NCERT Solutions for Class-XII Maths

Chapter-4.1

the determinants in Exercises 1 and 2.

1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ expanding along R_1, we get

$= 2 \times (-1) - 4 \times (-5) = -2 + 20 = 18$

2. (i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

2. We know that determinant of A is calculated as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Now, $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$= \cos\theta(\cos\theta) - (-\sin\theta)(\sin\theta)$

$= \cos^2\theta + \sin^2\theta$

$= 1 \qquad [\because \cos^2\theta + \sin^2\theta = 1]$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

We know that determinant of A is calculated as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Now, $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

$= (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$

$= (x^3 - x^2 + x + x^2 - x + 1) - (x^2 - 1)$

$= x^3 + 1 - x^2 + 1$

$= x^3 - x^2 + 2$

3. If $A=\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$, then show that |2A| = 4|A|

3. $|2A|=\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$ expanding along R_1, we get

= 2 × 4 – 4 × 8 = 8 – 32 = – 24 ….(1)

$4|A|=4\begin{vmatrix}1 & 2\\4 & 2\end{vmatrix}$ Expanding along R_1, we get

= 4(1 × 2 – 2 × 4) = 4 (–6) = –24 ….(2)

From the equation (1) and (2), we get, |2A| = 4 |A|

4. If $A=\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$, then show that |2A| = 4 |A|

4. $|A|=\begin{vmatrix}1 & 0 & 1\\0 & 1 & 2\\0 & 0 & 4\end{vmatrix}$

We know that a determinant of a 3 x 3 matrix is calculated as

$$|A|=\begin{vmatrix}i & j & k\\a & b & c\\d & e & f\end{vmatrix}=i\begin{vmatrix}b & c\\e & f\end{vmatrix}-j\begin{vmatrix}a & c\\d & f\end{vmatrix}+k\begin{vmatrix}a & b\\d & e\end{vmatrix}$$

$$=1\begin{vmatrix}1 & 2\\0 & 4\end{vmatrix}-0\begin{vmatrix}0 & 2\\0 & 4\end{vmatrix}+1\begin{vmatrix}0 & 1\\0 & 0\end{vmatrix}$$

= 1[1(4) - 2(0)] – 0 + 1[0-0]

= 1[4 - 0] – 0 + 0

= 4

|A|= 4

<u>LHS</u>: |3A|

$$3A=3\begin{bmatrix}1 & 0 & 1\\0 & 1 & 2\\0 & 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}3 & 0 & 3\\0 & 3 & 6\\0 & 0 & 12\end{bmatrix}$$

$$|3A|=\begin{vmatrix}3 & 0 & 3\\0 & 3 & 6\\0 & 0 & 12\end{vmatrix}$$

$$= 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 6 \\ 0 & 12 \end{vmatrix} + 3\begin{vmatrix} 0 & 3 \\ 0 & 0 \end{vmatrix}$$

= 3[3(12) - 0(6)] – 0 + 3[0 – 0]

= 3(36) – 0 + 0

= 108

|3A|=108 ----LHS

<u>RHS</u>: 27|A|

27|A|=27(4)

=108

27|A|=108 ----RHS

LHS=RHS

Hence proved.

5. Evaluate the determinants:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

5. (i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expending along R_1, we get

= 3(0 – 5) + 1(0 + 3) – 2(0 - 0) = –15 + 3 – 0 = –12

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ Expanding along R_1, we get

= 3(1 + 6) + 4(1 + 4) + 5(3 - 2) = 21 + 20 + 5 = 46

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 0(0 + 9) - 1(0 - 6) + 2(-3 - 0) = 0 + 6 - 6 = 0$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 2(0 - 5) + 1(0 + 3) - 2(0 - 6) = -10 + 3 + 12 = 5$

6. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find |A|

6. given:

$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

We know that a determinant of a 3x3 matrix is calculated as

$|A| = \begin{vmatrix} i & j & k \\ a & b & c \\ d & e & f \end{vmatrix} = i\begin{vmatrix} b & c \\ e & f \end{vmatrix} - j\begin{vmatrix} a & c \\ d & f \end{vmatrix} + k\begin{vmatrix} a & b \\ d & e \end{vmatrix}$

$= 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

$= 1[-9 - (-3)(4)] - 1[2(-9) - (-3)(5)] - 2[2(4) - 1(5)]$

$= 1[-9 + 12] - 1[-18 + 15] - 2[8 - 5]$

$= 1[3] - 1[-3] - 2[3]$

$= 3 + 3 - 6$

$= 0$

7. Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

7. (i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 - 20 = 2x^2 - 24 \Rightarrow x^2 = 3 \quad \Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 10 - 12 = 5x - 6x \quad \Rightarrow -2 = -x \quad \Rightarrow x = 2$

8. if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to:

(a) 6

(b) ± 6

(c) –6

(d) 0

8. The correct answer is (B) ±6

Explanation:

We have $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

We know that determinant of A is calculated as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$\Rightarrow x(x) - 2(18) = 6(6) - 2(18)$

$\Rightarrow x^2 - 38 = 36 - 38$

$\Rightarrow x^2 = 36 - 38 + 38$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$