EXERCISE 2.2

Answer the following and justify:

(i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?

Solution:

No, $x^2 - 1$ cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5.

Justification:

When a degree 6 polynomial is divided by degree 5 polynomial,
The quotient will be of degree 1.
Assume that $(x^2 - 1)$ divides the degree 6 polynomial with and the quotient obtained is degree 5 polynomial (1)
According to our assumption,
(degree 6 polynomial) = $(x^2 - 1)$(degree 5 polynomial) + r(x)   [Since, (a = bq + r)]
= (degree 7 polynomial) + r(x)   [Since, (x^2 term × x^5 term = x^7 term)]
= (degree 7 polynomial)
From the above equation, it is clear that, our assumption is contradicted.
$x^2 - 1$ cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5
Hence Proved.

(ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s$, $p \neq 0$?

Solution:

Degree of the polynomial $px^3 + qx^2 + rx + s$ is 3
Degree of the polynomial $ax^2 + bx + c$ is 2
Here, degree of $px^3 + qx^2 + rx + s$ is greater than degree of the $ax^2 + bx + c$
Therefore, the quotient would be zero,
And the remainder would be the dividend = $ax^2 + bx + c$.

(iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

Solution:

We know that,
$p(x)= g(x) \times q(x)+r(x)$
According to the question,
$q(x) = 0$
When $q(x)=0$, then $r(x)$ is also = 0
So, now when we divide p(x) by g(x),
Then p(x) should be equal to zero
Hence, the relation between the degrees of p (x) and g (x) is the degree $p(x) < \text{degree } g(x)$

(iv) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

Solution:

In order to divide p(x) by g(x)

We know that,
Degree of p(x) > degree of g(x)
or
Degree of p(x)= degree of g(x)
Therefore, we can say that,
The relation between the degrees of *p* (*x*) and *g* (*x*) is degree of p(x) ≥ degree of g(x)

(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?
Solution:

A Quadratic Equation will have equal roots if it satisfies the condition:

$b^2 - 4ac = 0$

Given equation is $x^2 + kx + k = 0$

$a = 1, b = k, x = k$

Substituting in the equation we get,

$k^2 - 4 (1) (k) = 0$

$k^2 - 4k = 0$

$k (k - 4) = 0$

$k = 0 , k = 4$

But in the question, it is given that k is greater than 1.
Hence the value of k is 4 if the equation has common roots.
Hence if the value of k = 4, then the equation ($x^2 + kx + k$) will have equal roots.