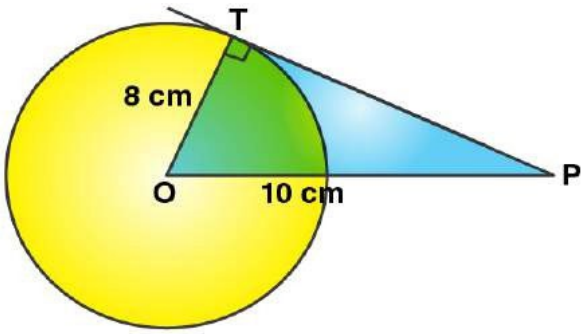
Exercise 18(A)

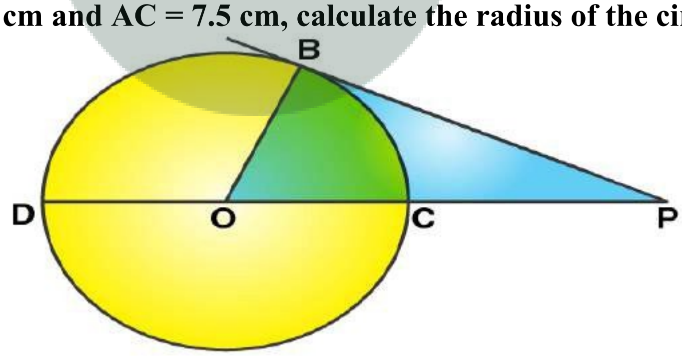
The radius of a circle is 8cm. Calculate the length of a tangent drawn to this circle from a point at a distance of 10cm from its centre.

Solution:



Given, a circle with centre O and radius 8 cm.
An external point P from where a tangent is drawn to meet the circle at T.
OP = 10 cm; radius OT = 8 cm
As OT ⊥ PT
In right ΔOTP, we have
$OP^2 = OT^2 + PT^2$ [By Pythagoras Theorem]
$10^2 = 8^2 + PT^2$
$PT^2 = 100 – 64 = 36$
So, PT = 6
Therefore, length of tangent = 6 cm.

1. In the given figure, O is the centre of the circle and AB is a tangent to the circle at B. If AB = 15 cm and AC = 7.5 cm, calculate the radius of the circle.

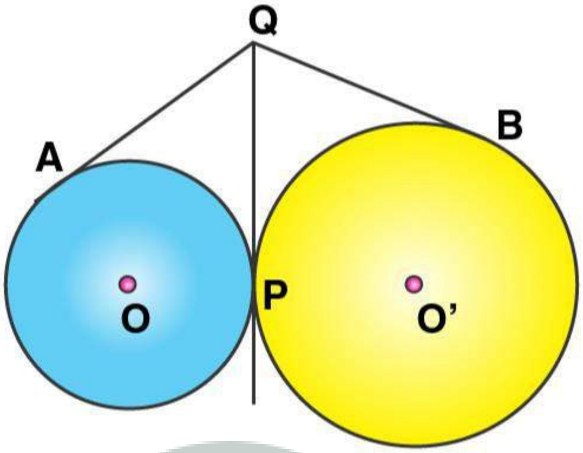


Solution:

Given,
AB = 15 cm, AC = 7.5 cm
Let's assume the radius of the circle to be 'r'.
So, AO = AC + OC = 7.5 + r
In right ΔAOB, we have
$AO^2 = AB^2 + OB^2$ [By Pythagoras Theorem]
$(7.5 + r)^2 = 15^2 + r^2$
$56.25 + r^2 + 15r = 225 + r^2$
$15r = 225 – 56.25$

r = 168.75/ 15
Thus,
r = 11.25 cm

2. Two circles touch each other externally at point P. Q is a point on the common tangent through P. Prove that the tangents QA and QB are equal.

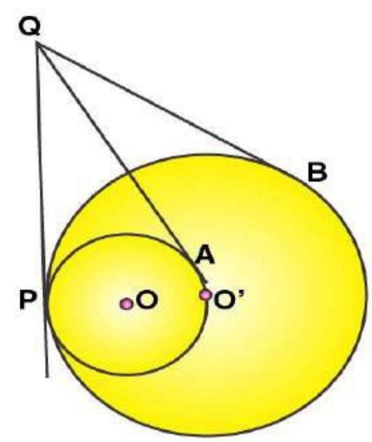


Solution:

Let Q be the point from which, QA and QP are two tangents to the circle with centre O
So, QA = QP .....(a)
Similarly, from point Q, QB and QP are two tangents to the circle with centre O'
So, QB = QP ......(b)
From (a) and (b), we have
QA = QB
Therefore, tangents QA and QB are equal.
- Hence Proved

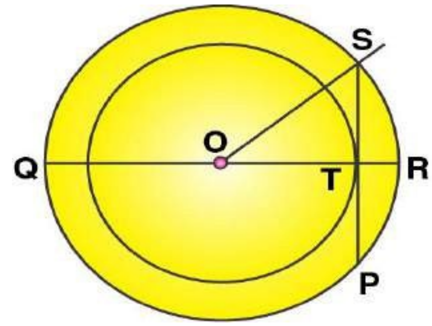
3. Two circles touch each other internally. Show that the tangents drawn to the two circles from any point on the common tangent are equal in length.
Solution:

Let Q be the point on the common tangent from which, two tangents QA and QP are drawn to the circle with centre O.
So, QA = QP ....(1)
Similarly, from point Q, QB and QP are two tangents to the circle with centre O'
So, QB = QP.........(2)
From (1) and (2), we have
QA = QB
Therefore, tangents QA and QB are equal.
- Hence Proved



4. Two circles of radii 5 cm and 3 cm are concentric. Calculate the length of a chord of the outer circle which touches the inner.

Solution:



Given,

OS = 5 cm and OT = 3 cm

In right triangle OST, we have

$ST^2 = OS^2 - OT^2$

$= 25 - 9$

$= 16$

So, ST = 4 cm

As we know, OT is perpendicular to SP and OT bisects chord SP

Hence, SP = 2 x ST = 8 cm

5. Three circles touch each other externally. A triangle is formed when the centers of these circles are joined together. Find the radii of the circles, if the sides of the triangle formed are 6 cm, 8 cm and 9 cm.

Solution:

Let ABC be the triangle formed when centres of 3 circles are joined.

Given,

AB = 6 cm, AC = 8 cm and BC = 9 cm

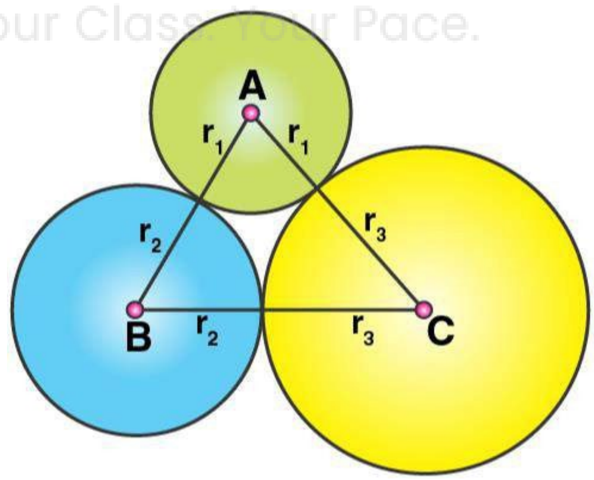
And let the radii of the circles having centres A, B and C be r_1, r_2 and r_3 respectively.

So, we have

$r_1 + r_3 = 8$

$r_3 + r_2 = 9$

$r_2 + r_1 = 6$



Adding all the above equations, we get

$r_1 + r_3 + r_3 + r_2 + r_2 + r_1 = 8 + 9 + 6$

$2(r_1 + r_2 + r_3) = 23$

So,

$r_1 + r_2 + r_3 = 11.5$ cm

Now,

$r_1 + 9 = 11.5$ (As $r_2 + r_3 = 9$)

$r_1 = 2.5$ cm

And,

$r_2 + 6 = 11.5$ (As $r_1 + r_3 = 6$)

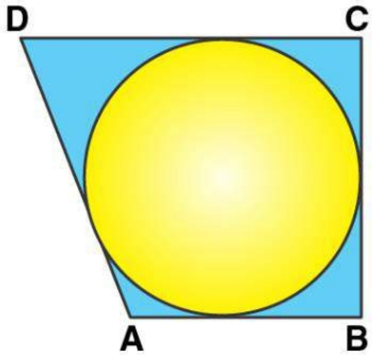
$r_2 = 5.5$ cm

Lastly, $r_3 + 8 = 11.5$ (As $r_2 + r_1 = 8$)

$r_3 = 3.5$ cm

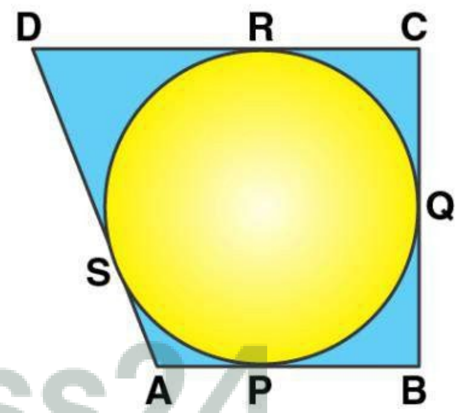
Therefore, the radii of the circles are $r_1 = 2.5$ cm, $r_2 = 5.5$ cm and $r_3 = 3.5$ cm.

6. If the sides of a quadrilateral ABCD touch a circle, prove that AB + CD = BC + AD.



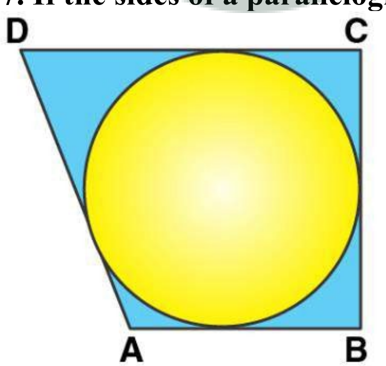
Solution:

Let a circle touch the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively.



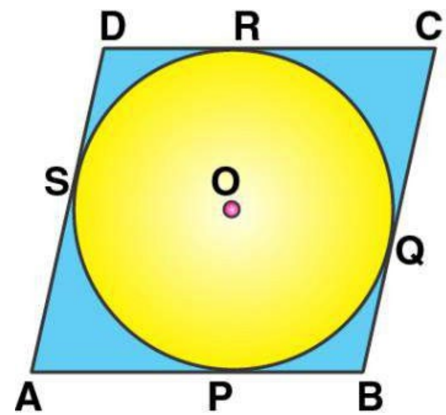
As, AP and AS are tangents to the circle from an external point A, we have
AP = AS .... (1)
Similarly, we also get
BP = BQ........ (2)
CR = CQ.........(3)
DR = DS......... (4)
Adding (1), (2), (3) and (4), we get
AP + BP + CR + DR = AS + DS + BQ + CQ
AB + CD = AD + BC
Therefore,
AB + CD = AD + BC
- Hence Proved

7. If the sides of a parallelogram touch a circle, prove that the parallelogram is a rhombus.



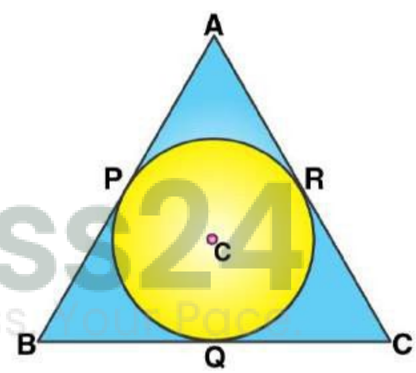
Solution:

Let a circle touch the sides AB, BC, CD and DA of parallelogram ABCD at P, Q, R and S respectively.
Now, from point A, AP and AS are tangents to the circle.
So, AP = AS. ...... (1)
Similarly, we also have
BP = BQ.......... (2)



CR = CQ...........(3)
DR = DS.......... (4)
Adding (1), (2), (3) and (4), we get
AP + BP + CR + DR = AS + DS + BQ + CQ
AB + CD = AD + BC
Therefore,
AB + CD = AD + BC
But AB = CD and BC = AD....... (5) [Opposite sides of a parallelogram]
Hence,
AB + AB = BC + BC
2AB = 2 BC
AB = BC..........(6)
From (5) and (6), we conclude that
AB = BC = CD = DA
Thus, ABCD is a rhombus.

8. From the given figure prove that:
AP + BQ + CR = BP + CQ + AR.



Also, show that AP + BQ + CR = ½ x perimeter of triangle ABC.
Solution:

As from point B, BQ and BP are the tangents to the circle
We have, BQ = BP.............. (1)
Similarly, we also get
AP = AR.................. (2)
And, CR = CQ …… (3)
Adding (1), (2) and (3) we get,
AP + BQ + CR = BP + CQ + AR..............(4)
Now, adding AP + BQ + CR to both sides in (4), we get
2(AP + BQ + CR) = AP + PQ + CQ + QB + AR + CR
2(AP + BQ + CR) = AB + BC + CA
Therefore, we get
AP + BQ + CR = ½ x (AB + BC + CA)
i.e.
AP + BQ + CR = ½ x perimeter of triangle ABC

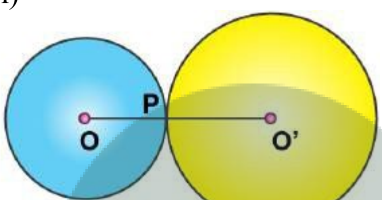
9. In the figure, if AB = AC then prove that BQ = CQ.
Solution:

As, from point A
AP and AR are the tangents to the circle
So, we have AP = AR
Similarly, we also have
BP = BQ and CR = CQ [From points B and C]

Now adding the above equations, we get
AP + BP + CQ = AR + BQ + CR
(AP + BP) + CQ = (AR + CR) + BQ
AB + CQ = AC + BQ ….. (i)
But, as AB = AC [Given]
Therefore, from (i)
CQ = BQ or BQ = CQ

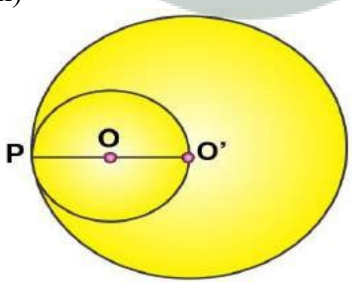
10. Radii of two circles are 6.3 cm and 3.6 cm. State the distance between their centers if -
i) they touch each other externally.
ii) they touch each other internally.
Solution:

Given,
Radius of bigger circle = 6.3 cm and of smaller circle = 3.6 cm
i)



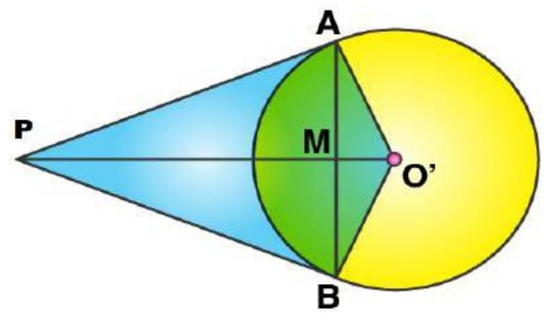
When the two circles touch each other at P externally. O and O' are the centers of the circles. Join OP and O'P.
So, OP = 6.3 cm, O'P = 3.6 cm
Hence, the distance between their centres (OO') is given by
OO' = OP + O'P = 6.3 + 3.6 = 9.9 cm

ii)



When the two circles touch each other at P internally. O and O' are the centers of the circles. Join OP and O'P
So, OP = 6.3 cm, O'P = 3.6 cm
Hence, the distance between their centres (OO') is given by
OO' = OP - O'P = 6.3 - 3.6 = 2.7 cm

11. From a point P outside the circle, with centre O, tangents PA and PB are drawn. Prove that:
i) ∠AOP = ∠BOP
ii) OP is the ⊥ bisector of chord AB.
Solution:



i) In ΔAOP and ΔBOP, we have
AP = BP [Tangents from P to the circle]
OP = OP [Common]
OA = OB [Radii of the same circle]
Hence, by SAS criterion of congruence
ΔAOP ≅ ΔBOP
So, by C.P.C.T we have
∠AOP = ∠BOP

ii) In ΔOAM and ΔOBM, we have
OA = OB [Radii of the same circle]
∠AOM = ∠BOM [Proved ∠AOP = ∠BOP]
OM = OM [Common]
Hence, by SAS criterion of congruence
ΔOAM ≅ ΔOBM
So, by C.P.C.T we have
AM = MB
And ∠OMA = ∠OMB
But,
∠OMA + ∠OMB = 180º
Thus, ∠OMA = ∠OMB = 90º
Therefore, OM or OP is the perpendicular bisector of chord AB.
- Hence Proved