

### Exercise 4(C)

#### 1. Expand:

(i)  $(x + 8)(x + 10)$

(ii)  $(x + 8)(x - 10)$

(iii)  $(x - 8)(x + 10)$

(iv)  $(x - 8)(x - 10)$

**Solution:**

Using the identity,  $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) We have,  $(x + 8)(x + 10)$

$$= x^2 + (8 + 10)x + 8 \times 10$$

$$= x^2 + 18x + 80$$

(ii) We have,  $(x + 8)(x - 10)$

$$= x^2 + (8 - 10)x + 8 \times (-10)$$

$$= x^2 - 2x - 80$$

(iii) (ii) We have,  $(x - 8)(x + 10)$

$$= x^2 + (-8 + 10)x + (-8) \times 10$$

$$= x^2 + 2x - 80$$

(iv) We have,  $(x - 8)(x - 10)$

$$= x^2 + (-8 - 10)x + (-8) \times (-10)$$

$$= x^2 - 18x + 80$$

#### 2. Expand:

(i)  $(2x - 1/x)(3x + 2/x)$

(ii)  $(3a + 2/b)(2a - 3/b)$

**Solution:**

(i) We have,  $(2x - 1/x)(3x + 2/x)$

$$= (2x)(3x) + (2x)(2/x) - (1/x)(3x) - (1/x)(2/x)$$

$$= 6x^2 + 4 - 3 - 2/x^2$$

$$= 6x^2 + 1 - 2/x^2$$

(ii) We have,  $(3a + 2/b)(2a - 3/b)$

$$= (3a)(2a) - (3a)(3/b) + (2/b)(2a) - (2/b)(3/b)$$

$$= 6a^2 - 9a/b + 4a/b - 6/b^2$$

$$= 6a^2 - 5a/b - 6/b^2$$

#### 3. Expand:

(i)  $(x + y - z)^2$

(ii)  $(x - 2y + 2)^2$

(iii)  $(5a - 3b + c)^2$

(iv)  $(5x - 3y - 2)^2$

(v)  $(x - 1/x + 5)^2$

**Solution:**

$$(i) (x + y - z)^2 = x^2 + y^2 + z^2 + 2(x)(y) - 2(y)(z) - 2(z)(x) \\ = x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$$

$$(ii) (x - 2y + 2)^2 = x^2 + (-2y)^2 + 2^2 + 2(x)(-2y) + 2(-2y)(2) + 2(2)(x) \\ = x^2 + 4y^2 + 4 - 4xy - 8y + 4x$$

$$(iii) (5a - 3b + c)^2 = (5a)^2 + (-3b)^2 + c^2 + 2(5a)(-3b) + 2(-3b)(c) + 2(c)(5a) \\ = 25a^2 + 9b^2 + c^2 - 30ab - 6bc + 10ac$$

$$(iv) (5x - 3y - 2)^2 = (5x)^2 + (-3y)^2 + (-2)^2 + 2(5x)(-3y) + 2(-3y)(-2) + 2(-2)(5x) \\ = 25x^2 + 9y^2 + 4 - 30xy + 12y - 20x$$

$$(v) (x - 1/x + 5)^2 = (x)^2 + (-1/x)^2 + (5)^2 + 2(x)(-1/x) + 2(-1/x)(5) + 2(5)(x) \\ = x^2 + 1/x^2 + 25 - 2 - 10/x + 10x \\ = x^2 + 1/x^2 + 23 - 10/x + 10x$$

**4. If  $a + b + c = 12$  and  $a^2 + b^2 + c^2 = 50$ ; find  $ab + bc + ca$ .**

**Solution:**

Given,  $a + b + c = 12$  and  $a^2 + b^2 + c^2 = 50$

We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$12^2 = 50 + 2(ab + bc + ca)$$

$$144 = 50 + 2(ab + bc + ca)$$

$$ab + bc + ca = (144 - 50) / 2 \\ = 94/2$$

Thus,

$$ab + bc + ca = 47$$

**5. If  $a^2 + b^2 + c^2 = 35$  and  $ab + bc + ca = 23$ ; find  $a + b + c$ .**

**Solution:**

Given,  $a^2 + b^2 + c^2 = 35$  and  $ab + bc + ca = 23$

We know that,

$$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$$

$$(a + b + c)^2 = 35 + 2(23)$$

$$(a + b + c)^2 = 35 + 46$$

$$(a + b + c)^2 = 81$$

$$(a + b + c) = \pm\sqrt{81}$$

Thus,

$$a + b + c = \pm 9$$

**6. If  $a + b + c = p$  and  $ab + bc + ca = q$ ; find  $a^2 + b^2 + c^2$ .**

**Solution:**

Given,  $a + b + c = p$  and  $ab + bc + ca = q$

We know that,

$$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$$

$$(p)^2 = (a^2 + b^2 + c^2) + 2(q)$$

$$\Rightarrow a^2 + b^2 + c^2 = p^2 - 2q$$

**7. If  $a^2 + b^2 + c^2 = 50$  and  $ab + bc + ca = 47$ , find  $a + b + c$ .**

**Solution:**

Given,  $a^2 + b^2 + c^2 = 50$  and  $ab + bc + ca = 47$

We know that,

$$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$$

$$(a + b + c)^2 = 50 + 2(47)$$

$$(a + b + c)^2 = 50 + 94$$
$$= 144$$

$$\Rightarrow (a + b + c) = \sqrt{144}$$

Thus,

$$a + b + c = \pm 12$$

**8. If  $x + y - z = 4$  and  $x^2 + y^2 + z^2 = 30$ , then find the value of  $xy - yz - zx$ .**

**Solution:**

Given,  $x + y - z = 4$  and  $x^2 + y^2 + z^2 = 30$

We know that,

$$(x + y - z)^2 = x^2 + y^2 + z^2 + 2(xy - yz - zx)$$

$$4^2 = 30 + 2(xy - yz - zx)$$

$$16 - 30 = 2(xy - yz - zx)$$

$$xy - yz - zx = -14/2$$

Thus,

$$xy - yz - zx = -7$$