Chapter 6

EXERCISE 6.6 (Optional)*:

1. In Fig. 6.56, PS is the bisector of ∠QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$

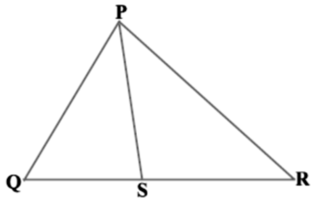


Fig. 6.56

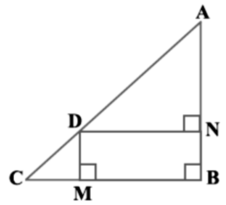
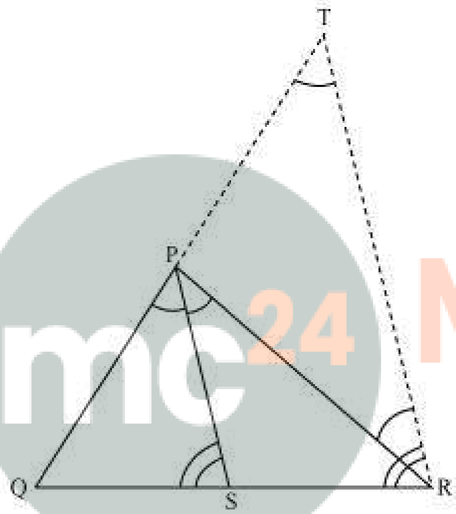


Fig. 6.57

Solution: Construct a line segment RT parallel to SP which intersects the extended line segment QP at point T



Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR (i)

By construction,

∠SPR = ∠PRT (As PS || TR) (ii)

∠QPS = ∠QTR (As PS || TR) (iii)

Using these equations, we get:

∠PRT = ∠QTR

PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

$$\frac{QS}{SR} = \frac{PQ}{PR}$$

2. In Fig. 6.57, D is a point on hypotenuse AC of Δ ABC, DM ⊥ BC and DN ⊥ AB. Prove that:

(i) $DM^2 = DN.MC$ (ii) $DN^2 = DM . AN$

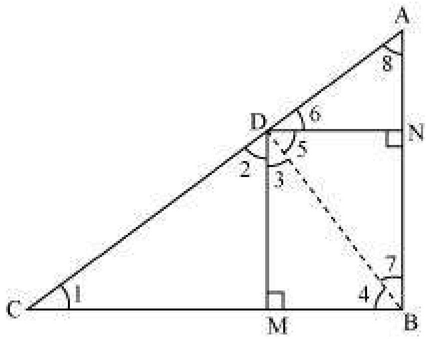
Solution: (i)Let us join DB

We have, DN || CB,

DM || AB,

And ∠B = 90°

DMBN is a rectangle



DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC

∠CDB = 90°

∠2 + ∠3 = 90° (i)

In ΔCDM,

∠1 + ∠2 + ∠DMC = 180°

∠1 + ∠2 = 90° (ii)

In ΔDMB,

∠3 + ∠DMB + ∠4 = 180°

⇒∠3 + ∠4 = 90° (iii)

From (i) and (ii), we get

∠1 = ∠3

From (i) and (iii), we get

∠2 = ∠4

In ΔDCM and ΔBDM,

∠1 = ∠3 (Proved above)

∠2 = ∠4 (Proved above)

ΔDCM similar to ΔBDM (AA similarity)

BM/DM = DM/MC

DN/DM = DM/MC (BM = DN)

DM^2 = DN × MC

(ii) In right triangle DBN,

∠5 + ∠7 = 90° (iv)

In right triangle DAN,

$\angle 6 + \angle 8 = 90°$ (v)

D is the foot of the perpendicular drawn from B to AC

$\angle ADB = 90°$

$\angle 5 + \angle 6 = 90°$ (vi)

From equation (iv) and (vi), we obtain

$\angle 6 = \angle 7$

From equation (v) and (vi), we obtain

$\angle 8 = \angle 5$

In ΔDNA and ΔBND,

$\angle 6 = \angle 7$ (Proved above)

$\angle 8 = \angle 5$ (Proved above)

Hence,

ΔDNA similar to ΔBND (AA similarity criterion)

AN/DN = DN/NB

$DN^2 = AN * NB$

$DN^2 = AN * DM$ (As NB = DM)

3. In Fig. 6.58, ABC is a triangle in which ∠ ABC > 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.

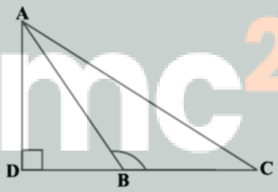


Fig. 6.58

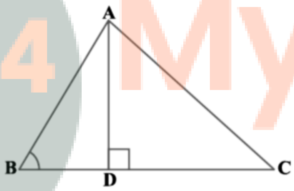


Fig. 6.59

Solution: Using Pythagoras theorem in ΔADB, we get:

$AB^2 = AD^2 + DB^2$ (i)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (i)]

###Topic###Maths||Triangles||Pythagoras Theorem### (Medium)

4. In Fig. 6.59, ABC is a triangle in which ∠ ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.

Solution: Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

$AD^2 = AB^2 – DB^2$ (i)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 – BD^2 + DC^2 = AC^2$ [Using equation (i)]

$AB^2 – BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 – BD^2 + BC^2 + BD^2 - 2BC \times BD$

$= AB^2 + BC^2 - 2BC \times BD$

5. In Fig. 6.60, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

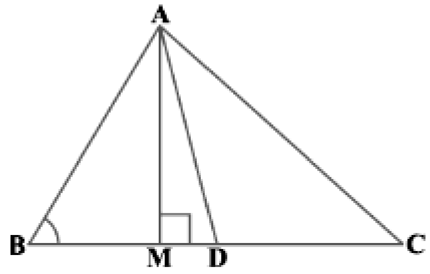


Fig. 6.60

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC \, . \, DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2\,AD^2 + \frac{1}{2}BC^2.$

Solution: (i) Using, Pythagoras theorem in ΔAMD, we get

$AM^2 + MD^2 = AD^2$ (i)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$[Using equation (i)]

Using, DC = BC/2 we get

$AD^2 + (BC/2)^2 + 2MD * (BC/2) = AC^2$

$AD^2 + (BC/2)^2 + MD * BC = AC^2$

(ii) Using Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 – DM^2) + MB^2$

$= (AD^2 – DM^2) + (BD - MD)^2$

$= AD^2 – DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$= AD^2 + (BC/2)^2 – 2\,(BC/2) * MD$

$= AD^2 + (BC/2)^2 – BC * MD$

(iii)Using Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2(2)$

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2+BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD (- BD + DC) = AB^2 + AC^2$

$2 (AM^2 + MD^2) + (BC/2)^2 + (BC/2)^2 + 2MD (-BC/2 + BC/2) = AB^2 + AC^2$

$2AD^2 + BC^2/2 = AB^2 + AC^2$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides

Solution: ABCD is a parallelogram in which AB = CD and AD = BC

Perpendicular AN is drawn on DC and perpendicular DM is drawn on AB extend up to M

In triangle AMD,

$AD^2 = DM^2 + AM^2$ (i)

In triangle BMD,

$BD^2 = DM^2 + (AM + AB)^2$

Or,

$BD^2 = DM^2 + AM^2 + AB^2 + 2AM * AB$ (ii)

Substituting the value of AM^2 from (i) in (ii), we get

$BD^2 = AD^2 + AB^2 + 2 * AM * AB$ (iii)

In triangle AND,

$AD^2 = AN^2 + DN^2$

In triangle ANC,

$AC^2 = AN^2 + (DC – DN)^2$

Or,

$AC^2 = AN^2 + DN^2 + DC^2 – 2 * DC * DN$ (v)

Substituting the value of AD^2 from (iv) in (v), we get

$AC^2 = AD^2 + DC^2 – 2 * DC * DN$ (vi)

We also have,

AM = DN and AB = CD

Substituting these values in (vi), we get

$AC^2 = AD^2 + DC^2 – 2 * AM *- AB$ (vii)

Adding (iii) and (vii), we get

$AC^2 + BD^2 = AD^2 + AB^2 + 2 * AM * AB + AD^2 + DC^2 – 2 * AM * AB$

Or,

$AC^2 + BD^2 = AB^2 + BC^2 + DC^2 + AD^2$

Hence, proved

7. In Fig. 6.61, two chords AB and CD intersect each other at the point P. Prove that:

(i) Δ APC ~ Δ DPB

(ii) AP . PB = CP . DP

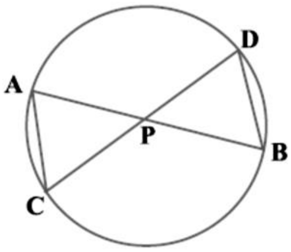


Fig. 6.61

Solution: (i) In triangle APC and DPB,

∠CAP = ∠BDP (Angles on the same side of a chord are equal)

∠APC = ∠DPB (Opposite angles)

Hence,

Δ APC ~ Δ DPB (By AAA similarity)

(ii) Since, the two triangles are similar

Hence,

$$\frac{AP}{CP} = \frac{DP}{PB}$$

Or,

AP * PB = CP * DP

Hence, proved

8. In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P(when produced) outside the circle. Prove that

(i) Δ PAC ~ Δ PDB

(ii) PA . PB = PC . PD

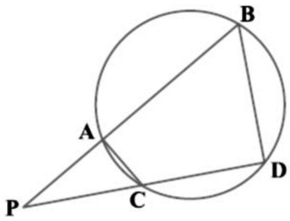


Fig. 6.62

Solution: (i) In triangle PAC and PDB

∠PAC + ∠CAB = 180° (Linear pair)

∠CAB + ∠BDC = 180° (Opposite angles of a cyclic quadrilateral are supplementary)

Hence,

∠PAC = ∠PDB

Similarly,
∠PCA = ∠PBD
Hence,
Δ PAC ~ Δ PDB

(ii) Since the two triangles are similar, so

$$\frac{PA}{PC} = \frac{PD}{PB}$$

Or,
PA * PB = PC * PD
Hence, proved

9. In Fig. 6.63, D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is thebisector of ∠ BAC

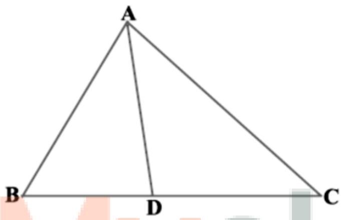


Fig. 6.63

Solution:Construct a line CM which meets BA extended up to AM so that AM = AC
This means, ∠AMC = ∠ACM (Angles opposite to equal sides)
$\frac{BD}{CD} = \frac{AB}{AC}$ (Given)

$$Or,$$

$\frac{BD}{CD} = \frac{AB}{AM}$BECAUSE AM = AC
Hence,
ΔABD ~ΔMBC
Hence, AD parallel MC
This means,
∠DAC = ∠ACM (Alternate angles)
Since,
∠AMC = ∠ACM
Hence,
∠DAC = ∠BDA
Hence, AD is the bisector of ∠BAC

10. Nazima is fly fishing in a stream. The tip ofher fishing rod is 1.8 m above the surfaceof the water and the fly at the end of thestring rests on the water 3.6 m away and2.4 m from a point directly under the tip ofthe rod. Assuming that her string(from the tip of

her rod to the fly) is taut,how much string does she have out(see Fig. 6.64)? If she pulls in the string atthe rate of 5 cm per second, what will bethe horizontal distance of the fly from herafter 12 seconds?

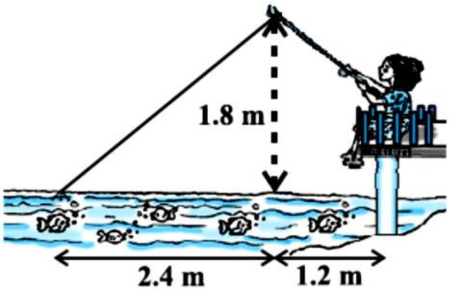


Fig. 6.64

Solution: As per the question:

AD = 1.8 m

BD = 2.4 m

CD = 1.2 m

Retracting speed of string = 5 cm per second

In triangle ABD, length of string i.e. AB can be calculated as follows

$AB^2 = AD^2 + BD^2$

$= (1.8)^2 + (2.4)^2$

$= 3.24 + 5.76$

$= 9$

Or, AB = 3

Let us assume that the string reaches at point M after 12 seconds

Length of refracted string in 12 inches = 5 * 12 = 60 cm

Remaining length = 3 – 0.6 = 2.4 m

In triangle AMD, we can find MD by using Pythagoras theorem,

$MD^2 = AM^2 – AD^2$

$= 2.4^2 – 1.8^2$

$= 5.76 – 3.24$

$= 2.52$

Or, MD = 1.58

Hence,

Horizontal distance between the girl and the fly = CD + MD

= 1.2 + 1.58

= 2.78 m