Complex Numbers & Quadratic Equations

Exercise 5A

Q. 1. Evaluate:
(i) i^{19}
(ii) i^{62}
(ii) i^{373}.

Answer : We all know that $i = \sqrt{(-1)}$.

And $i^{4n} = 1$

$i^{4n+1} = i$ (where n is any positive integer)

$i^{4n+2} = -1$

$i^{4n+3} = -1$

So,

(i) L.H.S = i^{19}

$= i^{4\times4+3}$

$= i^{4n+3}$

Since it is of the form i^{4n+3} so the solution would be simply $-i$

Hence the value of i^{19} is $-i$.

(ii) $L.H.S = i^{62}$

$\Rightarrow i^{4\times15+2}$

$\Rightarrow i^{4n+2} \Rightarrow i^2 = -1$

so it is of the form i^{4n+2} so its solution would be -1

(iii) $L.H.S. = i^{373}$

$\Rightarrow i^{4\times 93+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow$ i

So, it is of the form of i^{4n+1} so the solution would be i.

Q. 2. Evaluate:

(i) $\left(\sqrt{-1}\right)^{192}$

(ii) $\left(\sqrt{-1}\right)^{93}$

(iii) $\left(\sqrt{-1}\right)^{30}$.

Answer : Since i = $\sqrt{-1}$ so

(i) L.H.S. = $\left(\sqrt{-1}\right)^{192}$

$\Rightarrow i^{192}$

$\Rightarrow i^{4\times 48} = 1$

Since it is of the form i^{4n} = 1 so the solution would be 1

(ii) L.H.S.= $\left(\sqrt{-1}\right)^{93}$

$\Rightarrow i^{4\times 23+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^{1} = i$

Since it is of the form of $i^{4n+1} = i$ so the solution would be simply i.

(iii) L.H.S = $\left(\sqrt{-1}\right)^{30}$

$\Rightarrow i^{4\times7+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

Since it is of the form i^{4n+2} so the solution would be -1

Q. 3. Evaluate:
(i) i^{-50}
(ii) i^{-9}
(ii) i^{-131}.

Answer : (i) $\text{L.H.S.} = i^{-50}$

$\Rightarrow i^{-4\times13+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow -1$

Since it is of the form i^{4n+2} so the solution would be -1

(ii) $\text{L.H.S.} = i^{-9}$

$\Rightarrow i^{-4\times3+3}$

$\Rightarrow i^{4n+3}$

$\Rightarrow i^3 = -i$

Since it is of the form of i^{4n+3} so the solution would be simply -i.

(iii) $\text{L.H.S.} = i^{-131}$

$\Rightarrow i^{-4\times33+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^{1} = i$

Since it is of the form i^{4n+1}. so the solution would be i

Q. 4. Evaluate:

(i) $\left(i^{41} + \frac{1}{i^{71}}\right)$

(ii) $\left(i^{53} + \frac{1}{i^{53}}\right)$

Answer :

(i) $\left(i^{41} + \frac{1}{i^{71}}\right) = i^{41} + i^{-71}$

$\Rightarrow i^{4\times10+1} + i^{-4\times18+1}$ (Since i^{4n+1} = i)

$\Rightarrow i^{1} + i^{1}$

$\Rightarrow$ 2i

Hence, $\left(i^{41} + \frac{1}{i^{71}}\right)$ = 2i

(ii) $\left(i^{53} + \frac{1}{i^{53}}\right)$

$\Rightarrow i^{53} + i^{-53}$

$\Rightarrow$ $i^{4\times13+1} + i^{-4\times14+3}$ (Since i^{4n+1} = i

$\Rightarrow$ $i^1 + i^3$ i^{4n+3} = -1)

$\Rightarrow$ 0

Hence, $\left(i^{53} + \frac{1}{i^{53}}\right)$ = 0

Q. 5. Prove that 1 + i^2 + i^4 + i^6 = 0

Answer : L.H.S.= 1 + i^2 + i^4 + i^6

To Prove: 1 + i^2 + i^4 + i^6 = 0

$\Rightarrow$ 1 + (-1) +1 + i^2

Since, i^{4n} = 1

(Where n is any positive integer)

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

$\Rightarrow$ 1 + -1 + 1 + -1=0

⇒L.H.S = R.H.S

Hence proved.

Q. 6. Prove that $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i$.

Answer : Given: $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48}$

To prove: $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i$

$\Rightarrow$ $6i^{4\times12+2} + 5i^{4\times8+1} - 2i^{4\times3+3} + 6i^{4\times12}$

$\Rightarrow 6i^2 + 5i^1 - 2i^3 + 6i^0$

$\Rightarrow$ -6+5i+2i+6

$\Rightarrow$ 7i

$\Rightarrow$ L.H.S = R.H.S

Hence proved.

Q. 7. Prove that $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$ = 0.

Answer :

Given: $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$

To prove : $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$ = 0.

$\Rightarrow$ L.H.S. = $i^{-1} - i^{-2} + i^{-3} - i^{-4}$

$\Rightarrow i^{-4\times1+3} - i^{-4\times1+2} + i^{-4\times1+3} - i^{-4\times1}$

Since i^{4n} = 1

$\Rightarrow i^{4n+1}$ = i

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3}$ = -1

So,

$\Rightarrow i^1 - i^2 + i^3 - 1$

$\Rightarrow$ i +1 – i -1

$\Rightarrow$ 0

⇒L.H.S = R.H.S

Hence Proved

Q. 8. Prove that $(1 + i^{10} + i^{20} + i^{30})$ is a real number.

Answer : L.H.S = $(1 + i^{10} + i^{20} + i^{30})$

= $(1 + i^{4\times2+2} + i^{4\times5} + i^{4\times7+2})$

Since $\Rightarrow i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

= $1 + i^2 + 1 + i^2$

= 1 + -1 +1 + -1

= 0, which is a real no.

Hence, $(1 + i^{10} + i^{20} + i^{30})$ is a real number.

Q. 9. Prove that $\left\{i^{21} - \left(\frac{1}{i}\right)^{46}\right\}^2$ = 2i.

Answer : L.H.S.= $\left\{i^{21} - \left(\frac{1}{i}\right)^{46}\right\}^2$

= $\left\{i^{4\times5+1} - i^{-4\times12+2}\right\}^2$

Since i^{4n} = 1

i^{4n+1} = i

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3$ = -1

= $\left\{i^1 - i^2\right\}^2$

= $\{i+1\}^2$

Now, applying the formula $(a+b)^2$ = $a^2 + b^2 +$ 2ab

= $i^2 + 1 + 2i$.

= -1 + 1 + 2i

= 2i

L.H.S = R.H.S

Hence proved.

Q. 10. $\left\{i^{18} + \frac{1}{i^{25}}\right\}^3$ **= 2(1 – i).**

Answer : L.H.S = $\left\{i^{18} + \frac{1}{i^{25}}\right\}^3$

$\Rightarrow \left\{i^{4\times4+2} + i^{-4\times7+3}\right\}^3$

Since i^{4n} = 1

i^{4n+1} = i

i^{4n+2} = -1

i^{4n+3} = -1

= $\left\{i^2 + i^3\right\}^3$.

= $(-1-i)^3$.

Applying the formula $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We have,

+ $3i^2 + 3i + 1)$

i + 3 - 3i - 1

= 2(1-i)

L.H.S = R.H.S

Hence proved.

Q. 11. Prove that (1 – i)^n $\left(1-\frac{1}{i}\right)^n$ = 2^n for all values of n N

Answer : L.H.S = $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$

= $(1-i)^n \left(1-i^{-4*1+3}\right)^n$

= $(1-i)^n \left(1-i^3\right)^n$

Since, i^{4n+3}. = -1

= $(1-i)^n (1+i)^n$.

Applying $a^n b^n = (ab)^n$

= $((1-i)(1+i))^n$.

= $\left(1-i^2\right)^n$

= 2^n

L.H.S = R.H.S

Q. 12. Prove that $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$ = 0.

Answer : L.H.S = $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

Since we know that i = $\sqrt{-1}$.

So,

= $\sqrt{16}$ i + 3$\sqrt{25}$ i + $\sqrt{36}$ i $-$ $\sqrt{625}$ i

$=4i + 15i + 6i - 25i$

$= 0$

L.H.S = R.H.S

Hence proved.

Q. 13. Prove that $(1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20}) = 1$.

Answer : L.H.S = $(1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20})$

$= \sum_{n=0}^{n=20} i^n$

$= 1 + -1 + 1 + -1 + ... + 1$

As there are 11 times 1 and 6 times it is with positive sign as i^0 =1 as this is the extra term and there are 5 times 1 with negative sign.

So, these 5 cancel out the positive one leaving one positive value i.e. 1

$= \sum_{n=0}^{20} i^n = 1$

L.H.S = R.H.S

Hence proved.

Q. 14. Prove that $i^{53} + i^{72} + i^{93} + i^{102} = 2i$.

Answer : L.H.S = $i^{53} + i^{72} + i^{93} + i^{102}$

$= i^{4\times13+1} + i^{4\times18} + i^{4\times23+1} + i^{4\times25+2}$

Since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$ (where n is any positive integer)

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

$= i + 1 + i + i^2$

$= i+1+i-1$

$=2i$

L.H.S = R.H.S

Hence proved.

Q. 15. Prove that $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right) = (-1 + i),$ **n N.**

Answer : L.H.S = $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$

$= i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + \ldots + i^{13} + i^{14}$

Since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

$= i - 1 - i + 1 + i - 1 \ldots\ldots + i - 1$

As, all terms will get cancel out consecutively except the first two terms. So that will get remained will be the answer.

= i - 1

L.H.S = R.H.S

Hence proved.

Exercise 5B

Q. 1. A. Simplify each of the following and express it in the form a + ib :

2(3 + 4i) + i(5 – 6i)

Answer : Given: 2(3 + 4i) + i(5 – 6i)

Firstly, we open the brackets

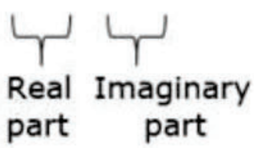
$2 \times 3 + 2 \times 4i + i \times 5 - i \times 6i$

$= 6 + 8i + 5i - 6i^2$

$= 6 + 13i - 6(-1)$ [∵, $i^2 = -1$]

$= 6 + 13i + 6$

$= 12 + 13i$



Q. 1. B. Simplify each of the following and express it in the form a + ib :

$$\left(3+\sqrt{-16}\right)-\left(4-\sqrt{-9}\right)$$

Answer : Given: $(3 + \sqrt{-16}) - (4 - \sqrt{-9})$

We re – write the above equation

$$(3 + \sqrt{(-1) \times 16})(-1)(4 - \sqrt{(-1) \times 9})$$

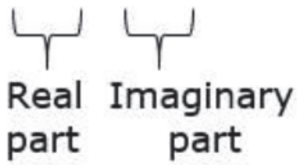
$= (3 + \sqrt{16i^2}) - (4 - \sqrt{9i^2})$ [∵ $i^2 = -1$]

= (3 + 4i) – (4 – 3i)

Now, we open the brackets, we get

3 + 4i – 4 + 3i

= -1 + 7i



Q. 1. C. Simplify each of the following and express it in the form a + ib :

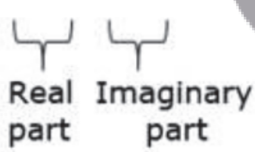
(–5 + 6i) – (–2 + i)

Answer : Given: (–5 + 6i) – (–2 + i)

Firstly, we open the brackets

-5 + 6i + 2 – i

= -3 + 5i



Q. 1. D. Simplify each of the following and express it in the form a + ib :

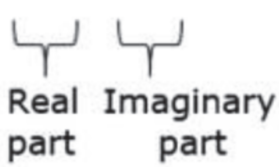
(8 – 4i) – (- 3 + 5i)

Answer : Given: (8 – 4i) – (- 3 + 5i)

Firstly, we open the brackets

8 – 4i + 3 – 5i

= 11 – 9i



Q. 1. E. Simplify each of the following and express it in the form a + ib :

$(1 - i)^2 (1 + i) - (3 - 4i)^2$

Answer : Given: $(1 - i)^2 (1 + i) - (3 - 4i)^2$

$= (1 + i^2 - 2i)(1 + i) - (9 + 16i^2 - 24i)$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= (1 - 1 - 2i)(1 + i) - (9 - 16 - 24i)$ $[\because i^2 = -1]$

$= (-2i)(1 + i) - (- 7 - 24i)$

Now, we open the brackets

$-2i \times 1 - 2i \times i + 7 + 24i$

$= -2i - 2i^2 + 7 + 24i$

$= -2(-1) + 7 + 22i$ $[\because, i^2 = -1]$

$= 2 + 7 + 22i$

$= 9 + 22i$

Real part Imaginary part

Q. 1. F. Simplify each of the following and express it in the form a + ib :

$$\left(5+\sqrt{-3}\right)\left(5-\sqrt{-3}\right)$$

Answer : Given: $(5 + \sqrt{-3})(5 - \sqrt{-3})$

We re – write the above equation

$$(5 + \sqrt{(-1) \times 3})(5 - \sqrt{(-1) \times 3})$$

$= (5 + \sqrt{3i^2})(5 - \sqrt{3i^2})$ $[\because, i^2 = -1]$

$= (5 + i\sqrt{3})(5 - i\sqrt{3})$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

Here, $a = 5$ and $b = i\sqrt{3}$

$= (5)^2 - (i\sqrt{3})^2$

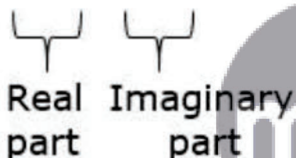
$= 25 - (3i^2)$

$= 25 - [3 \times (-1)]$

$= 25 + 3$

$= 28 + 0$

$= 28 + 0i$



Q. 1. G. Simplify each of the following and express it in the form a + ib :

(3 + 4i) (2 – 3i)

Answer : Given: (3 + 4i) (2 – 3i)

Firstly, we open the brackets

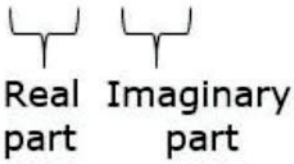
$3 \times 2 + 3 \times (-3i) + 4i \times 2 - 4i \times 3i$

$= 6 - 9i + 8i - 12i^2$

$= 6 - i - 12(-1)$ [$\because$, $i^2 = -1$]

$= 6 - i + 12$

$= 18 - i$



Q. 1. H. Simplify each of the following and express it in the form a + ib :

$$\left(-2+\sqrt{-3}\right)\left(-3+2\sqrt{-3}\right)$$

Answer : Given: $(-2+\sqrt{-3})(-3+2\sqrt{-3})$

We re – write the above equation

$$(-2+\sqrt{(-1)\times 3})(-3+2\sqrt{(-1)\times 3})$$

$= (-2+\sqrt{3i^2})(-3+2\sqrt{3i^2})$ [∵, $i^2 = -1$]

$= (-2+i\sqrt{3})(-3+2i\sqrt{3})$

Now, open the brackets,

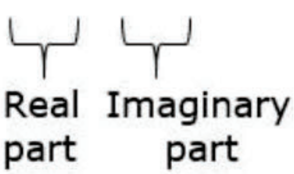
$= -2 \times (-3) + (-2) \times 2i\sqrt{3} + i\sqrt{3} \times (-3) + i\sqrt{3} \times 2i\sqrt{3}$

$= 6 - 4i\sqrt{3} - 3i\sqrt{3} + 6i^2$

$= 6 - 7i\sqrt{3} + [6 \times (-1)]$ [∵, $i^2 = -1$]

$= 6 - 7i\sqrt{3} - 6$

$= 0 - 7i\sqrt{3}$



Q. 2. A. Simplify each of the following and express it in the form (a + ib) :

$$\left(2+\sqrt{-3}\right)^2$$

Answer : Given: $(2 - \sqrt{-3})^2$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ …(i)

So, on replacing a by 2 and b by $\sqrt{-3}$ in eq. (i), we get

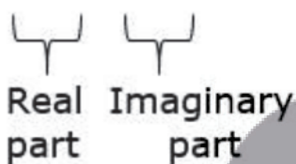
$(2)^2 + (\sqrt{-3})^2 - 2(2)(\sqrt{-3})$

$= 4 + (-3) - 4\sqrt{-3}$

$= 4 - 3 - 4\sqrt{-3}$

$= 1 - 4\sqrt{3}i^2$ [$\because i^2 = -1$]

$= 1 - 4i\sqrt{3}$



Q. 2. B. Simplify each of the following and express it in the form (a + ib) :

$(5 - 2i)^2$

Answer : Given: $(5 - 2i)^2$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ …(i)

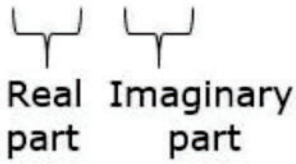
So, on replacing a by 5 and b by 2i in eq. (i), we get

$(5)^2 + (2i)^2 - 2(5)(2i)$

$= 25 + 4i^2 - 20i$

$= 25 - 4 - 20i$ [$\because i^2 = -1$]

$= 21 - 20i$



Q. 2. C. Simplify each of the following and express it in the form (a + ib) :

$(-3 + 5i)^3$

Answer : Given: $(-3 + 5i)^3$

We know that,

$(-a + b)^3 = - a^3 + 3a^2b - 3ab^2 + b^3$ …(i)

So, on replacing a by 3 and b by 5i in eq. (i), we get

$-(3)^3 + 3(3)^2(5i) - 3(3)(5i)^2 + (5i)^3$

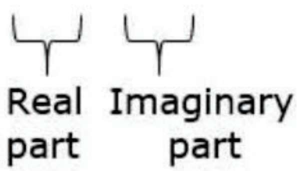
$= -27 + 3(9)(5i) - 3(3)(25i^2) + 125i^3$

$= -27 + 135i - 225i^2 + 125i^3$

$= -27 + 135i - 225 \times (-1) + 125i \times i^2$

$= -27 + 135i + 225 - 125i$ [$\because i^2 = -1$]

$= 198 + 10i$



Q. 2. D. Simplify each of the following and express it in the form (a + ib) :

$$\left(-2 - \frac{1}{3}i\right)^3$$

Answer : Given: $\left(-2 - \frac{1}{3}i\right)^3$

We know that,

$(-a-b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3$ …(i)

So, on replacing a by 2 and b by 1/3i in eq. (i), we get

$$-(2)^3 - 3(2)^2\left(\frac{1}{3}i\right) - 3(2)\left(\frac{1}{3}i\right)^2 - \left(\frac{1}{3}i\right)^3$$

$$= -8 - 4i - 6\left(\frac{1}{9}i^2\right) - \left(\frac{1}{27}i^3\right)$$

$$= -8 - 4i - \frac{2}{3}i^2 - \frac{1}{27}i(i^2)$$

$$= -8 - 4i - \frac{2}{3}(-1) - \frac{1}{27}i(-1)$$ [∵ $i^2 = -1$]

$$= -8 - 4i + \frac{2}{3} + \frac{1}{27}i$$

$$= \left(-8 + \frac{2}{3}\right) + \left(-4i + \frac{1}{27}i\right)$$

$$= \left(\frac{-24+2}{3}\right) + \left(\frac{-108i + i}{27}\right)$$

$$= -\frac{22}{3} + \left(-\frac{107}{27}i\right)$$

$$= \underbrace{-\frac{22}{3}}_{\text{Real part}} \underbrace{-\frac{107}{27}i}_{\text{Imaginary part}}$$

Q. 2. E. Simplify each of the following and express it in the form (a + ib) :

$(4 - 3i)^{-1}$

Answer : Given: $(4 - 3i)^{-1}$

We can re- write the above equation as

$$= \frac{1}{4 - 3i}$$

Now, rationalizing

$$= \frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}$$

$$= \frac{4+3i}{(4-3i)(4+3i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{4 + 3i}{(4)^2 - (3i)^2}$$

$$= \frac{4 + 3i}{16 - 9i^2}$$

$$= \frac{4+3i}{16-9(-1)} \quad [\because i^2 = -1]$$

$$= \frac{4 + 3i}{16 + 9}$$

$$= \frac{4 + 3i}{25}$$

$$= \frac{4}{25} + \frac{3}{25}i$$

Real part   Imaginary part

Q. 2. F. Simplify each of the following and express it in the form (a + ib) :

$$\left(-2 + \sqrt{-3}\right)^{-1}$$

Answer : Given: $(-2 + \sqrt{-3})^{-1}$

We can re- write the above equation as

$$= \frac{1}{-2+\sqrt{-3}}$$

$$= \frac{1}{-2+\sqrt{3i^2}}$$ [∵ $i^2 = -1$]

$$= \frac{1}{-2+i\sqrt{3}}$$

Now, rationalizing

$$= \frac{1}{-2+i\sqrt{3}} \times \frac{-2-i\sqrt{3}}{-2-i\sqrt{3}}$$

$$= \frac{-2-i\sqrt{3}}{(-2+i\sqrt{3})(-2-i\sqrt{3})}$$ …(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{-2-i\sqrt{3}}{(-2)^2-\left(i\sqrt{3}\right)^2}$$

$$= \frac{-2-i\sqrt{3}}{4-(3i^2)}$$

$$= \frac{-2-i\sqrt{3}}{4-3(-1)}$$ [∵ $i^2 = -1$]

$$= \frac{-2-i\sqrt{3}}{4+3}$$

$$= \frac{-2 - i\sqrt{3}}{7}$$

$$= -\frac{2 + i\sqrt{3}}{7}$$

$$= \underbrace{-\frac{2}{7}}_{\text{Real part}} \underbrace{-\frac{\sqrt{3}}{7}i}_{\text{Imaginary part}}$$

Q. 2. G. Simplify each of the following and express it in the form (a + ib) :

$(2 + i)^{-2}$

Answer : Given: $(2 + i)^{-2}$

Above equation can be re – written as

$$= \frac{1}{(2 + i)^2}$$

Now, rationalizing

$$= \frac{1}{(2 + i)^2} \times \frac{(2 - i)^2}{(2 - i)^2}$$

$$= \frac{(2 - i)^2}{(2 + i)^2 (2 - i)^2}$$

$$= \frac{4 + i^2 - 4i}{(4 + i^2 + 4i)(4 + i^2 - 4i)}$$ [$\because (a - b)^2 = a^2 + b^2 - 2ab$]

$$= \frac{4 - 1 - 4i}{(4 - 1 + 4i)(4 - 1 - 4i)}$$ [$\because i^2 = -1$]

$$= \frac{3 - 4i}{(3 + 4i)(3 - 4i)}$$ …(i)

Now, we know that,

(a + b)(a – b) = $(a^2 - b^2)$

So, eq. (i) become

$$= \frac{3 - 4i}{(3)^2 - (4i)^2}$$

$$= \frac{3 - 4i}{9 - 16i^2}$$

$$= \frac{3 - 4i}{9 - 16(-1)}$$

$$= \frac{3 - 4i}{25}$$

$$= \frac{3}{25} - \frac{4}{25}i$$

Real part Imaginary part

Q. 2. H. Simplify each of the following and express it in the form (a + ib) :

$(1 + 2i)^{-3}$

Answer : Given: $(1 + 2i)^{-3}$

Above equation can be re – written as

$$= \frac{1}{(1 + 2i)^3}$$

Now, rationalizing

$$= \frac{1}{(1 + 2i)^3} \times \frac{(1 - 2i)^3}{(1 - 2i)^3}$$

$$= \frac{(1 - 2i)^3}{(1 + 2i)^3(1 - 2i)^3}$$

We know that,

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$= \frac{(1)^3 - 3(1)^2(2i) + 3(1)(2i)^2 - (2i)^3}{[(1)^3 + 3(1)^2(2i) + 3(1)(2i)^2 + (2i)^3][(1)^3 - 3(1)^2(2i) + 3(1)(2i)^2 - (2i)^3]}$$

$$= \frac{1 - 6i + 6i^2 - 8i^3}{[1 + 6i + 6i^2 + 8i^3][1 - 6i + 6i^2 - 8i^3]}$$

$$= \frac{1-6i+6(-1)-8i(-1)}{[1+6i+6(-1)+8i(-1)][1-6i+6(-1)-8i(-1)]}$$ $[\because i^2 = -1]$

$$= \frac{1 - 6i - 6 + 8i}{[1 + 6i - 6 - 8i][1 - 6i - 6 + 8i]}$$

$$= \frac{-5 + 2i}{[-5 - 2i][-5 + 2i]}$$

$$= \frac{-5 + 2i}{-5(-5) - 5(2i) - 2i(-5) - 2i(2i)}$$

$$= \frac{-5 + 2i}{25 - 10i + 10i - 4i^2}$$

$$= \frac{-5+2i}{25-4(-1)}$$ $[\because i^2 = -1]$

$$= \frac{-5 + 2i}{29}$$

$$= \underbrace{-\frac{5}{29}}_{\text{Real part}} + \underbrace{\frac{2}{29}i}_{\text{Imaginary part}}$$

Q. 2. I. Simplify each of the following and express it in the form (a + ib) :

$(1 + i)^3 - (1 - i)^3$

Answer : Given: $(1 + i)^3 - (1 - i)^3$ …(i)

We know that,

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

By applying the formulas in eq. (i), we get

$(1)^3 + 3(1)^2(i) + 3(1)(i)^2 + (i)^3 - [(1)^3 - 3(1)^2(i) + 3(1)(i)^2 - (i)^3]$

$= 1 + 3i + 3i^2 + i^3 - [1 - 3i + 3i^2 - i^3]$

$= 1 + 3i + 3i^2 + i^3 - 1 + 3i - 3i^2 + i^3$

$= 6i + 2i^3$

$= 6i + 2i(i^2)$

$= 6i + 2i(-1)$ [$\because i^2 = -1$]

$= 6i - 2i$

$= 4i$

$= 0 + 4i$

Real part (0), Imaginary part (4)

Q. 3. A. Express each of the following in the form (a + ib):

$$\frac{1}{(4 + 3i)}$$

Answer : Given: $\frac{1}{4+3i}$

Now, rationalizing

$$= \frac{1}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{4-3i}{(4+3i)(4-3i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{4-3i}{(4)^2-(3i)^2}$$

$$= \frac{4-3i}{16-9i^2}$$

$$= \frac{4-3i}{16-9(-1)} \quad [\because i^2 = -1]$$

$$= \frac{4-3i}{16+9}$$

$$= \frac{4-3i}{25}$$

$$= \frac{4}{25} - \frac{3}{25}i$$

Real part: $\frac{4}{25}$, Imaginary part: $-\frac{3}{25}$

Q. 3. B. Express each of the following in the form (a + ib):

$$\frac{(3+4i)}{(4+5i)}$$

Answer : Given: $\frac{3+4i}{4+5i}$

Now, rationalizing

$$= \frac{3+4i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{(3+4i)(4-5i)}{(4+5i)(4-5i)} \quad ...(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(3+4i)(4-5i)}{(4)^2-(5i)^2}$$

$$= \frac{3(4)+3(-5i)+4i(4)+4i(-5i)}{16-25i^2}$$

$$= \frac{12-15i+16i-20i^2}{16-25(-1)} \quad [\because i^2 = -1]$$

$$= \frac{12+i-20(-1)}{16+25}$$

$$= \frac{12+i+20}{41}$$

$$= \frac{32+i}{41}$$

$$= \underbrace{\frac{32}{41}}_{\text{Real part}} + \underbrace{\frac{1}{41}i}_{\text{Imaginary part}}$$

Q. 3. C. Express each of the following in the form (a + ib):

$$\frac{\left(5+\sqrt{2}i\right)}{\left(1-\sqrt{2}i\right)}$$

Answer : Given: $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$

Now, rationalizing

$$=\frac{5+\sqrt{2}i}{1-\sqrt{2}i}\times\frac{1+\sqrt{2}i}{1+\sqrt{2}i}$$

$$=\frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1-\sqrt{2}i)(1+\sqrt{2}i)}\quad \text{...(i)}$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$=\frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1)^2-\left(\sqrt{2}i\right)^2}$$

$$=\frac{5(1)+5\left(\sqrt{2}i\right)+\sqrt{2}i(1)+\sqrt{2}i\left(\sqrt{2}i\right)}{1-2i^2}$$

$$=\frac{5+5\sqrt{2}i+\sqrt{2}i+2i^2}{1-2(-1)}\quad [\because i^2 = -1]$$

$$=\frac{5+6i\sqrt{2}+2(-1)}{1+2}$$

$$=\frac{3+6i\sqrt{2}}{3}$$

$$=\frac{3(1+2i\sqrt{2})}{3}$$

$= 1 + 2i\sqrt{2}$

Real part Imaginary part

Q. 3. D. Express each of the following in the form (a + ib):

$$\frac{(-2+5i)}{(3-5i)}$$

Answer : Given: $\frac{-2+5i}{3-5i}$

Now, rationalizing

$$= \frac{-2+5i}{3-5i} \times \frac{3+5i}{3+5i}$$

$$= \frac{(-2+5i)(3+5i)}{(3-5i)(3+5i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(-2+5i)(3+5i)}{(3)^2-(5i)^2}$$

$$= \frac{-2(3)+(-2)(5i)+5i(3)+5i(5i)}{9-25i^2}$$

$$= \frac{-6-10i+15i+25i^2}{9-25(-1)} \quad [\because i^2 = -1]$$

$$= \frac{-6+5i+25(-1)}{9+25}$$

$$= \frac{-31+5i}{34}$$

$$= -\frac{31}{34} + \frac{5}{34}i$$

Real part Imaginary part

Q. 3. E. Express each of the following in the form (a + ib):

$$\frac{(3-4i)}{(4-2i)(1+i)}$$

Answer : Given: $\frac{3-4i}{(4-2i)(1+i)}$

Solving the denominator, we get

$$\frac{3-4i}{(4-2i)(1+i)} = \frac{3-4i}{4(1)+4(i)-2i(1)-2i(i)}$$

$$= \frac{3-4i}{4+4i-2i-2i^2}$$

$$= \frac{3-4i}{4+2i-2(-1)}$$

$$= \frac{3-4i}{6+2i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 6 + 2i

$$= \frac{3-4i}{6+2i} \times \frac{6-2i}{6-2i}$$

$$= \frac{(3-4i)(6-2i)}{(6+2i)(6-2i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(3-4i)(6-2i)}{(6)^2-(2i)^2}$$

$$= \frac{3(6)+3(-2i)+(-4i)(6)+(-4i)(-2i)}{36-4i^2}$$

$= \frac{18-6i-24i+8i^2}{36-4(-1)}$ [∵ $i^2 = -1$]

$= \frac{18-30i+8(-1)}{36+4}$

$= \frac{18-30i-8}{40}$

$= \frac{10-30i}{40}$

$= \frac{10(1-3i)}{40}$

$= \frac{1-3i}{4}$

$= \frac{1}{4} - \frac{3}{4}i$

Real part Imaginary part

Q. 3. F. Express each of the following in the form (a + ib):

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

Answer : Given: $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Firstly, we solve the given equation

$$= \frac{3(2)+3(3i)-2i(2)+(-2i)(3i)}{(1)(2)+1(-i)+2i(2)+2i(-i)}$$

$$= \frac{6 + 9i - 4i - 6i^2}{2 - i + 4i - 2i^2}$$

$$= \frac{6 + 5i - 6(-1)}{2 + 3i - 2(-1)}$$

$$= \frac{6 + 6 + 5i}{2 + 3i + 2}$$

$$= \frac{12 + 5i}{4 + 3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 4 + 3i

$$= \frac{12 + 5i}{4 + 3i} \times \frac{4 - 3i}{4 - 3i}$$

$$= \frac{(12+5i)(4-3i)}{(4+3i)(4-3i)} \quad ...(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(12 + 5i)(4 - 3i)}{(4)^2 - (3i)^2}$$

$$= \frac{12(4) + 12(-3i) + 5i(4) + 5i(-3i)}{16 - 9i^2}$$

$$= \frac{48-36i+20i-15i^2}{19-9(-1)} \quad [\because i^2 = -1]$$

$$= \frac{48-16i-15(-1)}{16+9} \quad [\because i^2 = -1]$$

$$= \frac{48 - 16i + 15}{25}$$

$$= \frac{63 - 16i}{25}$$

$$= \frac{63}{25} - \frac{16}{25}i$$

Real part — Imaginary part

Q. 3. G. Express each of the following in the form (a + ib):

$$\frac{(2+3i)^2}{(2-i)}$$

Answer : Given: $\frac{(2+3i)^2}{(2-i)}$

Now, we rationalize the above equation by multiply and divide by the conjugate of (2 – i)

$$= \frac{(2+3i)^2}{(2-i)} \times \frac{(2+i)}{(2+i)}$$

$$= \frac{(2+3i)^2(2+i)}{(2-i)(2+i)}$$

$$= \frac{(4+9i^2+12i)(2+i)}{(2)^2-(i)^2}$$

[∵(a + b)(a – b) = $(a^2 - b^2)$]

$$= \frac{[4+9(-1)+12i](2+i)}{4-i^2}$$ [∵i^2 = -1]

$$= \frac{[4-9+12i](2+i)}{4-(-1)}$$

$$= \frac{(-5+12i)(2+i)}{5}$$

$$= \frac{-10 - 5i + 24i + 12i^2}{5}$$

$$= \frac{-10 + 19i + 12(-1)}{5}$$

$$= \frac{-10 - 12 + 19i}{5}$$

$$= \frac{-22 + 19i}{5}$$

$$= -\frac{22}{5} + \frac{19}{5}i$$

Real part Imaginary part

Q. 3. H. Express each of the following in the form (a + ib):

$$\frac{(1-i)^3}{(1-i^3)}$$

Answer : Given: $\frac{(1-i)^3}{(1-i^3)}$

The above equation can be re-written as

$$= \frac{(1)^3 - (i)^3 - 3(1)^2(i) + 3(1)(i)^2}{(1 - i \times i^2)}$$

[∵(a – b)^3 = a^3 – b^3 – 3a^{2}b + 3ab^2]

$= \frac{1-i^3-3i+3i^2}{[1-i(-1)]}$ [∵i^2 = -1]

$$= \frac{1 - i \times i^2 - 3i + 3(-1)}{(1 + i)}$$

$$= \frac{1 - i(-1) - 3i - 3}{1 + i}$$

$$= \frac{-2 + i - 3i}{1 + i}$$

$$= \frac{-2 - 2i}{1 + i}$$

$$= \frac{-2(1 + i)}{1 + i}$$

Real part Imaginary part

= -2 + 0i

Q. 3. I. Express each of the following in the form (a + ib):

$$\frac{(1+2i)^3}{(1+i)(2-i)}$$

Answer : Given: $\frac{(1+2i)^3}{(1+i)(2-i)}$

We solve the above equation by using the formula

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$$= \frac{(1)^3 + (2i)^3 + 3(1)^2(2i) + 3(1)(2i)^2}{1(2) + 1(-i) + i(2) + i(-i)}$$

$$= \frac{1 + 8i^3 + 6i + 12i^2}{2 - i + 2i - i^2}$$

$$= \frac{1+8i\times i^2+6i+12(-1)}{2+i-(-1)}$$ [∵ $i^2 = -1$]

$$= \frac{1 + 8i(-1) + 6i - 12}{2 + i + 1}$$

$$= \frac{1 - 8i + 6i - 12}{3 + i}$$

$$= \frac{-11 - 2i}{3 + i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 + i

$$= \frac{-11 - 2i}{3 + i} \times \frac{3 - i}{3 - i}$$

$$= \frac{(-11-2i)(3-i)}{(3+i)(3-i)} \quad ...(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(-11 - 2i)(3 - i)}{(3)^2 - (i)^2}$$

$$= \frac{-11(3) + (-11)(-i) + (-2i)(3) + (-2i)(-i)}{9 - i^2}$$

$$= \frac{-33+11i-6i+2i^2}{9-(-1)} \quad [\because i^2 = -1]$$

$$= \frac{-33+5i+2(-1)}{9+1} \quad [\because i^2 = -1]$$

$$= \frac{-33 + 5i - 2}{10}$$

$$= \frac{-35 + 5i}{10}$$

$$= \frac{5(-7 + i)}{10}$$

$$= \frac{-7 + i}{2}$$

Real part   Imaginary part $= \frac{-7}{2} + \frac{1}{2}i$

Q. 4. Simplify each of the following and express it in the form (a + ib):

(i) $\left(\frac{5}{-3+2i} + \frac{2}{1-i}\right)\left(\frac{4-5i}{3+2i}\right)$

(ii) $\left(\frac{1}{1+4i} - \frac{2}{1+i}\right)\left(\frac{1-i}{5+3i}\right)$

Answer : Given:

$$\left(\frac{5}{-3+2i} + \frac{2}{1-i}\right)\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{5(1-i)+2(-3+2i)}{(-3+2i)(1-i)}\right]\left(\frac{4-5i}{3+2i}\right)$$ [Taking the LCM]

$$= \left[\frac{5-5i-6+4i}{(-3)(1-i)+2i(1-i)}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{-1-i}{-3+3i+2i-2i^2}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{-(1+i)}{-3+5i-2(-1)}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left(\frac{-(1+i)}{-1+5i}\right)\left(\frac{4-5i}{3+2i}\right)$$

$$= \frac{-1(4-5i)-i(4-5i)}{-1(3+2i)+5i(3+2i)}$$

$$= \frac{-4+5i-4i+5i^2}{-3-2i+15i+10i^2}$$

$= \frac{-4+i+5(-1)}{-3+13i+10(-1)}$ [Putting $i^2 = -1$]

$= \frac{-9+i}{-13+13i}$

$= \frac{-(9-i)}{-(13-13i)}$

$= \frac{9-i}{13-13i}$

Now, rationalizing by multiply and divide by the conjugate of (13 – 13i)

$= \frac{9-i}{13-13i} \times \frac{13+13i}{13+13i}$

$= \frac{(9-i)(13+13i)}{(13-13i)(13+13i)}$

$= \frac{117+117i-13i-13i^2}{(13)^2-(13i)^2}$ [$\because (a-b)(a+b) = (a^2-b^2)$]

$= \frac{117+104i-13(-1)}{169-169i^2}$ [$\because i^2 = -1$]

$= \frac{130+104i}{169(1-i^2)}$

$= \frac{13(10+8i)}{169[1-(-1)]}$ [Taking 13 common]

$= \frac{10+8i}{13 \times 2}$

$= \frac{5+4i}{13}$

$= \frac{5}{13} + \frac{4}{13}i$

(ii) Given:

$$\left(\frac{1}{1+4i}-\frac{2}{1+i}\right)\left(\frac{1-i}{5+3i}\right)$$

$$=\left[\frac{1(1+i)-2(1+4i)}{(1+4i)(1+i)}\right]\left(\frac{1-i}{5+3i}\right)$$ [Taking the LCM]

$$=\left[\frac{1+i-2-8i}{(1)(1+i)+4i(1+i)}\right]\left(\frac{1-i}{5+3i}\right)$$

$$=\left[\frac{-1-7i}{1+i+4i+4i^2}\right]\left(\frac{1-i}{5+3i}\right)$$

$$=\left[\frac{-1-7i}{1+5i+4(-1)}\right]\left(\frac{1-i}{5+3i}\right)$$

$$=\left(\frac{-1-7i}{-3+5i}\right)\left(\frac{1-i}{5+3i}\right)$$

$$=\frac{-1(1-i)-7i(1-i)}{-3(5+3i)+5i(5+3i)}$$

$$=\frac{-1+i-7i+7i^2}{-15-9i+25i+15i^2}$$

$$=\frac{-1-6i+7(-1)}{-15+16i+15(-1)}$$

$$=\frac{-6i-8}{16i-30}$$

$$=\frac{-2(4+3i)}{-2(15-8i)}$$

$$=\frac{4+3i}{15-8i}$$

Now, rationalizing by multiply and divide by the conjugate of (15 + 8i)