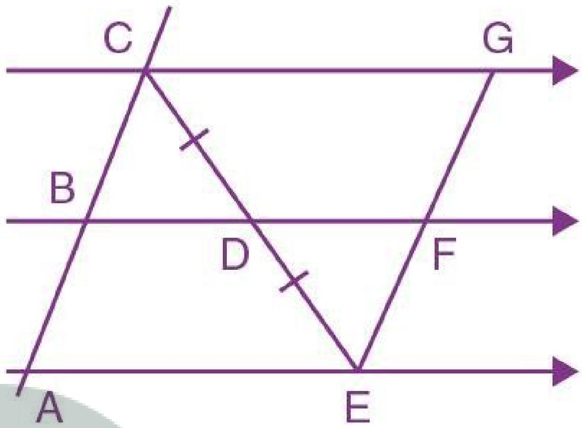
Exercise 12(B)

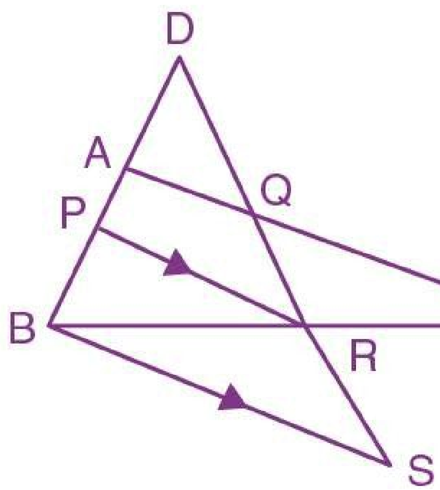
1. Use the following figure to find:
(i) BC, if AB = 7.2 cm.
(ii) GE, if FE = 4 cm.
(iii) AE, if BD = 4.1 cm
(iv) DF, if CG = 11 cm.



Solution:

According to equal intercept theorem, as CD = DE
⇒ AB = BC and EF = GF
Thus,
(i) BC = AB = 7.2cm
(ii) GE = EF + GF
= 2EF
= 2 × 4
= 8 cm
Since B, D and F are the midpoints and AE || BF || CG
Therefore, AE = 2BD and CG = 2DF
(iii) AE = 2BD
= 2 × 4.1
= 8.2 cm
(iv) DF = ½ CG
= ½ × 11
= 5.5 cm

2. In the figure, give below, 2AD = AB, P is mid-point of AB, Q is mid-point of DR and PR // BS. Prove that:
(i) AQ // BS
(ii) DS = 3 Rs.



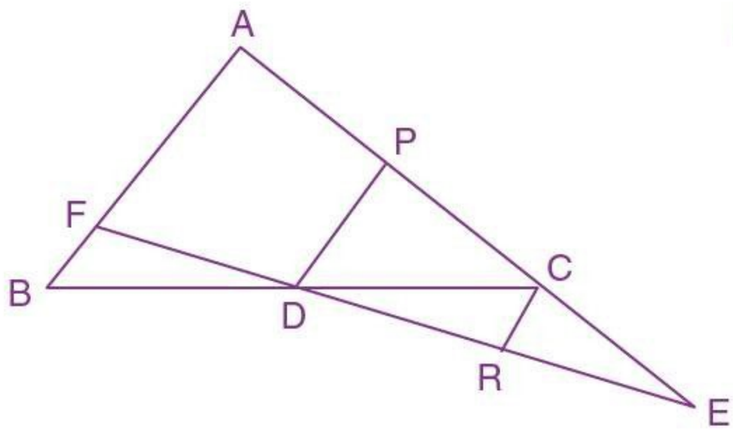
Solution:

Given, AD = AP = PB as 2AD = AB and P is the midpoint of AB
(i) In ∆DPR, we have
A and Q are the midpoints of DP and DR
Therefore, AQ || PR
Now, as PR || BS
∴ AQ || BS

(ii) In ∆ABC, P is the midpoint and PR || BS
Therefore, R is the midpoint of BC
Now, in ∆BRS and ∆QRC, we have
∠BRS = ∠QRC
BR = RC
∠RBS = ∠RCQ
∴ ∆BRS ≅ ∆QRC by SAS Congruence criterion
Hence, by CPCT
QR = RS
Thus, DS = DQ + QR + RS
= QR + QR + RS
= 3RS

3. The side AC of a triangle ABC is produced to point E so that CE = ½ AC. D is the mid-point of BC and ED produced meets AB at F. Lines through D and C are drawn parallel to AB which meet AC at point P and EF at point R respectively. Prove that:
(i) 3DF = EF (ii) 4CR = AB.
Solution:

Let’s consider the figure as below:

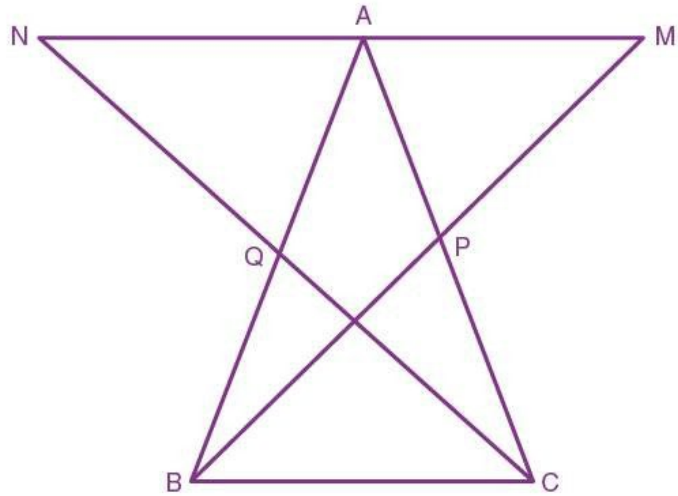


Here D is the midpoint of *BC* and *DP* is parallel to *AB*,
Therefore, P is the midpoint of AC and PD = ½ AB
(i) Again, in ∆AEF we have AE || PD || CR and AP = 1/3 AE
Therefore, DF = 1/3 EF
⇒ 3DF = EF
- Hence proved

(ii) In ∆PED, we have PD || CR and C is the midpoint of PE
So, CR = ½ PD
Now,
PD = ½ AB
½ PD = ¼ AB
CR = ¼ AB
4CR = AB
- Hence proved

4. In triangle ABC, the medians BP and CQ are produced upto points M and N respectively such that BP = PM and CQ = QN. Prove that:
(i) M, A and N are collinear.
(ii) A is the mid-point of MN.
Solution:

The figure is shown as below:

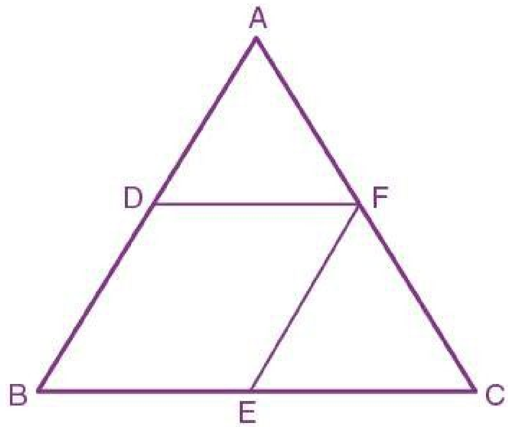


(i) In ΔBPC and ΔMPA, we have
∠BPC = ∠APN [Vertically opposite angle]
BP = MP
PC = PA
∴ ΔBPC ≅ ΔMPA by SAS congruence postulate
Thus, by CPCT
∠PCB = ∠PAM … (1)
And, BC = AM … (2)
Similarly,
Considering ΔCQB and ΔNQA, we have
∠QBC = ∠QAN … (3)
And, BC = AN … (4)
Now, by angle sum property of ΔABC
∠ABC + ∠ACB + ∠BAC = 180°
⇒ ∠QBC + ∠PCB + ∠BAC = 180°
∠QAN + ∠PAM + ∠BAC = 180° [From (1) and (3)]
Therefore, it's a straight angle and M, A, N must be collinear.

(ii) Now, from (2) and (4) we have
AM = AN
Hence, A is the midpoint of MN.

5. In triangle ABC, angle B is obtuse. D and E are mid-points of sides AB and BC respectively and F is a point on side AC such that EF is parallel to AB. Show that BEFD is a parallelogram.
Solution:

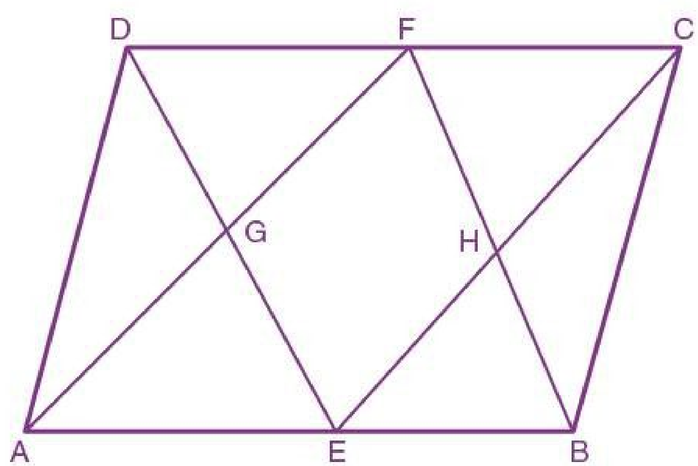
The figure is shown as below:



We have,
EF||AB and E is the midpoint of BC
Therefore, F is the midpoint of AC
And,
EF = BD as D is the midpoint of AB
Now, as BE || DF
BE = DF as E is the midpoint of BC.
Therefore, BEFD is a parallelogram.

6. In parallelogram ABCD, E and F are mid-points of the sides AB and CD respectively. The line segments AF and BF meet the line segments ED and EC at points G and H respectively. Prove that:
(i) Triangles HEB and FHC are congruent
(ii) GEHF is a parallelogram.
Solution:

The figure is shown as below:



(i) In ∆HEB and ∆HCF, we have
BE = FC [Given]

∠EHB = ∠FHC [Vertically opposite angles]

∠HBE = ∠HFC [Alternate angles]

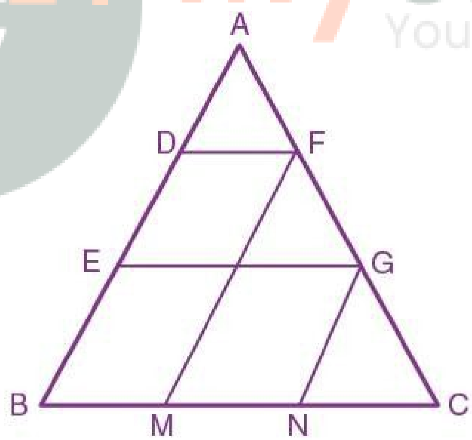
∴ ΔHEB ≅ ΔHCF by ASA congruence criterion

∴ EH = CH, BH = FH

(ii) Similarly, AG = GF and EG = DG ... (1)
In ΔECD, we have
F and H are the midpoints of CD and EC respectively
Therefore, HF || DE and HF = ½ DE ... (2)
From (1) and (2), we get
HF = EG and HF || EG
Similarly, we can show
EH = GF and EH || GF
Therefore, GEHF is a parallelogram.

7. In triangle ABC, D and E are points on side AB such that AD = DE = EB. Through D and E, lines are drawn parallel to BC which meet side AC at points F and G respectively. Through F and G, lines are drawn parallel to AB which meet side BC at points M and N respectively. Prove that: BM = MN = NC.
Solution:

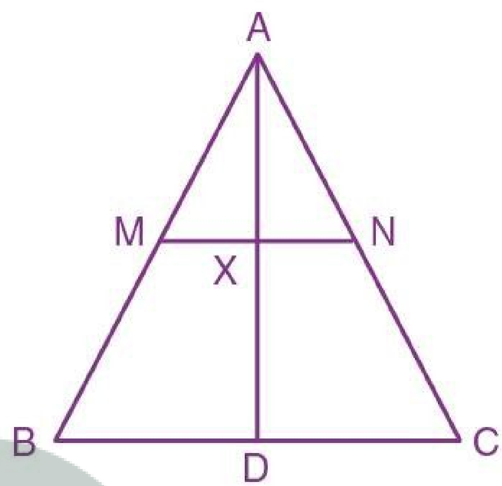
The figure is shown as below:



In ΔAEG, we have
D is the midpoint of AE and DF || EG || BC
Therefore, F is the midpoint of AG
⇒ AF = GF ... (1)
Again, we have DF || EG || BC and DE = BE
Therefore, GF = GC … (2)
From (1) and (2), we get
AF = GF = GC
Similarly, as GN || FM || AB and AF = GF
Therefore, BM = MN = NC
- Hence proved.

8. In triangle ABC; M is mid-point of AB, N is mid-point of AC and D is any point in base BC. Use intercept theorem to show that MN bisects AD.
Solution:

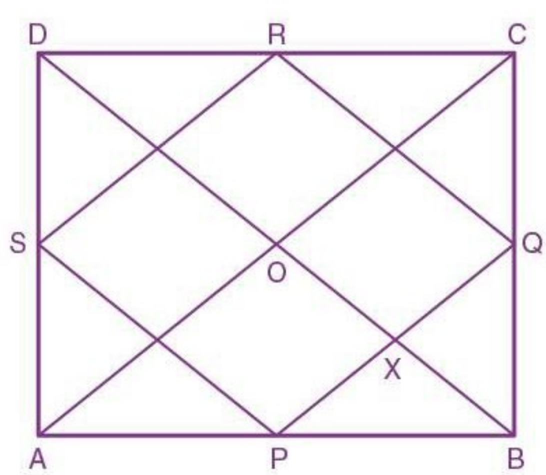
The figure is shown as below



As M and N are the midpoint of AB and AC respectively and MN || BC
Then according to intercept theorem, we have
AM = BM,
And therefore, AX = DX.
- Hence proved

9. If the quadrilateral formed by joining the mid-points of the adjacent sides of quadrilateral ABCD is a rectangle, show that the diagonals AC and BD intersect at right angle.
Solution:

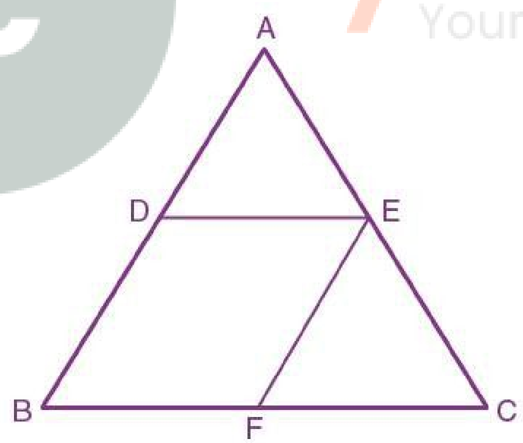
The figure is shown as below:



Let ABCD be a quadrilateral where P, Q, R and S are the midpoints of AB, BC, CD and DA.
And, PQRS is a rectangle
Diagonal AC and BD intersect at point O.
Required to show: AC and BD intersect at right angle.
Proof:
As PQ || AC,
⇒ ∠AOD = ∠PXO [Corresponding angles] ... (1)
Again, as BD||RQ,
⇒ ∠PXO = ∠RQX = 90° [Corresponding angle and angle of rectangle] ... (2)
From (1) and (2), we get
∠AOD = 90°
Similarly,
∠AOB = ∠BOC = ∠DOC = 90°
Therefore, diagonals AC and BD intersect at right angle
- Hence proved.

10. In triangle ABC; D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If AB = 16 cm, AC = 12 cm and BC = 18 cm, find the perimeter of the parallelogram BDEF.
Solution:

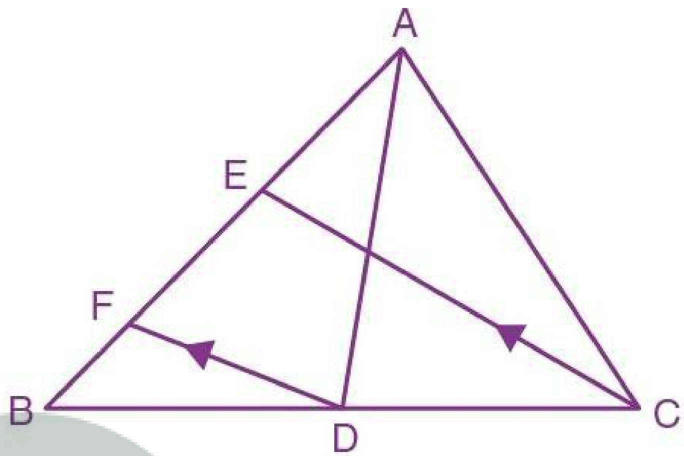
The figure is shown below as:



As E is the midpoint of AC and EF||AB, we have
Thus, F is the midpoint of BC and
2DE = BC or DE = BF
Again, as D and E are midpoints, we have
DE||BF and EF = BD
Hence, BDEF is a parallelogram.
Now,
BD = EF = ½ AB
= ½ ×16
= 8cm

BF = DE = ½ BC
= ½ ×18
= 9cm
Therefore, perimeter of BDEF = 2(BF + EF) = 2(9 + 8) = 34cm

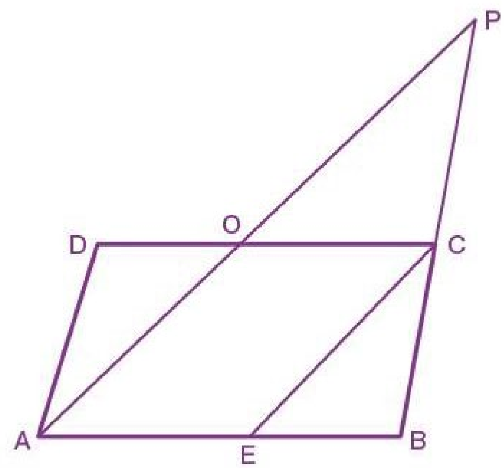
11. In the given figure, AD and CE are medians and DF||CE. Prove that: FB = ¼ AB



Solution:

Given, AD and CE are medians and DF || CE
We know that from the midpoint theorem,
If two lines are parallel and the starting point of segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.
Consider ∆BEC,
Given that DF || CE and D is midpoint of BC
So, F must be the midpoint of BE
⇒ FB = ½ BE … (i)
But, BE = ½ AB
On substituting the value of BE in (i), we get
FB = ¼ AB
- Hence Proved

12. In parallelogram ABCD, E is the mid-point of AB and AP is parallel to EC which meets DC at point O and BC produced at P.
Prove that:
(i) BP = 2AD
(ii) O is the mid-point of AP.



Solution:

Given ABCD is parallelogram,
⇒ AD = BC and AB = CD

(i) Now, in ΔAPB
Given, EC is parallel to AP and E is midpoint of side AB
So, by midpoint theorem,
C is the midpoint of BP
So, BP = 2BC
But,
BC = AD as ABCD is a parallelogram.
Hence, BP = 2AD

(ii) In ΔAPB, we have
AB || OC as ABCD is a parallelogram
So, by midpoint theorem
O is the midpoint of AP
Hence Proved.

13. In trapezium ABCD, sides AB and DC are parallel to each other. E is mid-point of AD and F is mid-point of BC. Prove that: AB + DC = 2EF.
Solution:

In trapezium ABCD, we have
E and F are the midpoints of sides AD and BC respectively

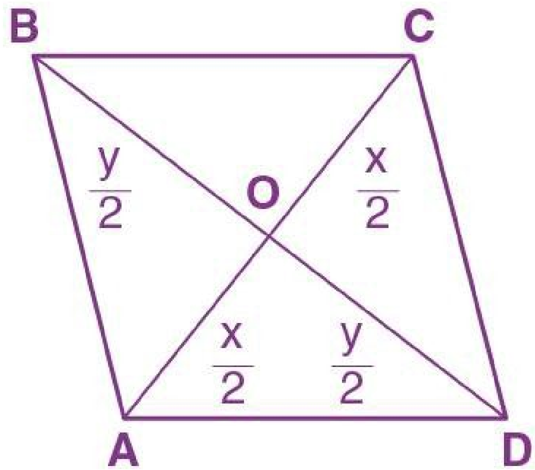
We know that, AB = GH = IJ
From midpoint theorem, we have
EG = ½ DI and
HF = ½ JC
Now, consider the L.H.S, we have
AB + CD = AB + CJ + JI + ID
= AB + 2HF + AB + 2EG
So, AB + CD = 2(AB + HF + EG)
= 2(EG + GH + HF)
= 2EF
∴ AB + CD = 2EF
- Hence Proved.

14. In ΔABC, AD is the median and DE is parallel to BA, where E is a point in AC. Prove that BE is also a median.
Solution:

Given, ΔABC and AD is the median
So, D is the midpoint of side BC
Also, given DE || AB
By the midpoint theorem,
E has to be midpoint of AC
So, line joining the vertex and midpoint of the opposite side is always the median.
Thus, BE is also median of ΔABC

15. Adjacent sides of a parallelogram are equal and one of the diagonals is equal to any one of the sides of this parallelogram. Show that its diagonals are in the ratio √3:1.
Solution:

If adjacent sides of a parallelogram are equal, then it is rhombus.
Now, the diagonals of a rhombus bisect each other and are perpendicular to each other.
Let's consider the lengths of the diagonals to be x and y
Diagonal of length y be equal to the sides of rhombus.
Thus, each side of rhombus = y



Now, in right angled ΔBOC
By Pythagoras theorem,
$OB^2 + OC^2 = BC^2$
$(y/2)^2 + (x/2)^2 = y^2$
$x^2/4 = y^2 - y^2/4$
$x^2/4 = (4y^2 - y^2)/4$
$x^2/4 = 3y^2/4$
$x^2 = 3y^2$
$x^2/y^2 = 3/1$
$x/y = \sqrt{3}/1$
Thus, the diagonals are in the ratio $\sqrt{3} : 1$