EXERCISE 21.2

Question. 1

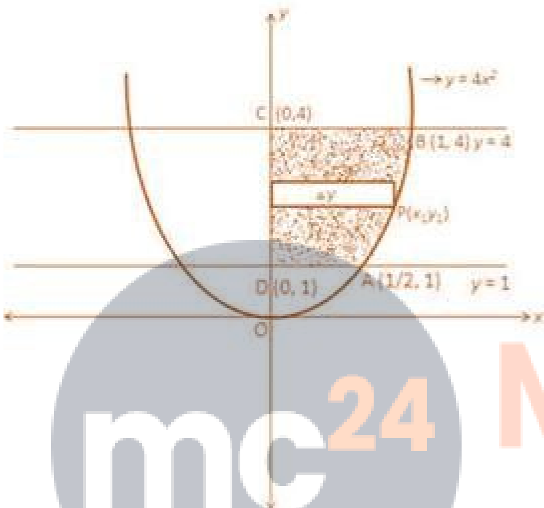
Solution:

From the question it is given that,

Lines, x = 0, y = 1, y = 4

Parabola $y = 4x^2$ … [equation (i)]

So, equation (i) represents a parabola with vertex (0, 0) and axis as y – axis. x = 0 is y – axis and y = 1, y = 4 are line parallel to x – axis passing through (0, 1) and (0, 4) respectively, as shown in the rough sketch below,



Now, we have to find the area of ABCDA,

Then, the area can be found by taking a small slice in each region of width Δy,

And length = x

The area of sliced part will be as it is a rectangle = x Δy

So, this rectangle can move horizontal from y = 1 to x = 4

The required area of the region bounded between the lines = Region ABCDA

$$= \int_1^4 x\,dy$$

Given, $y = 4x^2$

$x = \sqrt{(y/4)}$

$$= \int_1^4 \sqrt{\frac{y}{4}}\,dy$$

$$= \frac{1}{2}\int_1^4 \sqrt{y}\,dy$$

On integrating we get,

$$= \frac{1}{2}\left[\frac{2}{3}y\sqrt{y}\right]_1^4$$

Now, applying limits we get,

$= \frac{1}{2} [((2/3) \times 4 \times \sqrt{4}) - ((2/3) \times 1 \times \sqrt{1})]$

$= \frac{1}{2} [(16/3) - (2/3)]$

$= \frac{1}{2} [(16 - 2)/3]$

$= \frac{1}{2} [14/3]$

$= 7/3$

Therefore, the required area is 7/3 square units.

Question. 2

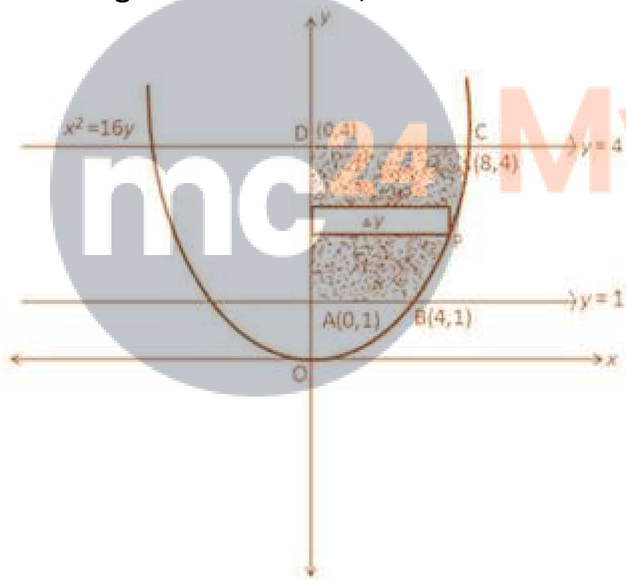
Solution:

From the question it is given that,

Region in first quadrant bounded by $y = 1$, $y = 4$

Parabola $x^2 = 16y$ … [equation (i)]

So, equation (i) represents a parabola with vertex (0, 0) and axis as y – axis, as shown in the rough sketch below,



Now, we have to find the area of ABCDA,

Then, the area can be found by taking a small slice in each region of width Δy,

And length = x

The area of sliced part will be as it is a rectangle = x Δy

So, this rectangle can move horizontal from y = 1 to x = 4

The required area of the region bounded between the lines = Region ABCDA

$= \int_1^4 x\,dy$

Given, $x^2 = 16y$

$x = \sqrt{16y}$

$x = 4\sqrt{y}$

$$= \int_1^4 4\sqrt{y}\,dy$$

On integrating we get,

$$= 4\left[\frac{2}{3}y\sqrt{y}\right]_1^4$$

Now, applying limits we get,

= 4 [((2/3) × 4 × √4) - ((2/3) × 1 × √1)]

= 4 [(16/3) – (2/3)]

= 4 [(16 - 2)/3]

= 4 [14/3]

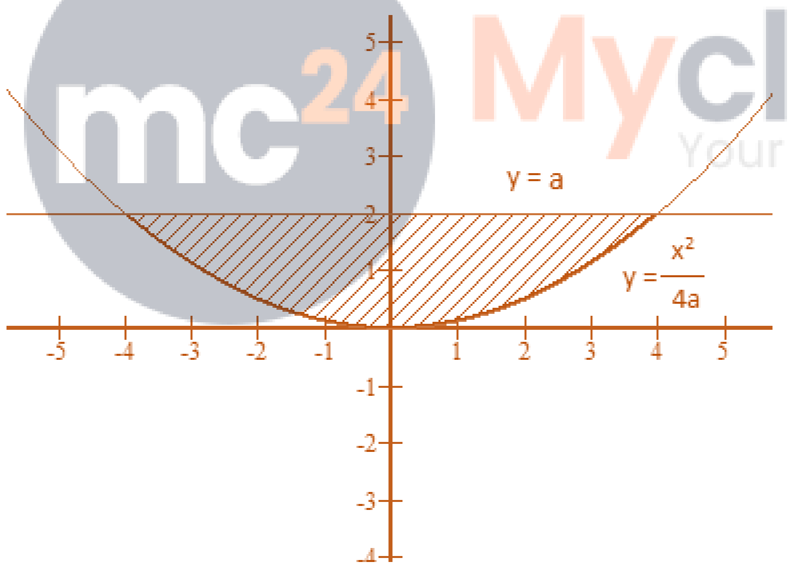
= 56/3

Therefore, the required area is 56/3 square units.

Question. 3

Solution:

We have to find the area of the region bounded by $x^2 = 4ay$



Then,

Area of the region = $2 \times \int_0^{2a} \left(a - \frac{x^2}{4a}\right) dx$

On integrating we get,

$$= 2 \times \left[ax - \frac{x^3}{12a}\right]_0^{2a}$$

Now applying limits,

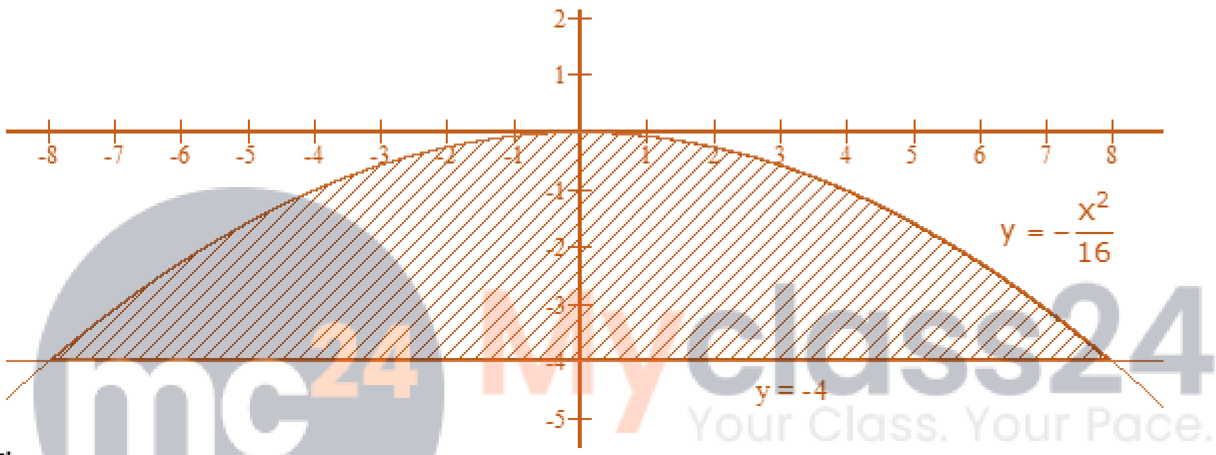
$= 2 \times [(a\,(2a - 0)) - ((2a)^3 - 0^3)/12a)]$

$= 2 \times [(2a^2) – (8a^3/12a)]$
$= 2 \times [(24a^3 – 8a^3)/12a]$
$= 2 \times [16a^3/12a]$
$= 2 \times [4a^2/3]$
$= 8a^2/3$

Therefore, the area of the region is $8a^2/3$ square units.

Question. 4
Solution:

We have to find the area of the region bounded by $x^2 + 16y = 0$



Then,

Area of the region $= 2\times\int_0^8\left[-\frac{x^2}{16}-(-4)\right]dx$

On integrating we get,

$$= 2\times\left[4x-\frac{x^3}{48}\right]_0^8$$

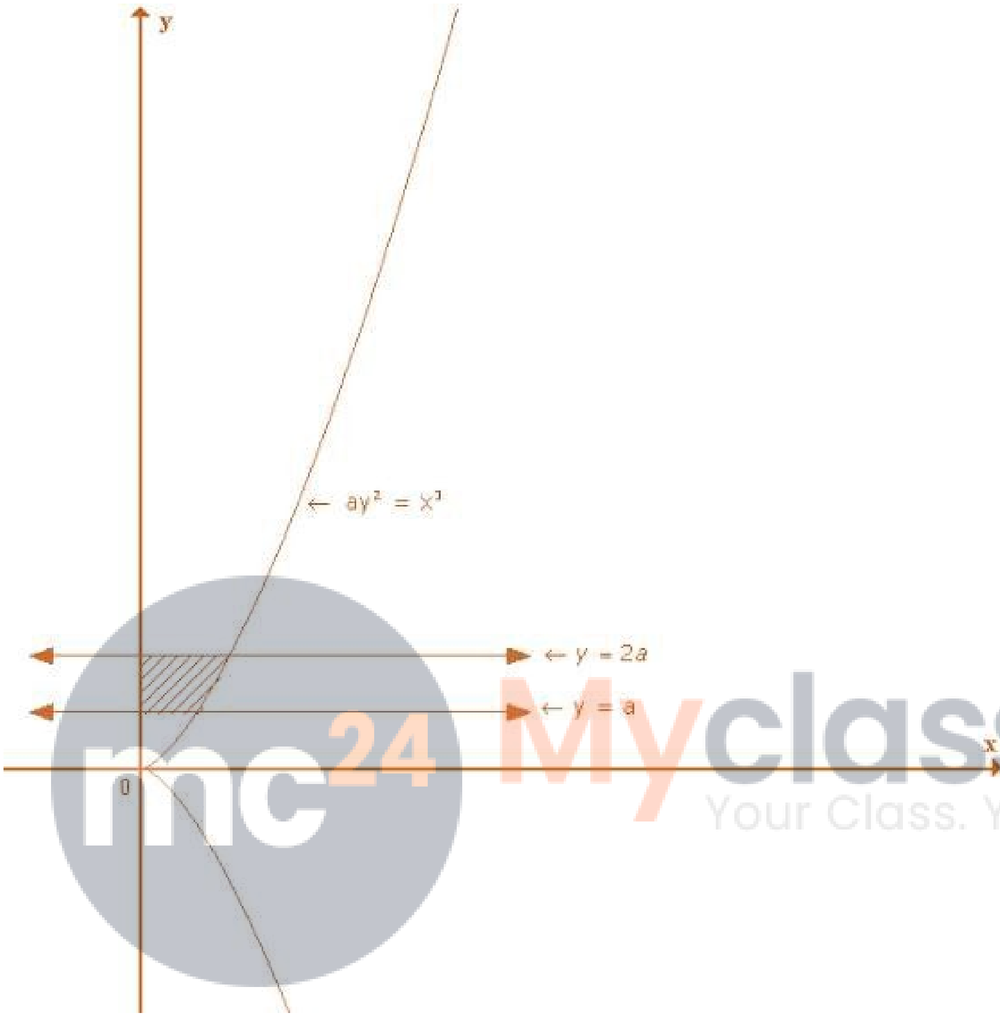
Now applying limits,

$= 2 \times [(4(8 – 0)) – ((8)^3 – 0^3)/48)]$
$= 2 \times [(32) – (512/48)]$
$= 2 \times [(32) – (32/3)]$
$= 2 \times [(96 – 32)/3]$
$= 2 \times [64/3]$
$= 128/3$

Therefore, the area of the region is 128/3 square units.

Question. 5
Solution:

We have to find the area of the region bounded by curve $ay^2 = x^3$, and lines y = a and y = 2a.



Then,

Area of the region = $\int_a^{2a} (ay^2)^{\frac{1}{3}}\, dy$

$= a^{\frac{1}{3}} \int_a^{2a} y^{\frac{2}{3}}\, dy$

On integrating we get,

$= a^{\frac{1}{3}} \left[\frac{3}{5} y^{\frac{5}{3}}\right]_0^{2a}$

Now applying limits we get,

$= \frac{3}{5}\left(2^{\frac{5}{3}} - 1\right) a^2$ sq units