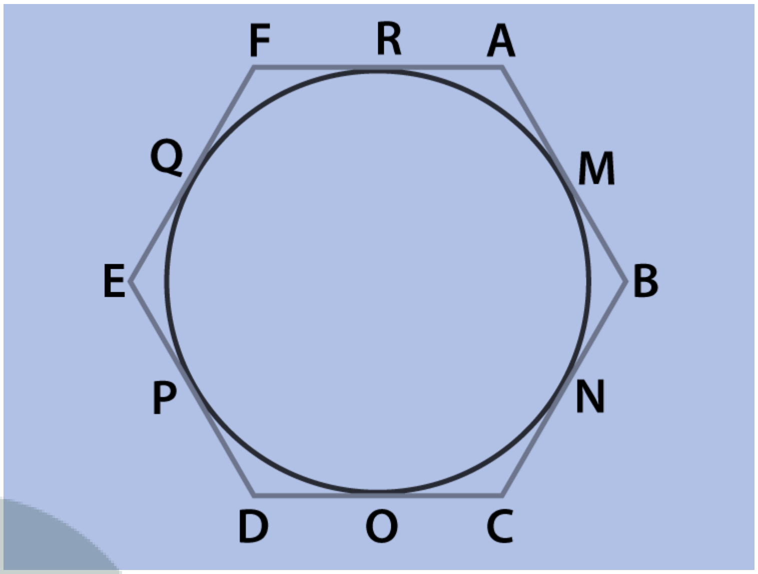
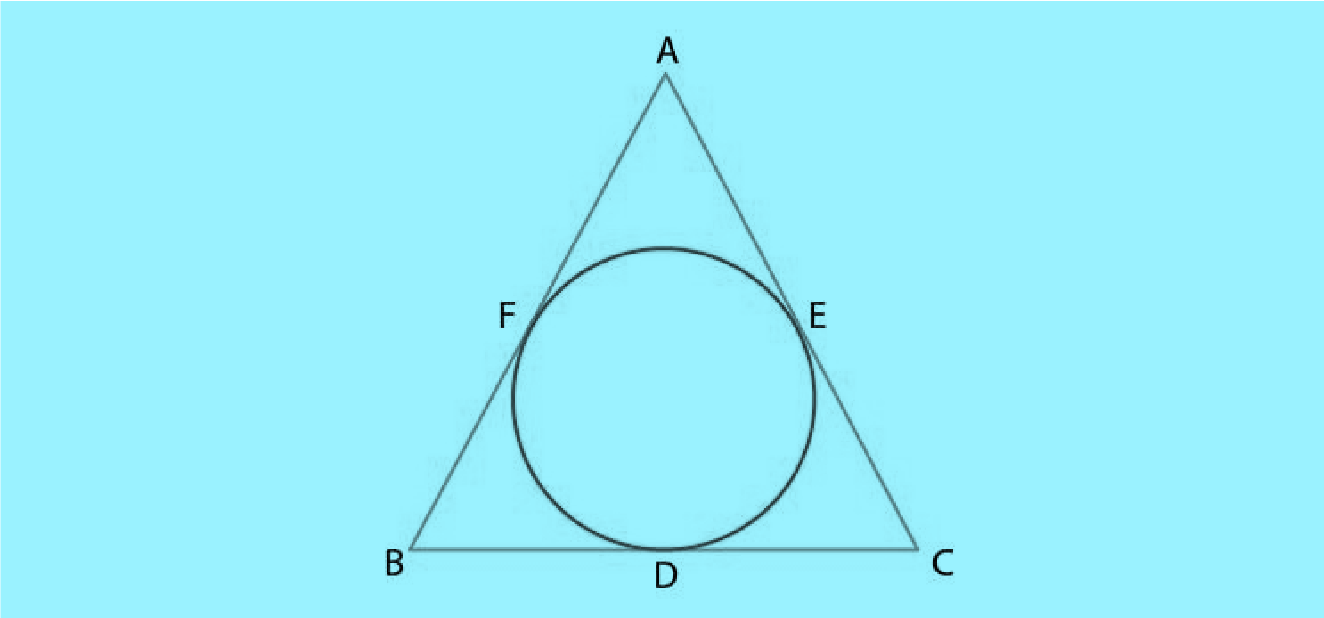
EXERCISE 9.4

1. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA.
Solution:



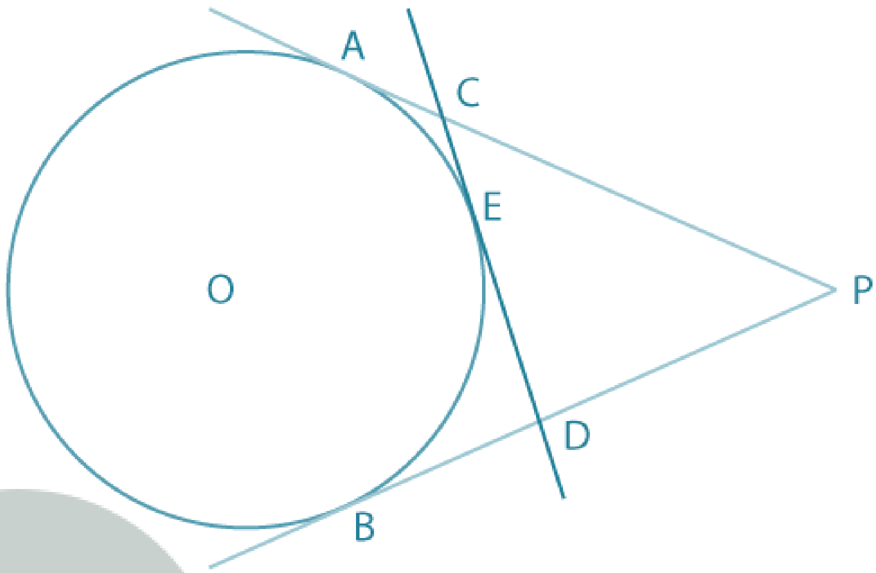
According to the question,
A Hexagon ABCDEF circumscribe a circle.
To prove:
AB + CD + EF = BC + DE + FA
Proof:
Tangents drawn from an external point to a circle are equal.
Hence, we have
AM = RA … eq 1 [tangents from point A]
BM = BN … eq 2 [tangents from point B]
CO = NC … eq 3 [tangents from point C]
OD = DP … eq 4 [tangents from point D]
EQ = PE … eq 5 [tangents from point E]
QF = FR … eq 6 [tangents from point F]
[eq 1]+[eq 2]+[eq 3]+[eq 4]+[eq 5]+[eq 6]
AM + BM + CO + OD + EQ + QF = RA + BN + NC + DP + PE + FR
On rearranging, we get,
(AM + BM) + (CO + OD) + (EQ + QF) = (BN + NC) + (DP + PE) + (FR + RA)
AB + CD + EF = BC + DE + FA
Hence Proved!

2. Let *s* denote the semi-perimeter of a triangle ABC in which BC = *a*, CA = *b*, AB = *c*. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = *s* – *b*.
Solution:



According to the question,
A triangle ABC with BC = a , CA = b and AB = c . Also, a circle is inscribed which touches the sides BC, CA and AB at D, E and F respectively and s is semi- perimeter of the triangle
To Prove: BD = s – b
Proof:
According to the question,
We have,
Semi Perimeter = s
Perimeter = 2s
2s = AB + BC + AC [1]
As we know,
Tangents drawn from an external point to a circle are equal
So we have
AF = AE [2] [Tangents from point A]
BF = BD [3] [Tangents From point B]
CD = CE [4] [Tangents From point C]
Adding [2] [3] and [4]
AF + BF + CD = AE + BD + CE
AB + CD = AC + BD
Adding BD both side
AB + CD + BD = AC + BD + BD
AB + BC - AC = 2BD
AB + BC + AC - AC - AC = 2BD
2s - 2AC = 2BD [From 1]
2BD = 2s - 2b [as AC = b]
BD = s – b
Hence Proved.

3. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.
Solution:



According to the question,
From an external point P, two tangents, PA and PB are drawn to a circle with center O.
At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 10 cm
To Find : Perimeter of △PCD
As we know that, Tangents drawn from an external point to a circle are equal.
So we have
AC = CE [1] [Tangents from point C]
ED = DB [2] [Tangents from point D]
Now Perimeter of Triangle PCD
= PC + CD + DP
= PC + CE + ED + DP
= PC + AC + DB + DP [From 1 and 2]
= PA + PB
Now,
PA = PB = 10 cm as tangents drawn from an external point to a circle are equal
So we have
Perimeter = PA + PB = 10 + 10 = 20 cm

4. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig. 9.17. Prove that ∠BAT = ∠ACB

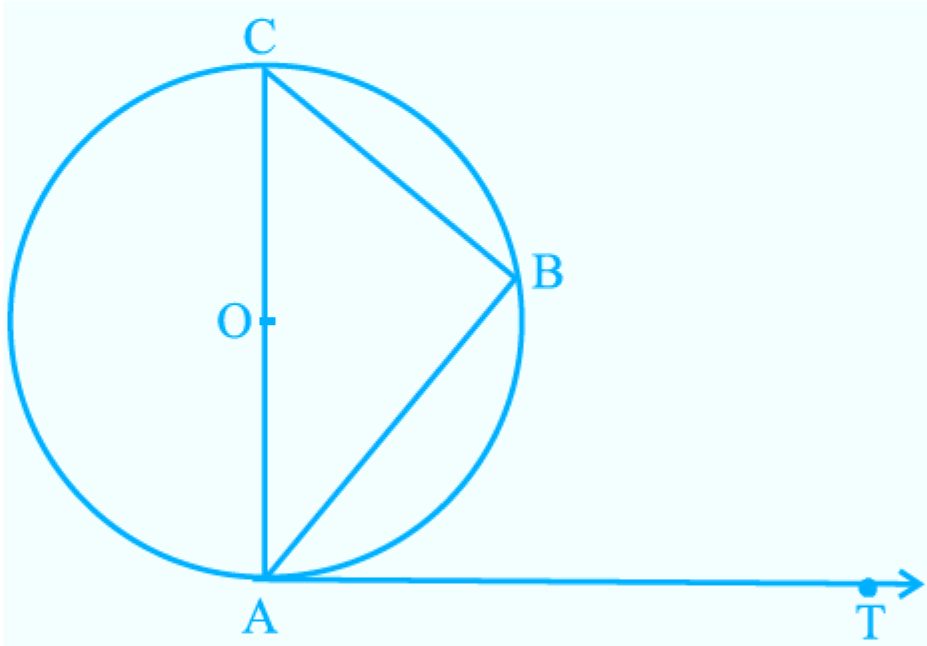


Fig. 9.17

Solution:

According to the question,

A circle with center O and AC as a diameter and AB and BC as two chords also AT is a tangent at point A

To Prove : ∠BAT = ∠ACB

Proof :

∠ABC = 90° [Angle in a semicircle is a right angle]

In △ABC By angle sum property of triangle

∠ABC + ∠ BAC + ∠ACB = 180 °

∠ACB + 90° = 180° - ∠BAC

∠ACB = 90 - ∠BAC [1]

Now,

OA ⊥ AT [Tangent at a point on the circle is perpendicular to the radius through point of contact]

∠OAT = ∠CAT = 90°

∠BAC + ∠BAT = 90°

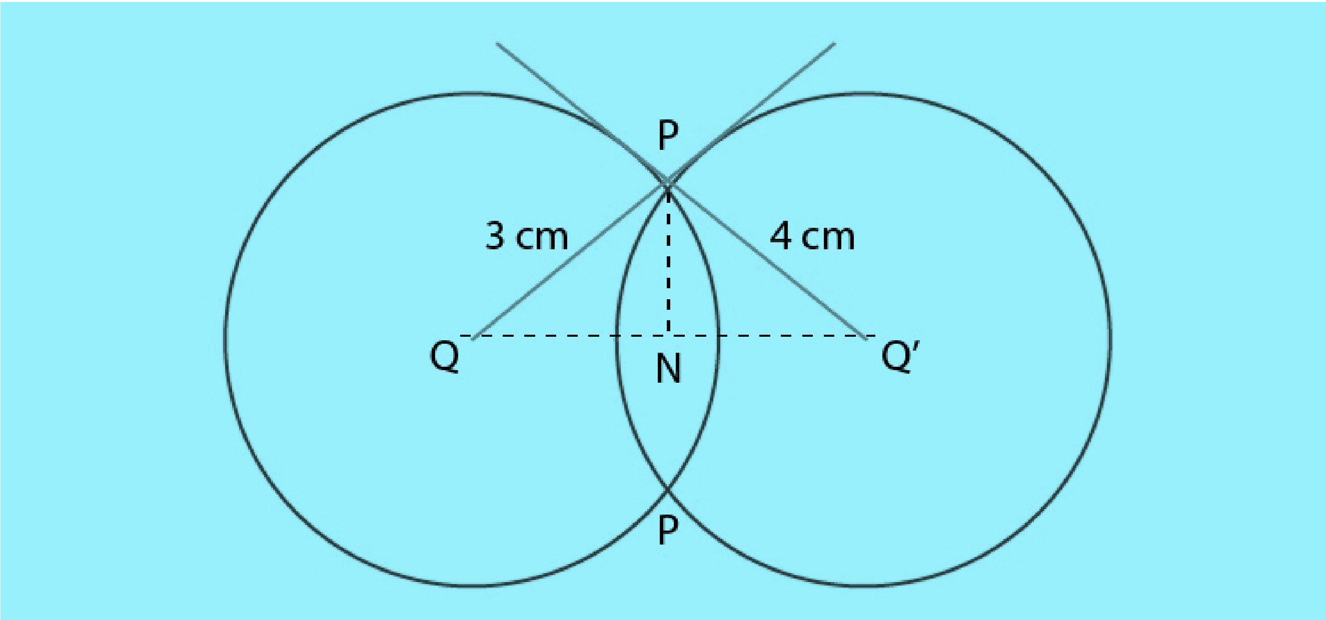
∠BAT = 90° - ∠BAC [2]

From [1] and [2]

∠BAT = ∠ACB [Proved]

5. Two circles with centres O and O′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O′P are tangents to the two circles. Find the length of the common chord PQ.

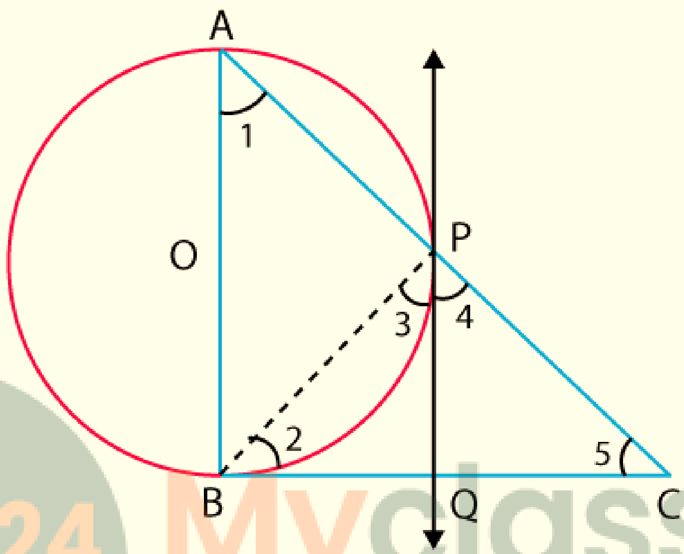
Solution:



According to the question,
Two circles with centers O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles and PQ is a common chord.
To Find: Length of common chord PQ
∠OPO' = 90° [Tangent at a point on the circle is perpendicular to the radius through point of contact]
So OPO is a right-angled triangle at P
Using Pythagoras in △ OPO', we have
$(OO')^2 = (O'P)^2 + (OP)^2$
$(OO')^2 = (4)^2 + (3)^2$
$(OO')^2 = 25$
OO' = 5 cm
Let ON = x cm and NO' = 5 - x cm
In right angled triangle ONP
$(ON)^2 + (PN)^2 = (OP)^2$
$x^2 + (PN)^2 = (3)^2$
$(PN)^2 = 9 - x^2$ [1]
In right angled triangle O'NP
$(O'N)^2 + (PN)^2 = (O'P)^2$
$(5 - x)^2 + (PN)^2 = (4)^2$
$25 - 10x + x^2 + (PN)^2 = 16$
$(PN)^2 = -x^2 + 10x - 9$[2]
From [1] and [2]
$9 - x^2 = -x^2 + 10x – 9$
$10x = 18$
$x = 1.8$
From (1) we have
$(PN)^2 = 9 - (1.8)^2$

=9 - 3.24 = 5.76
PN = 2.4 cm
PQ = 2PN = 2(2.4) = 4.8 cm

6. In a right triangle ABC in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.
Solution:



According to the question,
In a right angle ΔABC is which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Also PQ is a tangent at P
To Prove: PQ bisects BC i.e. BQ = QC
Proof:
∠APB = 90° [Angle in a semicircle is a right-angle]
∠BPC = 90° [Linear Pair]
∠3 + ∠4 = 90 [1]
Now, ∠ABC = 90°
So in △ABC
∠ABC + ∠BAC + ∠ACB = 180°
90 + ∠1 + ∠5 = 180
∠1 + ∠5 = 90 [2]
Now,
∠ 1 = ∠ 3[angle between tangent and the chord equals angle made by the chord in alternate segment]
Using this in [2] we have
∠3 + ∠5 = 90 [3]
From [1] and [3] we have
∠3 + ∠4 = ∠3 + ∠5
∠4 = ∠5
QC = PQ [Sides opposite to equal angles are equal]

But Also PQ = BQ [Tangents drawn from an external point to a circle are equal]
So, BQ = QC
i.e. PQ bisects BC .

7. In Fig. 9.18, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find the ∠RQS.
[Hint: Draw a line through Q and perpendicular to QP.]

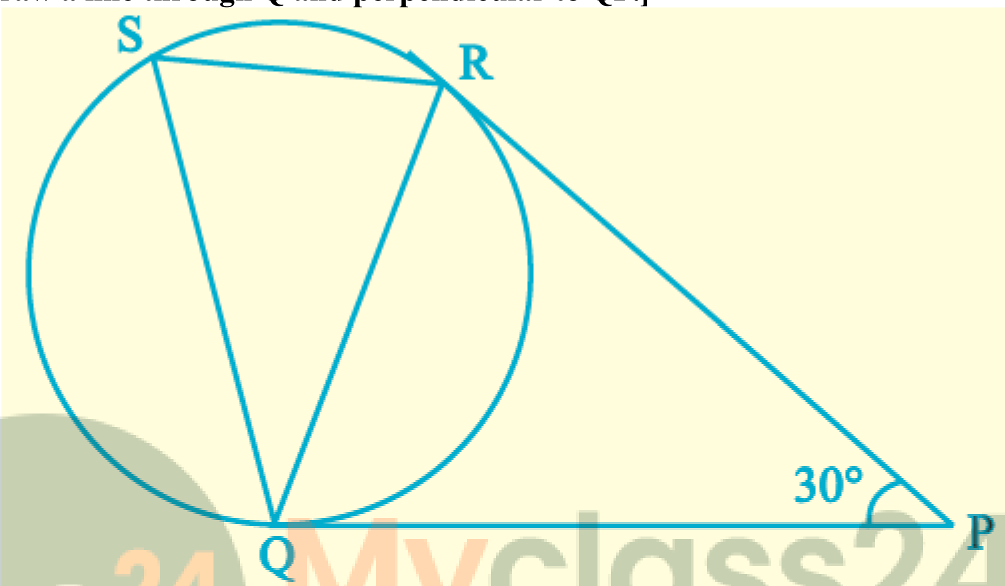
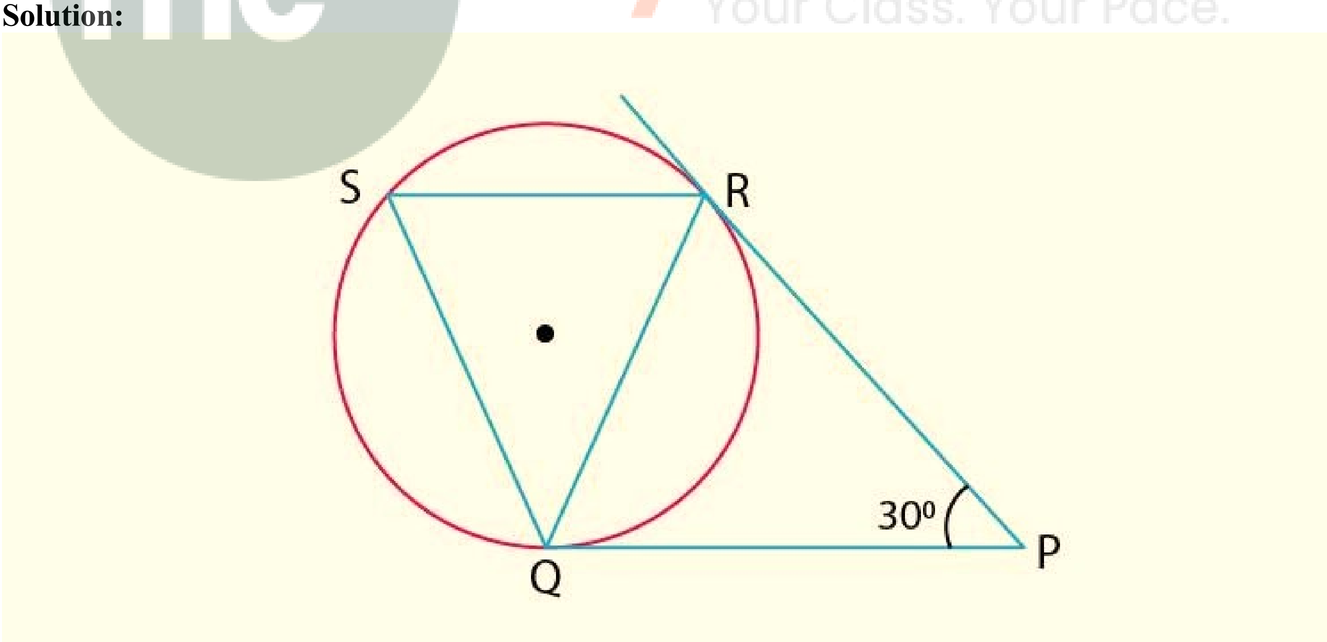


Fig. 9.18

Solution:



According to the question,
Tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ.
To Find : ∠RQS

PQ = PR [Tangents drawn from an external point to a circle are equal]
∠PRQ = ∠PQR [Angles opposite to equal sides are equal] [1]
In △PQR
∠PRQ + ∠PQR + ∠QPR = 180°
∠PQR + ∠PQR + ∠QPR = 180° [Using 1]
2∠PQR + ∠RPQ = 180°
2∠PQR + 30 = 180
2∠PQR = 150
∠PQR = 75°
∠QRS = ∠PQR = 75° [Alternate interior angles]
∠QSR = ∠PQR = 75° [angle between tangent and the chord equals angle made by the chord in alternate segment]
Now In △RQS
∠RQS + ∠QRS + ∠QSR = 180
∠RQS + 75 + 75 = 180
∠RQS = 30°