25. Product of Three Vectors

Exercise 25A

Q. 1

Prove that

i. $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

ii. $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Answer :

i. $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Formulae :

a) Dot Products :

i) $\hat{\imath} . \hat{\imath} = \hat{\jmath} . \hat{\jmath} = \hat{k} . \hat{k} = 1$

ii) $\hat{\imath} . \hat{\jmath} = \hat{\jmath} . \hat{k} = \hat{k} . \hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a} \quad \bar{b} \quad \bar{c}] = \bar{a} . (\bar{b} \times \bar{c})$

Now,

(i) $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = \hat{\imath} \,.(\hat{\jmath} \times \hat{k})$

$= \hat{\imath} \,.\, \hat{\imath}$ .... $(\because \hat{\jmath} \times \hat{k} = \hat{\imath})$

= 1 .... $(\because \hat{\imath} \,.\, \hat{\imath} = 1)$

$\therefore \; [\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$ .... eq(1)

(ii) $[\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = \hat{\jmath} \,.(\hat{k} \times \hat{\imath})$

$= \hat{\jmath} \,.\, \hat{\jmath}$ .... $(\because \hat{k} \times \hat{\imath} = \hat{\jmath})$

= 1 .... $(\because \hat{\jmath} \,.\, \hat{\jmath} = 1)$

$\therefore \; [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = 1$ .... eq(2)

(iii) $[\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = \hat{k} \,.(\hat{\imath} \times \hat{\jmath})$

$= \hat{k} \,.\, \hat{k}$ .... $(\because \hat{\imath} \times \hat{\jmath} = \hat{k})$

= 1 .... $(\because \hat{k} \,.\, \hat{k} = 1)$

$\therefore \; [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$ .... eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Hence Proved.

Notes :

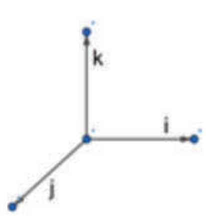
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a} \quad \bar{b} \quad \bar{c}] = [\bar{b} \quad \bar{c} \quad \bar{a}] = [\bar{c} \quad \bar{a} \quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$

$[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = -1$



ii. $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Formulae :

a) Dot Products :

i) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a} \quad \bar{b} \quad \bar{c}] = \bar{a}.(\bar{b} \times \bar{c})$

Answer :

(i) $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = \hat{\imath}.(\hat{k} \times \hat{\jmath})$

$= \hat{\imath}.(-\hat{\imath})$ .... $(\because \hat{k} \times \hat{\jmath} = -\hat{\imath})$

$= -\hat{\imath}.\hat{\imath}$

= -1 .... $(\because \hat{\imath}.\hat{\imath} = 1)$

$\therefore [\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = -1$ .... eq(1)

(ii) $[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = \hat{k}.(\hat{\jmath} \times \hat{\imath})$

$= \hat{k}.(-\hat{k})$ .... $(\because \hat{j} \times \hat{i} = -\hat{k})$

$= -\hat{k}.\hat{k}$

= -1 .... $(\because \hat{k}.\hat{k} = 1)$

$\therefore\ [\hat{k} \quad \hat{j} \quad \hat{i}] = -1$ .... eq(2)

(iii) $[\hat{j} \quad \hat{i} \quad \hat{k}] = \hat{j}.(\hat{i} \times \hat{k})$

$= \hat{j}.(-\hat{j})$ .... $(\because \hat{i} \times \hat{k} = -\hat{j})$

$= -\hat{j}.\hat{j}$

= -1 .... $(\because \hat{j}.\hat{j} = 1)$

$\therefore\ [\hat{j} \quad \hat{i} \quad \hat{k}] = -1$ .... eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{i} \quad \hat{k} \quad \hat{j}] = [\hat{k} \quad \hat{j} \quad \hat{i}] = [\hat{j} \quad \hat{i} \quad \hat{k}] = -1$

Hence Proved.

Notes :

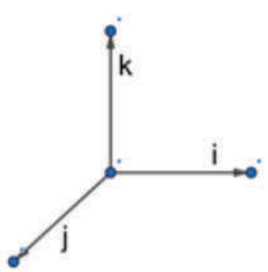
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a} \quad \bar{b} \quad \bar{c}] = [\bar{b} \quad \bar{c} \quad \bar{a}] = [\bar{c} \quad \bar{a} \quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{i} \quad \hat{j} \quad \hat{k}] = 1$

$[\hat{k} \quad \hat{j} \quad \hat{i}] = -1$



Q. 2

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, **when**

i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ **and .** $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ **.**

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ **and** $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$ **and** $\vec{c} = 3\hat{i} - \hat{k}$

Answer :

i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

2) $\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

3) $\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - 1 \times 1) - 1((-1) \times 2 - 3 \times 1) + 3((-1) \times 1 - 3 \times 2)$

$= 2(3) - 1(-5) + 3(-7)$

= 6 + 5 - 21

= - 10

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -10$$

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

2) $\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - (-1) \times (-1)) - (-3)(1 \times 2 - 3 \times (-1)) + 4(1 \times (-1) - 3 \times 2)$

$= 2(3) + 3(5) + 4(-7)$

= 6 + 15 – 28

= - 7

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -7$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath}$

2) $\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 0\hat{k}$

$\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} + 0\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$= 2(1 \times (-1) - (-1) \times 0) - (-3)(1 \times (-1) - 3 \times (-1)) + 0(1 \times 0 - 3 \times 1)$

$= 2(-1) + 3(2) + 0$

= - 2 + 6

= 4

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 4$$

Q. 3

Find the volume of the parallelepiped whose conterminous edges are represented by the vectors

i. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

ii. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \ \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Answer :

i. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

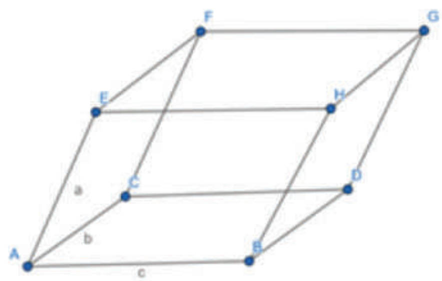
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 1((-1) \times (-1) - 2 \times 1) - 1(1 \times (-1) - 1 \times 1) + 1(1 \times 2 - 1 \times (-1))$$

= 1(-1) -1(-2) + 1(3)

= -1+2+3

= 4

Therefore,

Volume of parallelepiped = 4 cubic unit

ii. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$

$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

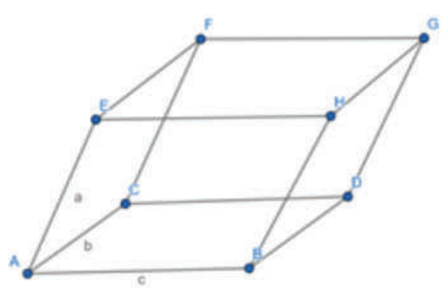
Answer :

Volume of parallelopiped with coterminous edges

$$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$$

$$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$$

$$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$$= -3\big(7 \times (-3) - (-5) \times (-3)\big) - 7\big((-5) \times (-3) - 7 \times (-3)\big) + 5((-5) \times (-5) - 7 \times 7)$$

= -3(-36) -7(36) + 5(-24)

= 108 – 252 – 120

= -264

As volume is never negative

Therefore,

Volume of parallelepiped = 264 cubic unit

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\jmath} + \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

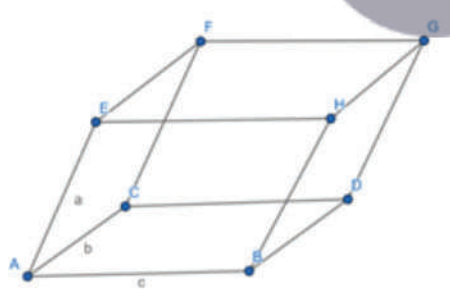
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 0\hat{\imath} + \hat{\jmath} + \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1\big(1 \times 1 - 1 \times (-1)\big) - (-2)\big(2 \times 1 - 0 \times (-1)\big) + \ 3(2 \times 1 - 0 \times 1)$$

= 1(2) +2(2) + 3(2)

= 2 + 4 + 6

= 12

Therefore,

Volume of parallelepiped = 12 cubic unit

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = 6\hat{\imath}$

$\bar{b} = 2\hat{\jmath}$

$\bar{c} = 5\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

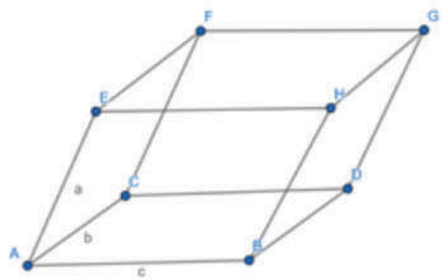
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = 6\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\bar{b} = 0\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$

$\bar{c} = 0\hat{\imath} + 0\hat{\jmath} + 5\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}$$

$= 6(2 \times 5 - 0 \times 0) - 0(0 \times 5 - 0 \times 0) + 0(0 \times 0 - 0 \times 2)$

= 6(10) + 0 + 0

= 60

Therefore,

Volume of parallelepiped = 60 cubic unit

Q. 4

Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ **are coplanar, when**

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ **and** $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ **and** $\vec{c} = 7\hat{j} + 3\hat{k}$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ **and** $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Answer :

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(3 \times 5 - (-3) \times (-4)) - (-2)((-2) \times 5 - 1 \times (-4)) + 3((-2) \times (-3) - 3 \times 1)$$

= 1(3) + 2(-6) + 3(3)

= 3 – 12 +9

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1((-1) \times 3 - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Given Vectors :

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

$$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

$$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 2 & -3 \\ 3 & -4 & 7 \end{vmatrix}$$

$$= 2(2 \times 7 - (-3) \times (-4)) - (-1)(1 \times 7 - 3 \times (-3)) + 2(1 \times (-4) - 3 \times 2)$$

= 2(2) + 1(16) + 2(-10)

= 4 + 16 -20

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Q. 5

Find the value of λ for which the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, when

i. $\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \lambda\hat{j} + 5\hat{k}\right)$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

iii. $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Answer :

i. . $\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$. and $\vec{c} = \left(3\hat{i} + \lambda\hat{j} + 5\hat{k}\right)$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & \lambda & 5 \end{vmatrix}$$

$= 2(2 \times 5 - 3 \times \lambda) - (-1)(1 \times 5 - 3 \times 3) + 1(1 \times \lambda - 3 \times 2)$

$= 2(10 - 3\lambda) - 4 + 1(\lambda - 6)$

$= 20 - 6\lambda - 4 + \lambda - 6$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 10 - 5\lambda$ ....eq(2)

From eq(1) and eq(2),

$10 - 5\lambda = 0$

$\therefore 5\lambda = 10$

$\boxed{\therefore \lambda = 2}$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$

$\bar{b} = -7\hat{\imath} - 5\hat{\jmath}$

$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$

$\bar{b} = -7\hat{\imath} - 5\hat{\jmath} + 0\hat{k}$

$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} \lambda & -10 & -5 \\ -7 & -5 & 0 \\ 1 & -4 & -3 \end{vmatrix}$$

$$= \lambda((-5) \times (-3) - 0 \times (-4)) - (-10)((-7) \times (-3) - 0 \times 1) + (-5)((-7) \times (-4) - 1 \times (-5))$$

$$= \lambda(15) + 10(21) - 5(33)$$

$= 15\lambda + 45$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 15\lambda + 45$ ....eq(2)

From eq(1) and eq(2),

$15\lambda + 45 = 0$

$\therefore 15\lambda = 45$

$\boxed{\therefore \lambda = -3}$

iii. . $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}$$

$= 1(1 \times \lambda - (-1) \times (-1)) - (-1)(2 \times \lambda - (-1) \times \lambda) + 1(2 \times (-1) - \lambda \times 1)$

$= 1(\lambda - 1) + 1(3\lambda) + 1(-\lambda - 2)$

$= \lambda - 1 + 3\lambda - 2 - \lambda$

$= 3\lambda - 3$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 3\lambda - 3$ ....eq(2)

From eq(1) and eq(2),

$3\lambda - 3 = 0$

$\therefore 3\lambda = 3$

$\boxed{\therefore \lambda = 1}$

Q. 6

If $\vec{a} = (2\hat{i} - \hat{j} + \hat{k}), \vec{b} = (\hat{i} - 3\hat{j} - 5\hat{k})$ **and** $\vec{c} = (3\hat{i} - 4\hat{j} - \hat{k})$, **find** $[\vec{a}\ \vec{b}\ \vec{c}]$ **and interpret the result.**

Answer :

Given Vectors :

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 2((-3) \times (-1) - (-4) \times (-5)) - (-1)((-1) \times 1 - 3 \times (-5)) + 1((-4) \times 1 - 3 \times (-3))$$

$$= 2(-17) + 1(14) + 1(5)$$

= - 34 + 14 + 5

= - 15

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -15$

Q. 7

The volume of the parallelepiped whose edges are $\left(-12\hat{i}+\lambda\hat{k}\right), \left(3\hat{j}-\hat{k}\right)$ **and** $\left(2\hat{i}+\hat{j}-15\hat{k}\right)$ **is 546 cubic units. Find the value of λ.**

Answer :

Given :

1) Coterminous edges of parallelepiped are

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$

2) Volume of parallelepiped,

V = 546 cubic unit

To Find : value of λ

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Given volume of parallelepiped,

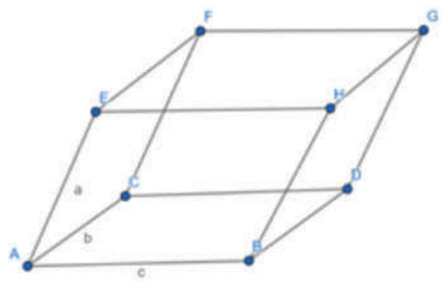
V = 546 cubic unit ....eq(1)

Volume of parallelopiped with coterminous edges

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix}$$

$$= -12(3 \times (-15) - 1 \times (-1)) - 0 + \lambda(0 \times 1 - 3 \times 2)$$

$$= 528 - 0 - 6\,\lambda$$

$$= 528 - 6\,\lambda$$

$\therefore V = (528 - 6\,\lambda)$ cubic unit ....eq(2)

From eq(1) and eq(2)

$$528 - 6\,\lambda = 546$$

$$\therefore -6\,\lambda = 18$$

$$\boxed{\therefore\ \lambda = -3}$$

Q. 8

Show that the vectors $\vec{a}=(\hat{i}+3\hat{j}+\hat{k}), \vec{b}=(2\hat{i}-\hat{j}-\hat{k})$ **and** $\vec{c}=(7\hat{j}+3\hat{k})$ **are parallel to the same plane.**

{HINT: Show that $[\vec{a}\ \vec{b}\ \vec{c}]=0$ **}**

Answer :

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are parallel to same plane.

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Vectors will be parallel to the same plane if they are coplanar.

For vectors $\bar{a}, \bar{b}, \bar{c}$ to be coplanar, $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Now, for given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1(3 \times (-1) - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Hence, given vectors are parallel to the same plane.

Q. 9

If the vectors $\left(a\hat{i} + a\hat{j} + c\hat{k}\right), \left(\hat{i} + \hat{k}\right)$ **and** $\left(c\hat{i} + c\hat{j} + b\hat{k}\right)$ **be coplanar, show that c^2 = ab.**

Answer :

Given : vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar. Where

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

To Prove : c^2 = ab

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}$$

$= a(0 \times b - c \times 1) - a(1 \times b - 1 \times c) + c(1 \times c - 0 \times c)$

= a.(- c) – a.(b - c) + c(c)

= – ac – ab + ac + c^2

= - ab + c^2

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -ab + c^2$ ....eq(2)

From eq(1) and eq(2),

- ab + c^2 = 0

Therefore,

c^2 = ab

Hence proved.

Note : Three vectors $\bar{a}, \bar{b}$ & $\bar{c}$ are coplanar if and only if

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$