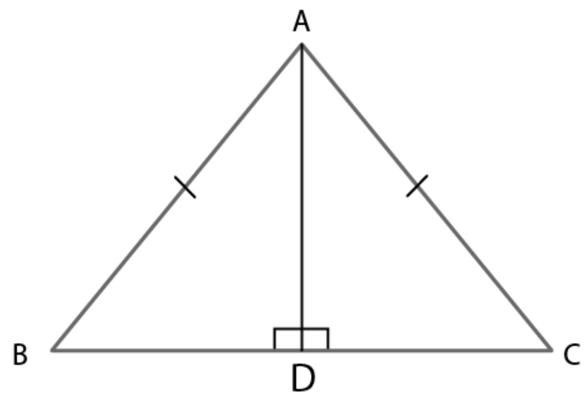


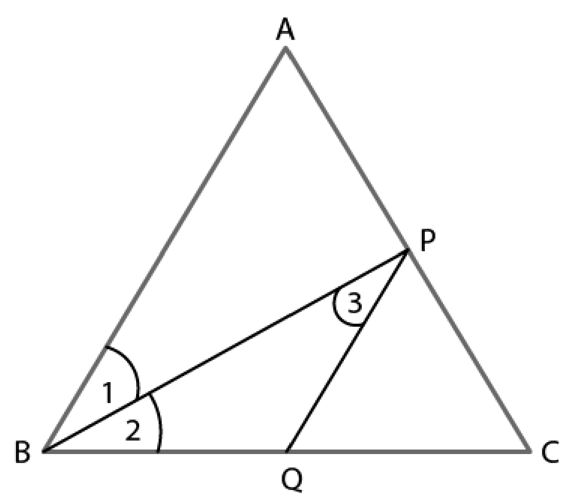
Fig. 7.13

What is the defect in the above arguments?
[Hint: Recall how ∠B = ∠C is proved when AB = AC].
Solution:



In Δ ABD and Δ ADC, we have
∠ADB = ∠ADC
According to the question,
AB = BC
AD = AD [Common side]
By RHS criterion of congruence,
We have,
Δ ABD ≅ Δ ACD
∠BAD = ∠CAD [CPCT]
Hence Proved.

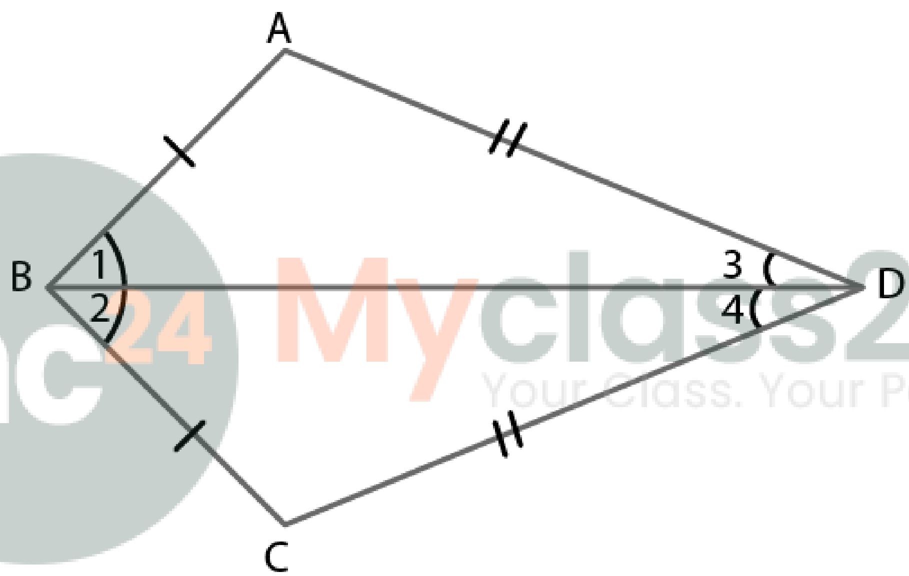
3. P is a point on the bisector of ∠ABC. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.
Solution:



To prove: BPQ is an isosceles triangle.
According to the question,
Since, BP is the bisector of ∠ABC,
∠1 = ∠2 … (1)
Now, PQ is parallel to BA and BP cuts them

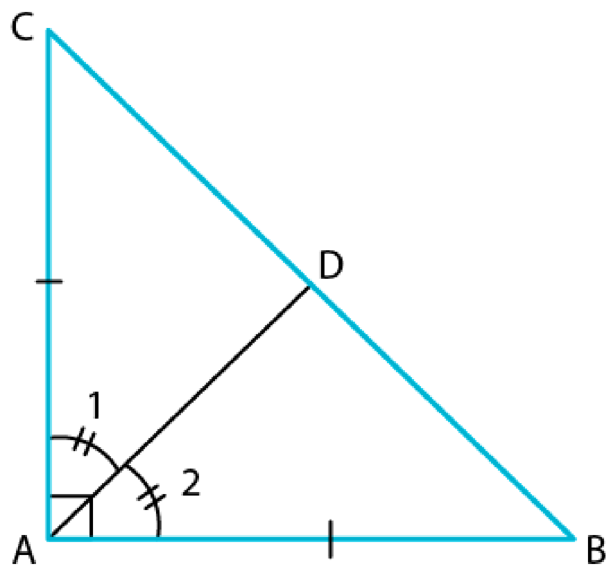
∠1 = ∠3 [Alternate angles] … (2)
From equations, (1) and (2),
We get
∠2 = ∠3
In Δ BPQ,
We have
∠2 = ∠3
PQ = BQ
Hence, BPQ is an isosceles triangle.

4. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.
Solution:



According to the question,
In ΔABC and ΔCBD,
We have
AB = BC
AD = CD
BD = BD [Common side]
ΔABC ≅ ΔCBD [By SSS congruence rule]
⇒ ∠1 = ∠2 [CPCT]
And ∠3 = ∠4
Hence, BD bisects both ∠ABC and ∠ADC.

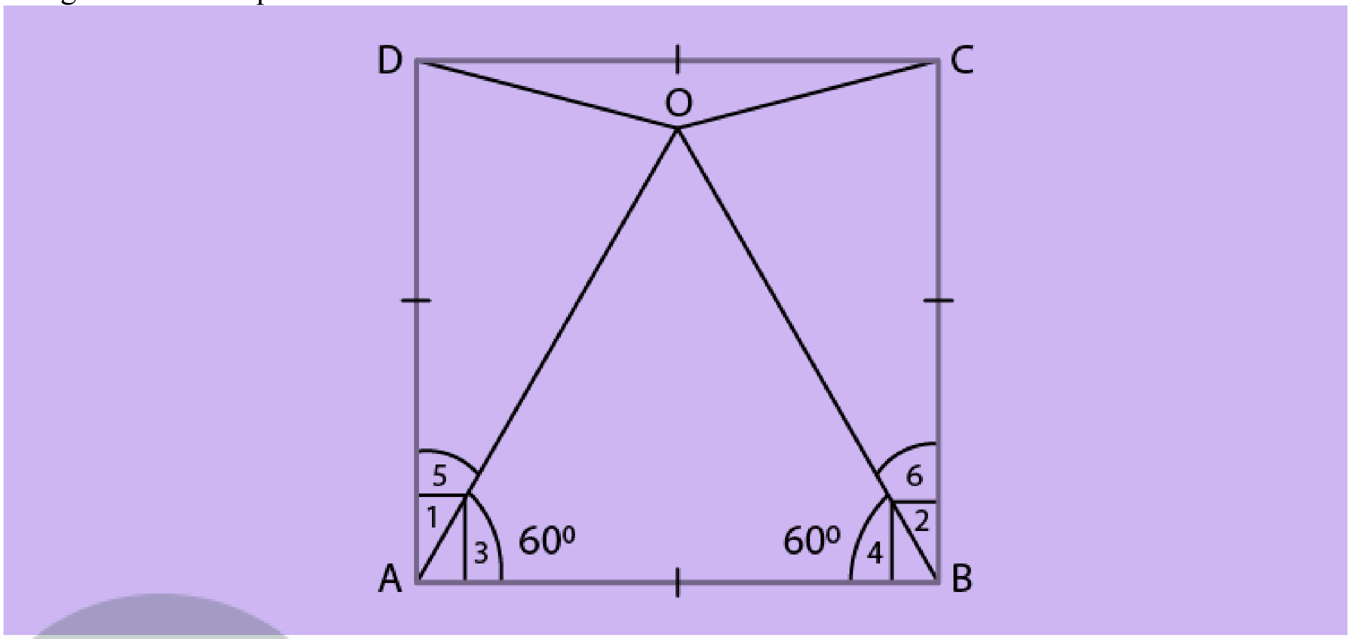
5. ABC is a right triangle with AB = AC. Bisector of ∠A meets BC at D. Prove that BC = 2 AD.
Solution:
Given: A right angles triangle with AB = AC bisector of ∠A meets BC at D.
To prove: BC = 2AD



Proof:
According to the question,
In right Δ ABC,
AB = AC
Since, hypotenuse is the longest side,
BC is hypotenuse
∠BAC = 90°
Now,
In Δ CAD and Δ BAD,
We have,
AC = AB
Since, AD is the bisector of ∠A,
∠1 = ∠2
AD = AD [Common side]
Now,
By SAS criterion of congruence,
We get,
Δ CAD ≅ Δ BAD
CD = BD [CPCT]
Since, Mid-point of hypotenuse of a right triangle is equidistant from the 3 vertices of a triangle.
AB = BD = CD …(1)
Now, BC = BD + CD
⇒ BC = AD + AD [Using eq.(1)]
⇒ BC = 2AD
Hence, proved.

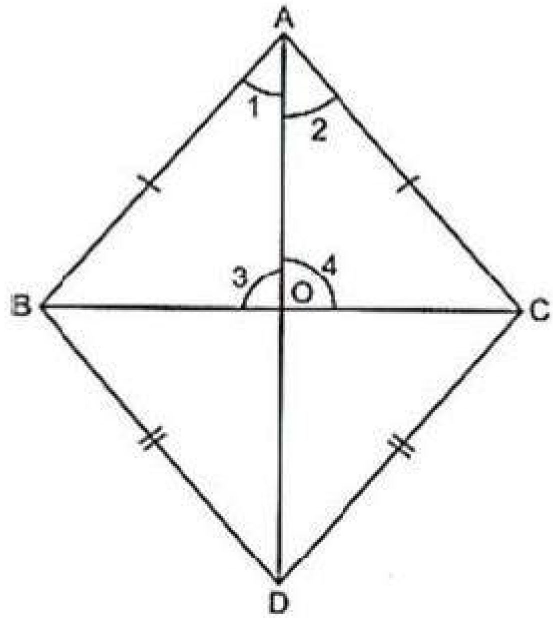
6. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that Δ OCD is an isosceles triangle.
Solution:

According to the question,
It is given that: A square ABCD and OA = OB = AB.



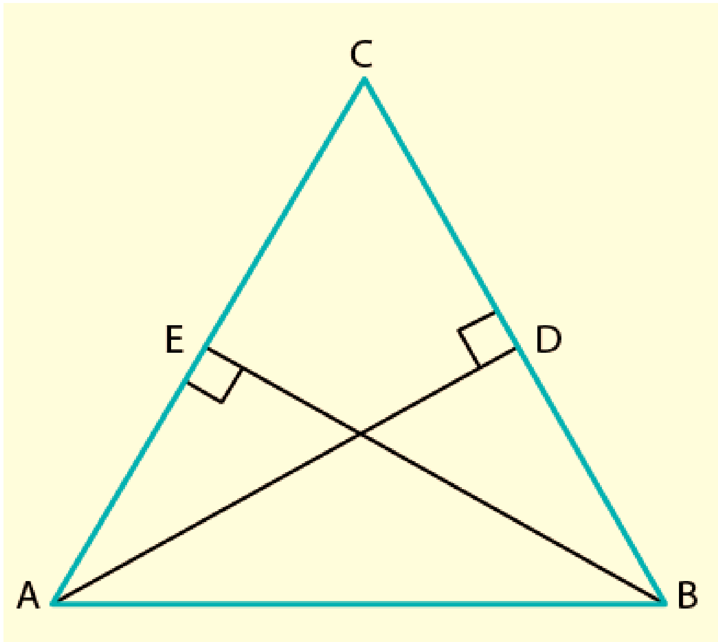
To prove: Δ OCD is an isosceles triangle.
Proof:
In square ABCD,
Since ∠1 and ∠2 is equal to 90°
∠1 = ∠2 …(1)
Now, in Δ OAB , we have
Since ∠3 and ∠4 is equal to 60°
∠3 = ∠4 …(2)
Subtracting equations (2) from (1),
We get
∠1−∠3 = ∠2 −∠4
⇒ ∠5 = ∠6
Now,
In Δ DAO and Δ CBO,
AD = BC [Given]
∠5 = ∠6 [Proved above]
OA = OB [Given]
By SAS criterion of congruence,
We have
Δ DAO ≅ Δ CBO
OD = OC
⇒ Δ OCD is an isosceles triangle.
Hence, proved.

7. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.
Solution:



Given: Δ ABC and Δ DBC on the same base BC. Also, AB = AC and BD = DC.
To prove: AD is the perpendicular bisector of BC i.e., OB = OC
Proof: In Δ BAD and Δ CADwe have
AB = AC [Given]
BD = CD [Given]
AD = AD [common side]
So, by SSS criterion of congruence, we have
Δ BAD ≅ Δ CAD
∠1 = ∠2 [CPCT]
Now, in Δ BAO and Δ CAO, we have
AB = AC [Given]
∠1 = ∠2 [Proved above]
AO = AO [Common side]
So, by SAS criterion of congruence, we have
Δ BAO ≅ Δ CAO
BO = CO [CPCT]
And, ∠3 = ∠4 [CPCT]
But, $\angle 3+\angle 4 = 180^{o}$ [Linear pair axiom]
⇒ $\angle 3+\angle 3 = 180$
⇒ $2\angle 3 = 180$
⇒ $\angle 3 = 180/2$
⇒ $\angle 3 = 90^{o}$
Since BO = CO and $\angle 3 = 90^{o}$,
AD is perpendicular bisector of BC.
Hence, proved.

8. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.
Solution:

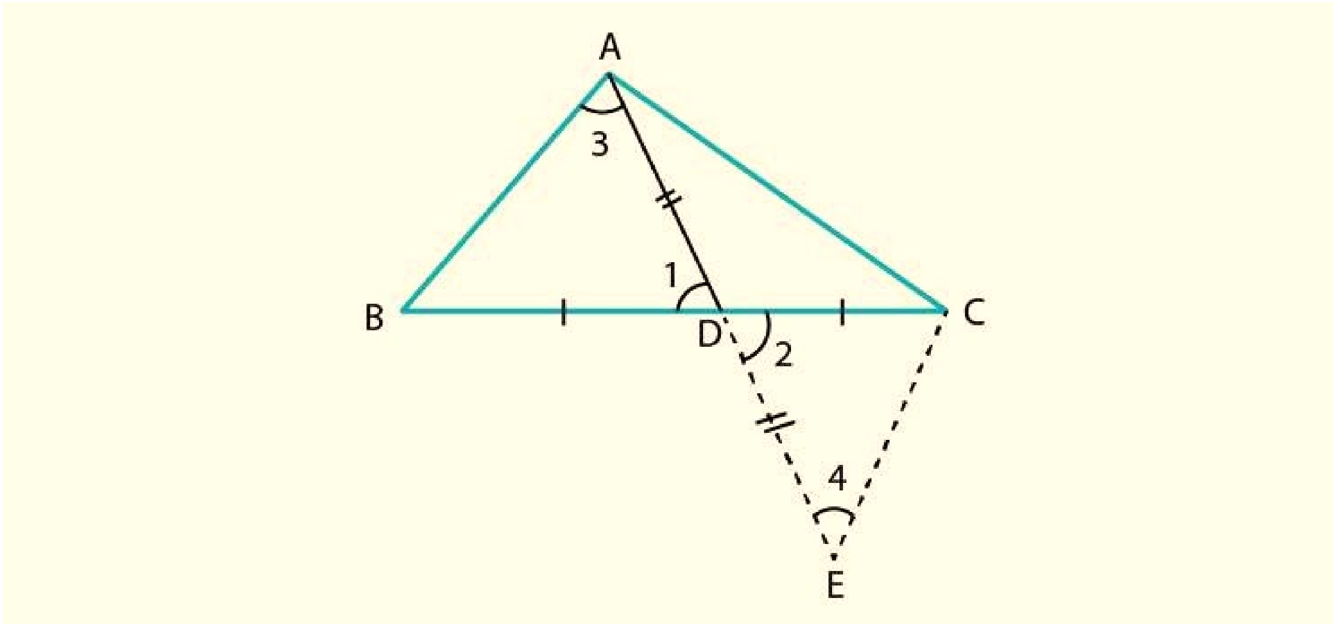


According to the question,
In Δ ADC and Δ BEC,
We have
AC = BC [Given] …(1)
Since ∠ADC and ∠BEC = 90°
∠ADC = ∠BEC
∠ACD = ∠BCE [Common angle]
Δ ADC ≅ Δ BEC [By ASA congruence rule]
CE = CD … (2) [CPCT]
Subtracting equation (2) from (1),
We get
AC – CE = BC – CD
⇒ AE = BD
Hence, proved.

9. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Solution:

According to the question,
We have, Δ ABC with median AD.
To prove:
AB + AC > 2AD
AB + BC > 2AD
BC + AC > 2AD
Construction:
Extend AD to E such that DE = AD
Join EC.
Proof:



In Δ ADB and Δ EDC ,

AD = ED [By construction]

∠1 = ∠2 [Vertically opposite angles are equal]

DB = DC [Given]

So, by SAS criterion of congruence, we have

Δ ADB ≅ Δ EDC

AB = EC [CPCT]

And ∠3 = ∠4 [CPCT]

Now, in Δ AEC,

Since sum of the lengths of any two sides of a triangle must be greater than the third side,

We have

AC + CE > AE

⇒ AC + CE > AD + DE

⇒ AC +CE > AD+ AD [\AD = DE]

⇒ AC + CE > 2AD

⇒ AC + AB > 2AD [∵AB = CE]

Similarly,

We get,

AB + BC > 2AD and BC + AC >2AD.

Hence, proved.