Exercise 9(B)

Solution 1:

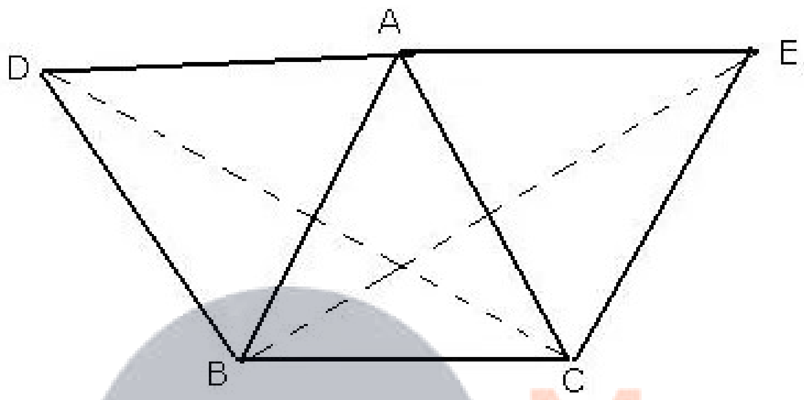
Given: ΔABD is an equilateral triangle

ΔACE is an equilateral triangle

We need to prove that

(i) $\angle CAD = \angle BAE$

(ii) $CD = BE$



Proof:

(i)

ΔABD is equilateral

$\therefore$ Each angle = $60°$

$\Rightarrow \angle BAD = 60°$ ...(1)

Similarly,

ΔACE is equilateral

$\therefore$ Each angle = $60°$

$\Rightarrow \angle CAE = 60°$ ...(2)

$\Rightarrow \angle BAD = \angle CAE$ [from (1) and (2)] ...(3)

Adding $\angle BAC$ to both sides, we have

$\angle BAD + \angle BAC = \angle CAE + \angle BAC$

$\Rightarrow \angle CAD = \angle BAE$ ...(4)

(ii)

In ΔCAD and ΔBAE

$AC = AE$ [ΔACE is equilateral]

$\angle CAD = \angle BAE$ [from (4)]

$AD = AB$ [ΔABD is equilateral]

$\therefore$ By Side-Angle-Side criterion of congruency,

$\Delta CAD \cong \Delta BAE$

The corresponding parts of the congruent triangles are congruent.

$\therefore CD = BE$ [by c.p.c.t]

Hence proved.

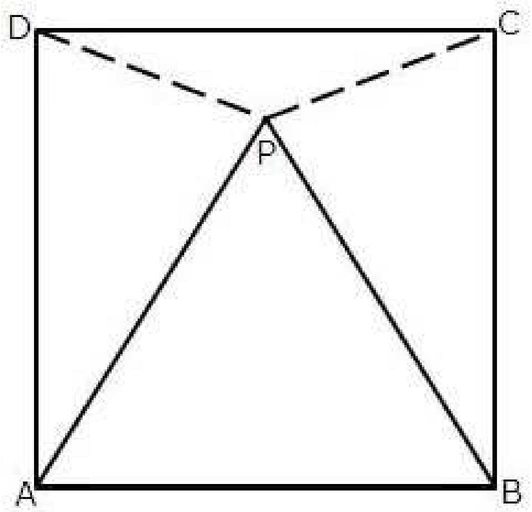
Solution 2:

Given: ABCD is a square and ΔAPB is an equilateral triangle.

We need to

(i)Prove that, $\Delta APD \cong \Delta BPC$

(ii) To find angles of ΔDPC



(a)

(i)Proof:

AP=PB=AB [ΔAPB is an equilateral triangle]

Also, we have,

$\angle PBA = \angle PAB = \angle APB = 60°$ ...(1)

Since ABCD is a square, we have

$\angle A = \angle B = \angle C = \angle D = 90°$ ...(2)

Since $\angle DAP = \angle A - \angle PAB$ ...(3)

$\Rightarrow \angle DAP = 90° - 60°$

$\Rightarrow \angle DAP = 30°$ [from (1) and (2)] ...(4)

Similarly $\angle CBP = \angle B - \angle PBA$

$\Rightarrow \angle CBP = 90° - 60°$

$\Rightarrow \angle CBP = 30°$ [from (1) and (2)] ...(5)

$\Rightarrow \angle DAP = \angle CBP$ [from (4) and (5)] ...(6)

In ΔAPD and ΔBPC

$AD = BC$ [Sides of square ABCD]

$\angle DAP = \angle CBP$ [from (6)]

$AP = BP$ [Sides of equilateral ΔAPB]

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$\Delta APD \cong \Delta BPC$

(ii)

$AP = PB = AB$ [ΔAPB is an equilateral triangle] ...(7)

$AB = BC = CD = DA$ [Sides of square ABCD] ...(8)

From (7) and (8), we have

$AP = DA$ and $PB = BC$ ...(9)

In ΔAPD,

$AP = DA$ [from (9)]

$\therefore \angle ADP = \angle APD$ [Angles opposite to equal sides are equal] ...(10)

$\angle ADP + \angle APD + \angle DAP = 180°$ [Sum of angles of a triangle $= 180°$]

$\Rightarrow \angle ADP + \angle ADP + 30° = 180°$ [from (3), $\angle DAP = 30°$; from (10), $\angle ADP = \angle APD$]

$\Rightarrow \angle ADP + \angle ADP = 180° - 30°$

$\Rightarrow 2\angle ADP = 150°$

$\Rightarrow \angle ADP = \dfrac{150°}{2}$

$\Rightarrow \angle ADP = 75°$

We have $\angle PDC = \angle D - \angle ADP$

$\Rightarrow \angle PDC = 90° - 75°$

$\Rightarrow \angle PDC = 15°$ ...(11)

In ΔBPC,

$PB = BC$ [from (9)]

$\therefore \angle PCB = \angle BPC$ [Angles opposite to equal sides are equal] ...(12)

$\angle PCB + \angle BPC + \angle CBP = 180°$ [Sum of angles of a triangle $= 180°$]

$\Rightarrow \angle PCB + \angle PCB + 30° = 180°$ [from (5), $\angle CBP = 30°$; from (12), $\angle PCB = \angle BPC$]

$\Rightarrow 2\angle PCB = 180° - 30°$

$\Rightarrow \angle PCB = \frac{150°}{2}$

$\Rightarrow \angle PCB = 75°$

We have $\angle PCD = \angle C - \angle PCB$

$\Rightarrow \angle PCD = 90° - 75°$

$\Rightarrow \angle PCD = 15°$ ...(13)

In ΔDPC,

$\angle PDC = 15°$

$\angle PCD = 15°$

$\angle PCD + \angle PDC + \angle DPC = 180°$ [Sum of angles of a triangle $= 180°$]

$\Rightarrow 15° + 15° + \angle DPC = 180°$

$\Rightarrow \angle DPC = 180° - 30°$

$\Rightarrow \angle DPC = 150°$

$\therefore$ Angles of ΔDPC, are: $15°, 150°, 15°$

(b)

(i)Proof: In ΔAPB

$AP = PB = AB$ [ΔAPB is an equilateral triangle]

Also, we have,

$\angle PBA = \angle PAB = \angle APB = 60°$ ...(1)

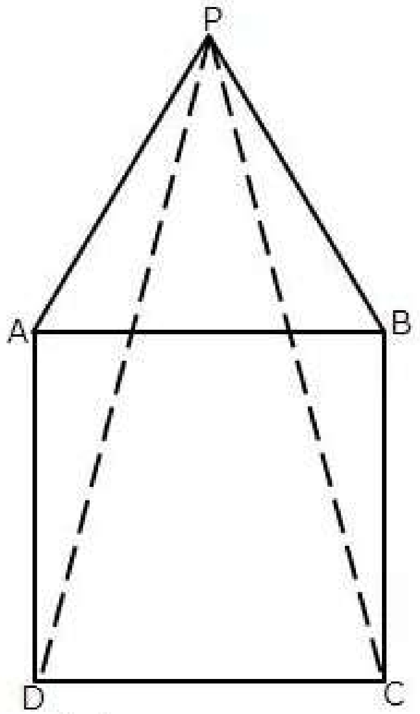
Since ABCD is a square, we have

$\angle A = \angle B = \angle C = \angle D = 90°$ ...(2)

Since $\angle DAP = \angle A + \angle PAB$ ...(3)

$\Rightarrow \angle DAP = 90° + 60°$

$\Rightarrow \angle DAP = 150°$ [from (1) and (2)] ...(4)



Similarly $\angle CBP = \angle B + \angle PBA$

$\Rightarrow \angle CBP = 90° + 60°$

$\Rightarrow \angle CBP = 150°$ [from (1) and (2)] ...(5)

$\Rightarrow \angle DAP = \angle CBP$ [from (4) and (5)] ...(6)

In ΔAPD and ΔBPC

$AD = BC$ [Sides of square ABCD]

$\angle DAP = \angle CBP$ [from (6)]

$AP = BP$ [Sides of equilateral ΔAPB]

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$\Delta APD \cong \Delta BPC$

(ii)

$AP = PB = AB$ [ΔAPB is an equilateral triangle] ...(7)

$AB = BC = CD = DA$ [Sides of square ABCD] ...(8)

From (7) and (8), we have

AP=DA and PB=BC ...(9)

In ΔAPD,

AP=DA [from (9)]

$\therefore \angle ADP = \angle APD$ [Angles opposite to equal sides are equal] ...(10)

$\angle ADP + \angle APD + \angle DAP = 180°$ [Sum of angles of a triangle =180°]

$\Rightarrow \angle ADP + \angle ADP + 150° = 180°$ [from (3), $\angle DAP = 150°$; from (10), $\angle ADP = \angle APD$]

$\Rightarrow \angle ADP + \angle ADP = 180° - 150°$

$\Rightarrow 2\angle ADP = 30°$

$\Rightarrow \angle ADP = \frac{30°}{2}$

$\Rightarrow \angle ADP = 15°$

We have $\angle PDC = \angle D - \angle ADP$

$\Rightarrow \angle PDC = 90° - 15°$

$\Rightarrow \angle PDC = 75°$ ...(11)

In ΔBPC,

PB=BC [from (9)]

$\therefore \angle PCB = \angle BPC$ [Angles opposite to equal sides are equal] ...(12)

$\angle PCB + \angle BPC + \angle CBP = 180°$ [Sum of angles of a triangle =180°]

$\Rightarrow \angle PCB + \angle PCB + 150° = 180°$ [from (5), $\angle CBP = 150°$; from (12), $\angle PCB = \angle BPC$]

$\Rightarrow 2\angle PCB = 180° - 150°$

$\Rightarrow \angle PCB = \frac{30°}{2}$

$\Rightarrow \angle PCB = 15°$

We have $\angle PCD = \angle C - \angle PCB$

$\Rightarrow \angle PCD = 90° - 15°$

$\Rightarrow \angle PCD = 75°$ ...(13)

In ΔDPC,

$\angle PDC = 75°$

$\angle PCD = 75°$

$\angle PCD + \angle PDC + \angle DPC = 180°$ $\begin{bmatrix} \text{Sum of angles of} \\ \text{a triangle } = 180° \end{bmatrix}$

$\Rightarrow 75° + 75° + \angle DPC = 180°$

$\Rightarrow \angle DPC = 180° - 150°$

$\Rightarrow \angle DPC = 30°$

$\therefore$ Angles of ΔDPC, are: $75°, 30°, 75°$

Solution 3:

Given: A ΔABC is right angled at B.

ABPQ and ACRS are squares

We need to prove that

(i) $\Delta ACQ \cong \Delta ASB$

(ii) $CQ = BS$

Proof:

(i)

$\angle QAB = 90°$ [$ABPQ$ is a square] ...(1)

$\angle SAC = 90°$ [$ACRS$ is a square] ...(2)

From (1) and (2), we have

$\angle QAB = \angle SAC$ ...(3)

Adding $\angle BAC$ to both sides of (3), we have

$\angle QAB + \angle BAC = \angle SAC + \angle BAC$

$\Rightarrow \angle QAC = \angle SAB$ ...(4)

In ΔACQ and ΔASB,

$QA = QB$ [sides of a square ABPQ]

$\angle QAC = \angle SAB$ [from (4)]

$AC = AS$ [sides of a square ACRS]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta ACQ \cong \Delta ASB$

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore CQ = BS$ [c.p.c.t]

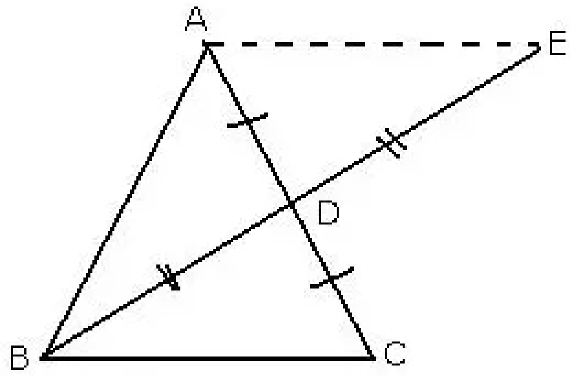
Solution 4:

Given: A ΔABC in which BD is the median to AC.

BD is produced to E such that BD=DE.

We need to prove that AE || BC.

Construction: Join AE



Proof:

AD=DC [*BD* is median to AC] ...(1)

In ΔBDC and ΔADE

$BD = DE$ [Given]

$\angle BDC = \angle ADE = 90^\circ$ [vertically opposite angles]

$AD = DC$ [from (1)]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta BDC \cong \Delta ADE$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle EAD = \angle BCD$ [c.p.c.t]

But these are alternate angles and AC is the transversal

Thus, AE || BC

Solution 5:

Given: A ΔPQR in which QX is the bisector of $\angle Q$ and RX is the bisector of $\angle R$.

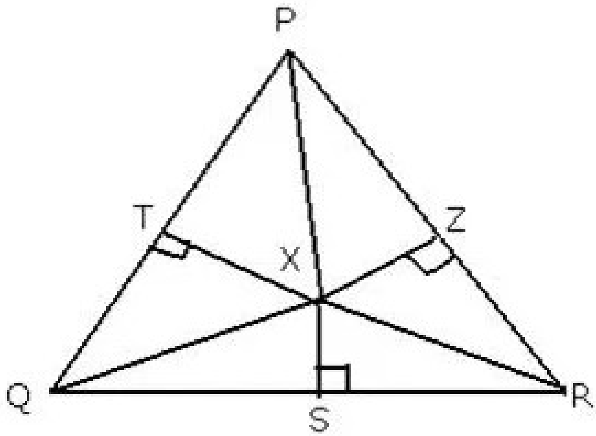
$XS \perp QR$ and $XT \perp PQ$.

We need to prove that

(i) $\Delta XTQ \cong \Delta XSQ$

(ii) PX bisects $\angle P$

Construction: Draw $XZ \perp PR$ and join PX.



Proof:

(i)

In ΔXTQ and ΔXSQ

$\angle QTX = \angle QSX = 90^\circ$ [$XS \perp QR$ and $XT \perp PQ$]

$\angle TQX = \angle SQX$ [QX is bisector of $\angle Q$]

$QX = QX$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta XTQ \cong \Delta XSQ$ ...(1)

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore XT = XS$ [c.p.c.t]

In ΔXSR and ΔXZR

$\angle XSR = \angle XZR = 90^\circ$ [$XS \perp QR$ and $\angle XSR = 90^\circ$]

$\angle SRX = \angle ZRX$ [RX is bisector of $\angle R$]

$RX = RX$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta XSR \cong \Delta XZR$

The corresponding parts of the congruent triangles are congruent.

$\therefore XS=XZ$ [c.p.c.t] ...(2)

From (1) and (2)

$XT=XZ$ ...(3)

In ΔXTP and ΔXZP

$\angle XTP=\angle XZP=90°$ [Given]

Hyp. XP=Hyp.XP [Common]

$XT = XZ$ [from (3)]

$\therefore$ By Right angle-Hypotenuse-Side criterion of congruence,

$\Delta XTP \cong \Delta XZP$

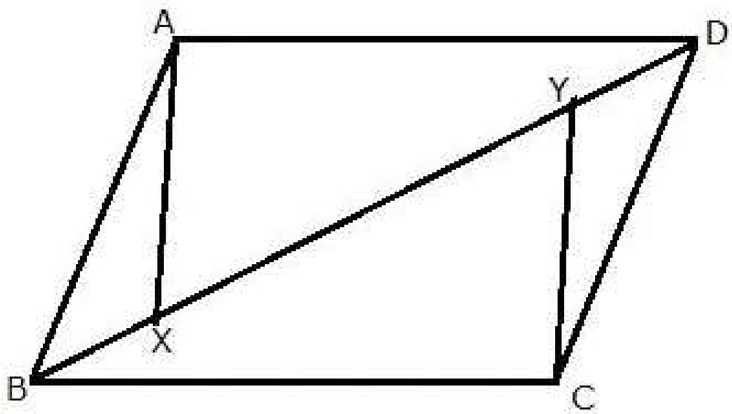
The corresponding parts of the congruent triangles are congruent.

$\therefore \angle XPT=\angle XPZ$ [c.p.c.t]

$\therefore PX$ bisects $\angle P$

Solution 6:

ABCD is a parallelogram in which ∠A and ∠C are obtuse.



Points X and Y are taken on the diagonal BD such that ∠XAD = ∠YCB = 90°.

We need to prove that XA = YC

Proof:

In ΔXAD and ΔYCB

$\angle XAD = \angle YCB = 90°$ [Given]

AD = BC [Opposite sides of a parallelogram]

$\angle ADX = \angle CBY$ [Alternate angles]

∴ By Angle-Side-Angle criterion of congruence,

$\Delta XAD \cong \Delta YCB$

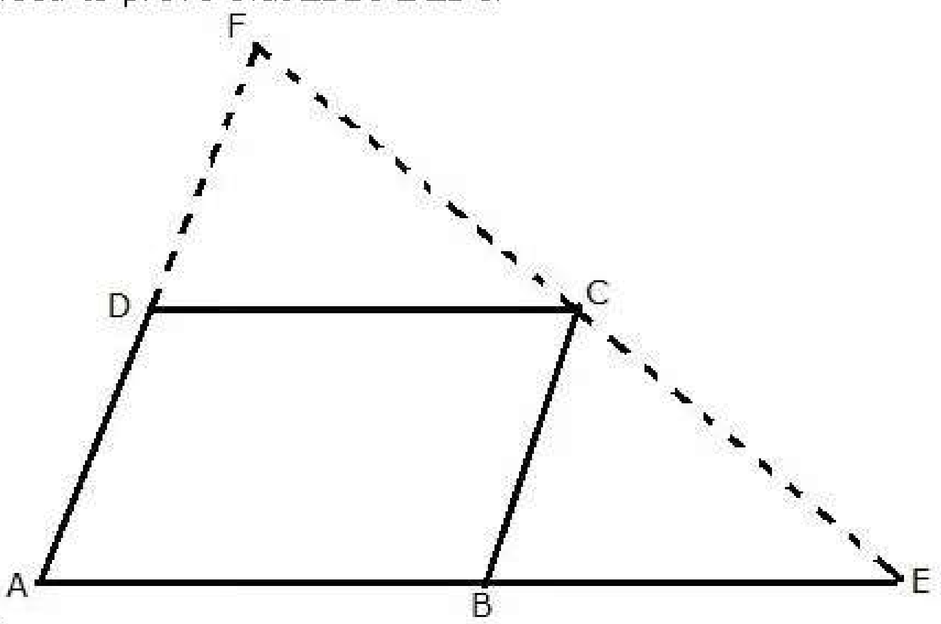
The corresponding parts of the congruent triangles are congruent.

∴ $XA = YC$ [c.p.c.t]

Hence proved.

Solution 7:

ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively,
such that AB = BE and AD = DF.
We need to prove that $\Delta BEC \cong \Delta DCF$



Proof:

$AB = DC$ $\left[\begin{array}{l}\text{Opposite sides of a} \\ \text{parallelogram}\end{array}\right]$ ...(1)

$AB = BE$ [Given] ...(2)

From (1) and (2), we have

$BE = DC$ ...(3)

$AD = BC$ $\left[\begin{array}{l}\text{Opposite sides of a} \\ \text{parallelogram}\end{array}\right]$ ...(4)

$AD = DF$ [Given] ...(5)

From (4) and (5), we have

$BC = DF$ ...(6)

Since $AD \parallel BC$, the corresponding angles are equal.

$\therefore \angle DAB = \angle CBE$ ...(7)

Since $AB \parallel DC$, the corresponding angles are equal.

$\therefore \angle DAB = \angle FDC$ ...(8)

From (7) and (8), we have

$\angle CBE = \angle FDC$ ...(9)

In ΔBEC and ΔDCF

In ΔBEC and ΔDCF

$BE = DC$ [from (3)]

$\angle CBE = \angle FDC$ [from (9)]

$BC = DF$ [from (6)]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta BEC \cong \Delta DCF$

Hence proved.

Solution 8:

Since, BC=QR, we have

BD=QS and DC=SR [D is the midpoint of BC and S is the midpoint of QR]

In ΔABD and ΔPQS

$AB = PQ$ ...(1)

$AD = PS$ ...(2)

$BD = QS$ ...(3)

Thus, by Side-Side-Side criterion of congruence,

we have $\Delta ABD \cong \Delta PQS$

Similarly, in ΔADC and ΔPSR

$AD = PS$ ...(4)

$AC = PR$ ...(5)

$DC = SR$ ...(6)

Thus, by Side-Side-Side criterion of congruence,

we have $\Delta ADC \cong \Delta PSR$

We have

$BC = BD + DC$ [D is the midpoint of BC]

$= QS + SR$ [from (3) and (6)]

$= QR$ [S is the midpoint of QR] ...(7)

Now consider the triangles ΔABC and ΔPQR

$AB = PQ$ [from (1)]

$BC = QR$ [from (7)]

$AC = PR$ [from (7)]

$\therefore$ By Side-Side-Side criterion of congruence, we have $\Delta ABC \cong \Delta PQR$

Hence proved.

Solution 9:

In the figure, AP and BQ are equal and parallel to each other. $\therefore$ AP=BQ and AP || BQ.

We need to prove that

(i) $\Delta AOP \cong \Delta BOQ$

(ii) AB and PQ bisect each other

(i) $\because$ AP || BQ

$\therefore \angle APO = \angle BQO$ [Alternate angles] ...(1)

and $\angle PAO = \angle QBO$ [Alternate angles] ...(2)

Now in ΔAOP and ΔBOQ,

$\angle APO = \angle BQO$ [from (1)]

AP=BQ [given]

$\angle PAO = \angle QBO$ [from (2)]

$\therefore$ By Angle-Side-Angle criterion of congruence, we have

$\Delta AOP \cong \Delta BOQ$

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore OP = OQ$ [c.p.c.t]

OA=OB [c.p.c.t]

Hence AB and PQ bisect each other.

Solution 10:

Given:

In the figure, OA=OC, AB=BC

We need to prove that,

(i) $\angle AOB = 90^\circ$

(ii) $\Delta AOD \cong \Delta COD$

(iii) $AD = CD$

(i) In ΔABO and ΔCBO,

$AB=BC$ [given]

$AO=CO$ [given]

$OB=OB$ [common]

$\therefore$ By Side-Side-Side criterion of congruence, we have

$\Delta ABO \cong \Delta CBO$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle ABO=\angle CBO$ [c.p.c.t]

$\Rightarrow \angle ABD=\angle CBD$

and $\angle AOB=\angle COB$ [c.p.c.t]

We have

$\angle AOB+\angle COB=180^\circ$ [Linear pair]

$\Rightarrow \angle AOB=\angle COB=90^\circ$ and $AC \perp BD$

(ii) In ΔAOD and ΔCOD,

$OD=OD$ [common]

$\angle AOD=\angle COD$ [each=90°]

$AO=CO$ [given]

$\therefore$ By Side-Angle-Side criterion of congruence, we have

$\Delta AOD \cong \Delta COD$

(iii)

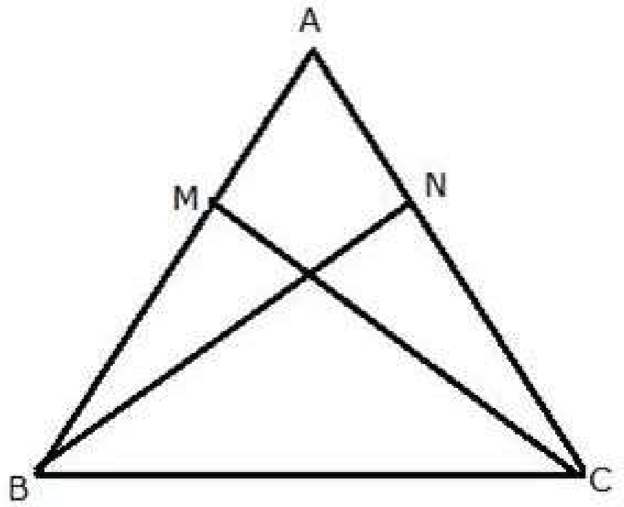
The corresponding parts of the congruent triangles are congruent.

$\therefore AD=CD$ [c.p.c.t]

Hence proved.

Solution 11:

In ΔABC, AB=AC. M and N are points on AB and AC such tht BM=CN.
BN and CM are joined.



(i) In ΔAMC and ΔANB

$AB = AC$ [Given] ...(1)

$BM = CN$ [Given] ...(2)

Subtracting (2) from (1), we have

$AB - BM = AC - CN$

$\Rightarrow AM = AN$ ...(3)

(ii) Consider the triangles ΔAMC and ΔANB

$AC = AB$ [given]

$\angle A = \angle A$ [common]

$AM = AN$ [from (3)]

$\therefore$ By Side-Angle-Side criterion of congruence, we have $\Delta AMC \cong \Delta ANB$

(iii)

The corresponding parts of the congruent triangles are congruent.

$\therefore CM = BN$ [c.p.c.t] ...(4)

(iv) Consider the triangles ΔBMC and ΔCNB

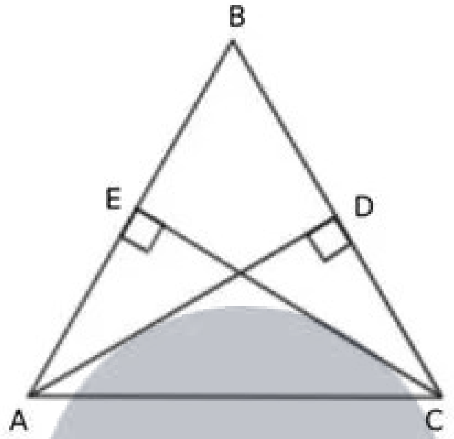
$BM = CN$ [given]

$BC = BC$ [common]

$CM = BN$ [from (4)]

$\therefore$ By Side-Side-Side criterion of congruence, we have $\Delta BMC \cong \Delta CNB$

Solution 12:



In ΔABD and ΔCBE,

$AB = BC$ (given)

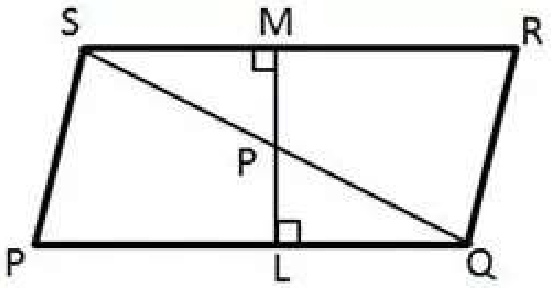
$\angle ADB = \angle CEB = 90^\circ$

$\angle B = \angle B$ (common angle)

$\therefore \Delta ABD \cong \Delta CBE$ (by SAS congruence)

$\Rightarrow AD = CE$ (cpct)

Solution 13:



Given : PL = RM

To prove : SP = PQ and MP = PL

Proof :

Since SR and PQ are opposite sides of a parallelogram,

PQ = SR ....(1)

Also, PL = RM ....(2)

Subtracting (2) from (1),

PQ – PL = SR – RM

$\Rightarrow$ LQ = SM ....(3)

Now, in ΔSMP and ΔQLP,

$\angle$MSP = $\angle$PQL (alternate interior angles)

$\angle$SMP = $\angle$PLQ (alternate interior angles)

SM = LQ [From (3)]

$\therefore$ $\Delta SMP \cong \Delta QLP$ (by ASA congruence)

$\Rightarrow$ SP = PQ and MP = PL (cpct)

$\Rightarrow$ LM and QS bisect each other.

Solution 14:

ΔABC is an equilateral triangle.

So, each of its angles equals 60°.

QP is parallel to AC,

$\Rightarrow \angle PQB = \angle RAQ = 60°$

In ΔQBP,

$\angle PBQ = \angle BQP = 60°$

So, $\angle PBQ + \angle BQP + \angle BPQ = 180°$ (angle sum property)

$\Rightarrow 60° + 60° + \angle BPQ = 180°$

$\Rightarrow \angle BPQ = 60°$

So, ΔBPQ is an equilateral triangle.

$\Rightarrow QP = BP$

$\Rightarrow QP = CR$ ...(i)

Now, $\angle QPM + \angle BPQ = 180°$ (linear pair)

$\Rightarrow \angle QPM + 60° = 180°$

$\Rightarrow \angle QPM = 120°$

Also, $\angle RCM + \angle ACB = 180°$ (linear pair)

$\Rightarrow \angle RCM + 60° = 180°$

$\Rightarrow \angle RCM = 120°$

In ΔRCM and ΔQMP,

$\angle RCM = \angle QPM$ (each is 120°)

$\angle RMC = \angle QMP$ (vertically opposite angles)

$QP = CR$ (from (i))

$\Rightarrow \Delta RCM \cong \Delta QMP$ (AAS congruence criterion)

So, $CM = PM$

$\Rightarrow$ QR bisects PC.