NCERT Solutions for Class-XII Maths

Chapter-13
Exercise- Miscellaneous

1. A and B are two events such that P (A) ≠ 0. Find P(B|A), if:
(i) A is a subset of B
(ii) A ∩ B = ϕ

1. (i) It is given in the question that,
A and B are two events such that $P\ (A) \neq 0$
We have, $A \cap B = A$
$\therefore P\ (A \cap B) = P\ (B \cap A) = P\ (A)$
Hence, $P(B|A) = \frac{P\ (B \cap A)}{P\ (A)} = \frac{P\ (A)}{P\ (A)} = 1$

(ii) We have,
$P\ (A \cap B) = 0$
$\therefore P(B|A) = \frac{P\ (A \cap B)}{P\ (A)}$
= 0

2. A couple has two children,
(i) Find the probability that both children are males, if it is known that at least one of the children is male.
(ii) Find the probability that both children are females, if it is known that the elder child is a female.

2. If a couple has two children, then the sample space is
$S = \{(b,b),(b,g),(g,b),(g,g)\}$
(i) Let E and F respectively denote the events that both children are males and at least one of the children is a male.

$$\therefore E \cap F = \{(b,b)\} \Rightarrow P(E \cap F) = \frac{1}{4}$$

$$P(E) = \frac{1}{4}$$

$$P(F) = \frac{3}{4}$$

$$\Rightarrow P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{3}{4}} = \frac{1}{3}$$

(ii) Let A and B respectively denote the events that both children are females and the elder child is a female.

$$A = \{(g, g)\} \Rightarrow P(A) = \frac{1}{4}$$

$$B = \{(g, b), (g, g)\} \Rightarrow P(B) = \frac{2}{4}$$

$$A \cap B = \{(g, g)\} \Rightarrow P(A \cap B) = \frac{1}{4}$$

$$P(AB) = \frac{P(A \cap B)}{P(B)} = \frac{4}{2} = \frac{1}{2}$$

3. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male?
Assume that there are equal number of males and females.

3. It is given in the question that,
5% of men and 0.25% of women have grey hair
∴ Total % of people having grey hair = 5 + 0.25
= 5.25 %
Hence, Probability of having a selected person male having grey hair, $P = \frac{5}{5.25} = \frac{20}{21}$

4. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

4. A person can be either right-handed or left-handed.
It is given that 90% of the people are right-handed.

$$\therefore p = P(\text{ right-handed }) = \frac{9}{10}$$

$$q = P(\text{ left-handed }) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed $= 1 - P$ (more than 6 are right-handed)

$$= 1 - \sum_{r=7}^{10} C_f (0.9)'(0.1)^{10-r}$$

5. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that:

(i) All will bear 'X' mark.

(ii) Not more than 2 will bear 'Y' mark.

(iii) At least one ball will bear 'Y' mark.

(iv) The number of balls with 'X' mark and 'Y' mark will be equal.

5. (i) It is given in the question that,

Total number of balls in the urn = 25

Number of balls bearing mark 'X' = 10

Number of balls bearing mark 'Y' = 15

Let p denotes the probability of balls bearing mark 'X' and q denotes the probability of balls bearing mark 'Y'

$\therefore p = \frac{10}{25} = \frac{2}{5}$

And, $q = \frac{15}{25} = \frac{3}{5}$

Now, 6 balls are drawn with replacement. Hence, the number of trials are Bernoulli triangle.

Let us assume, Z be the random variable that represents the number of balls bearing 'Y' mark in the trials

$\therefore$ Z has a binomial distribution where n = 6 and $p = \frac{2}{5}$

$P\ (Z = z) = {}^n C_z\, p^{n-z} q^z$

Hence, P (All balls will bear mark 'X') = P (Z = 0)

$= {}^6 C_0 \left(\frac{2}{5}\right)^6$

$= \left(\frac{2}{5}\right)^6$

(ii) Probability (Not more than 2 will bear 'Y' mark) $= P\ (Z \leq 2)$

= P (Z = 0) + P (Z = 1) + P (Z = 2)

$= {}^6C_0\,(p)^6(q)^0 + {}^6C_1(p)^5(q)^1 + {}^6C_2(p)^4(q)^2$

$= \left(\frac{2}{5}\right)^6 + 6\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right) + 15\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2$

$= \left(\frac{2}{5}\right)^4\left[\left(\frac{2}{5}\right)^2 + 6\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + 15\left(\frac{3}{5}\right)^2\right]$

$= \left(\frac{2}{5}\right)^4\left[\frac{175}{25}\right]$

$= 7\left(\frac{2}{5}\right)^4$

(iii)Now, Probability (At least one ball will bear 'Y' mark) = P (Z ≥ 1)

= 1 – P (Z = 0)

$= 1 - \left(\frac{2}{5}\right)^6$

(iv)Probability (Having equal number of balls with 'X' mark and 'Y' mark) = P (Z = 3)

$= {}^6C_3 \left(\frac{2}{54}\right)^3 \left(\frac{3}{5}\right)^3$

$= \frac{20\times8\times27}{15625}$

$= \frac{864}{3125}$

6. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

6. Let p and q respectively be the probabilities that the player will clear and knock down the hurdle.

$$\therefore p = \frac{5}{6}$$

$$\Rightarrow q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$$

Let X be the random variable that represents the number of times the player will knock down the hurdle.

Therefore, by binomial distribution, we obtain

$P(X = x) = {}^nC_x p^{k-1} q^{\varepsilon}$

P (player knocking down less than 2 hurdles) $= P(X < 2)$

$= P(X = 0) + P(X = 1)$

$= {}^{10}C_0 (q)^0 (p)^{14} + {}^{10}C_1 (q)(p)^9$

$= \left(\frac{5}{6}\right)^{10} + 10 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^9 = \left(\frac{5}{6}\right)^9 \left[\frac{5}{6} + \frac{10}{6}\right] = \frac{5}{2}\left(\frac{5}{6}\right)^9 = \frac{(5)^{90}}{2\times(6)^9}$

7. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

7. From the condition given in the question, it is clear that:

Probability of getting a six in a throw of die $= \frac{1}{6}$

And, probability of not getting a six $= \frac{5}{6}$

Let us assume, $p = \frac{1}{6} and\ q = \frac{5}{6}$

Now, we have:

Probability that the 2 sixes come in the first five throws of the die $= {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

$= \frac{10\times(5)^3}{(6)^5}$

Also, Probability that the six come in the sixth throw $= \frac{10\times(5)^3}{(6)^5} \times \frac{1}{6}$

$= \frac{10\times125}{(6)^6} = \frac{625}{23328}$

8. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

8. In a leap year, there are 366 days i.e., 52 weeks and 2 days.

In 52 weeks, there are 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be

Monday and Tuesday

Tuesday and Wednesday

Wednesday and Thursday

Thursday and Friday

Friday and Saturday

Saturday and Sunday

Sunday and Monday

Total number of cases = 7

Favourable cases = 2

∴ Probability that a leap year will have 53 Tuesdays $= \frac{2}{7}$

9. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

9. It is given in the question that,

Probability of failure = x

And, probability of success = 2x

∴ x + 2x = 1

3x = 1

$x = \frac{1}{3}$

$\therefore 2x = \frac{2}{3}$

Let us now assume $p = \frac{1}{3} and\ q = \frac{2}{3}$

Also, X be the random variable that represents the number of trials

Hence, by binomial distribution we have:

$P\ (X = x) = ^n C_x\, p^{n-x} q^x$

∴ Probability of having at least 4 successes $= P\ (X \geq 4)$

= P (X = 4) + P (X = 5) + P (X = 6)

$= ^6 C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + ^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + ^6C_6 \left(\frac{2}{3}\right)^6$

$= \frac{15\,(2)^4}{3^6} + \frac{6\,(2)^5}{3^6} + \frac{(2)^6}{3^6}$
$= \frac{31 \times (2)^4}{(3)^6}$
$= \frac{31}{9}\left(\frac{2}{3}\right)^4$

10. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

10. Let the man toss the coin n times. The n tosses are n Bernoulli trials. Probability (p) of getting a head at the toss of a coin is $1/2$.

$\therefore$ $P(X = x) = {}^{n}C_x p^{n-1} q^x = {}^{n}C_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x = "C, \left(\frac{1}{2}\right)^n$

It is given that,

$P(\text{ getting at least one head }) > \frac{90}{100}$

$P(x \geq 1) > 0.9$

$\therefore 1 - P(x = 0) > 0.9$

$1 - {}^{0}C_0 \cdot \frac{1}{2^p} > 0.9$

${}^{n}C_0 \cdot \frac{1}{2^a} < 0.1$

$\frac{1}{2^n} < 0.1$

$2^n > \frac{1}{0.1}$

$2^\pi > 10$

The minimum value of n that satisfies the given inequality is 4 .

Thus, the man should toss the coin 4 or more than 4 times.

11. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

11. For the situation given in the equation, we have:

Probability of getting a six in a throw of a die $= \frac{1}{6}$

Also, probability of not getting a 6 $= \frac{5}{6}$

Now, there are three cases from which the expected value of the amount which he wins can be calculated:

(i) First case is that, if he gets a six on his first through then the required probability will be $\frac{1}{6}$

$\therefore$ Amount received by him = Rs. 1

(ii) Secondly, if he gets six on his second throw then the probability $= \left(\frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{5}{36}$

∴ Amount received by him = - Rs. 1 + Rs. 1 = 0

(iii)Lastly, if he does not get six in first two throws and gets six in his third throw then the probability $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

∴ Amount received by him = - Rs. 1 – Rs. 1 + Rs. 1 = - 1

Hence, expected value that he can win $= \frac{1}{6} - \frac{25}{216} = \frac{36-25}{216} = \frac{11}{216}$

12. Suppose we have four boxes A, B, C and D containing coloured marbles as given below:

Box	**Marble colour**		
	Red	**White**	**Black**
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

12. Let R be the event of drawing the red marble.

Let E_A, E_B, and E_C respectively denote the events of selecting the box A, B, and C.

Total number of marbles $= 40$

Number of red marbles $= 15$

$$\therefore P(R) = \frac{15}{40} = \frac{3}{8}$$

Probability of drawing the red marble from box A is given by $P(E_A \mid R)$.

$$\therefore P(E_A R) = \frac{P(E_A \cap R)}{P(R)} = \frac{\frac{1}{40}}{\frac{3}{8}} = \frac{1}{15}$$

Probability that the red marble is from box B is $P(E_B \mid R)$.

$$\Rightarrow P(E_B \mid R) = \frac{P(E_B \cap R)}{P(R)} = \frac{\frac{6}{3}}{\frac{3}{8}} = \frac{2}{5}$$

Probability that the red marble is from box C is P (E $E_c \mid R$).

$$\Rightarrow P(E_c \mid R) = \frac{P(E_c \cap R)}{P(R)} = \frac{\frac{8}{3}}{\frac{3}{8}} = \frac{8}{15}$$

13. Assume that the chances of a patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

13. Let us assume, X denotes the events having a person heart attack

A_1 denote events having the selected person followed the course of yoga and meditation

And, A_2 denote the events having the person adopted the drug prescription

It is given in the question that,

P (X) = 0.40

And, $P\ (A_1) = P\ (A_2) = \frac{1}{2}$

$P\ (X|A_1) = 0.40 \times 0.70 = 0.28$

$P\ (X|A_2) = 0.40 \times 0.75 = 0.30$

∴ Probability (The patient suffering from a heart attack and followed a course of meditation and yoga):

$$P\ (A_1|X) = \frac{P(A_1)\ P\ (X|A_1)}{P(A_1)P\ (X|A_1) + P\ (A_2)P\ (X|A_2)} = \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30} = \frac{14}{29}$$

14. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

14. The total number of determinants of second order with each element being 0 or 1 is $(2)^4 = 16$

The value of determinant is positive in the following cases.

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

∴ Required probability $= \frac{3}{16}$

15. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

P (A fails) = 0.2

P (B fails alone) = 0.15

P (A and B fail) = 0.15

Evaluate the following probabilities:
(i) P (A fails | B has failed)
(ii) P (A fails alone)

15. (i) Let us assume the event which is failed by A is denoted by E_A
And, event which is failed by B is denoted by E_B
It is given in the question that,
Event failed by A, P $(E_A) = 0.2$
Event failed by both, $P(E_A \cap E_B) = 0.15$
And, event failed by B alone $= P(E_B) - P(E_A \cap E_B)$
$0.15 = P(E_B) - 0.15$
$\therefore P(E_B) = 0.30$
Hence, $P(E_A|E_B) = \frac{P(E_A \cap E_B)}{P(E_4)} = \frac{0.15}{0.3} = 0.5$
(ii) We have,
Probability where A fails alone $= P(E_A) - P(E_A \cap E_B)$
$= 0.2 - 0.15$
$= 0.05$

16. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

16. Let E_1 and E_2 respectively denote the events that a red ball is transferred from bag I to II and a black ball is transferred from bag I to II.

$P(E_1) = \frac{3}{7}$ and $P(E_2) = \frac{4}{7}$

Let A be the event that the ball drawn is red.
When a red ball is transferred from bag I to II,

$P(AE_1) = \frac{5}{10} = \frac{1}{2}$

When a black ball is transferred from bag I to II,

$$P(A \mid E_2) = \frac{4}{10} = \frac{2}{5}$$

$$\therefore P(E_2 \mid A) = \frac{P(E_1)P(A \mid E_2)}{P(E_1)P(AE_1) + P(E_2)P(A \mid E_2)}$$

$$= \frac{4}{3} \times \frac{2}{3} \times \frac{1}{2} + \frac{4}{7} \times \frac{2}{5}$$

$$= \frac{16}{31}$$

17. If A and B are two events such that $P(A) \neq 0$ and $P(B \mid A) = 1$, then

(a) $A \subset B$ (b) $B \subset A$

(c) $B = \phi$ (d) $A = \phi$

17. It is given in the question that,

A and B are two events where,

$P\ (A) \neq 0$

And, $P\ (B|A) = 1$

$\therefore P\ (B|A) = \frac{P\ (B \cap A)}{P\ (A)}$

$1 = \frac{P\ (B \cap A)}{P\ (A)}$

$P\ (A) = P\ (B \cap A)$

$\therefore A \subset B$

Hence, option A is correct

18. If $P\ (A|B) > P\ (A)$, then which of the following is correct:

(a) $P\ (B|A) < P\ (B)$ (b) $P\ (A \cap B) < P\ (A) . P\ (B)$

(c) $P\ (B|A) > P\ (B)$ (d) $P\ (B|A) = P\ (B)$

18. $P(AB) > P(A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$$

$$\Rightarrow P(A \cap B) > P(A) \cdot P(B)$$

$$\Rightarrow \frac{P(A \cap B)}{P(A)} > P(B)$$

$$\Rightarrow P(BA) > P(B)$$

Thus, the correct answer is C.

19. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(a) $P(B|A) = 1$ (b) $P(A|B) = 1$

(c) $P(B|A) = 0$ (d) $P(A|B) = 0$

19. It is given in the question that,

A and B are any two events where,

$P\ (A) + P\ (B) - P\ (A \text{ and } B) = P\ (A)$

$P\ (A) + P\ (B) - P\ (A \cap B) = P\ (A)$

$P\ (B) - P\ (A \cap B) = 0$

$P\ (A \cap B) = P\ (B)$

$\therefore P(A|B) = \frac{P\ (A \cap B)}{P\ (B)} = 1$

Hence, option B is correct