## Exercise 4(D)

### Solution 1:

Given that $x^3 + 4y^3 + 9z^3 = 18xyz$ and $x + 2y + 3z = 0$

\ $x + 2y = -3z$, $2y + 3z = -x$ and $3z + x = -2y$

Now

$$\frac{(x+2y)^2}{xy} + \frac{(2y+3z)^2}{yz} + \frac{(3z+x)^2}{zx} = \frac{(-3z)^2}{xy} + \frac{(-x)^2}{yz} + \frac{(-2y)^2}{zx}$$

$$= \frac{9z^2}{xy} + \frac{x^2}{yz} + \frac{4y^2}{zx}$$

$$= \frac{x^3 + 4y^3 + 9z^3}{xyz}$$

Given that $x^3 + 4y^3 + 9z^3 = 18xyz$

$$\therefore \frac{(x+2y)^2}{xy} + \frac{(2y+3z)^2}{yz} + \frac{(3z+x)^2}{zx} = \frac{18xyz}{xyz} = 18$$

$$\Rightarrow a^2 + \frac{1}{a^2} = m^2 - 2$$

Now consider the expansion of $\left(a - \frac{1}{a}\right)^2$ :

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = m^2 - 2 - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = m^2 - 4$$

$$\Rightarrow \left(a - \frac{1}{a}\right) = \pm\sqrt{m^2 - 4} \ldots(1)$$



(ii)

$(2x^2 - 8)(x - 4)^2$ $= (a+b)(a-b)]$

$= (2x^2 - 8)(x^2 - 8x + 16)$

$= 4x^4 - 16x^3 + 32x^2 - 8x^2 + 64x - 128$

$= 4x^4 - 16x^3 + 24x^2 + 64x - 128$

Hence,

coefficient of $x^3 = -16$

coefficient of $x^2 = 24$

constant term $= -128$

**Solution 4:**

Given that

$$x^2 + \frac{1}{9x^2} = \frac{25}{36}$$

$$\Rightarrow x^2 + \frac{1}{(3x)^2} = \frac{25}{36} \ldots.(1)$$

Now consider the expansion of $\left(x + \frac{1}{3x}\right)^2$:

$$\left(x + \frac{1}{3x}\right)^2 = x^2 + \frac{1}{(3x)^2} + 2 \times x \times \frac{1}{3x}$$

$$\Rightarrow \quad = x^2 + \frac{1}{(3x)^2} + \frac{2}{3}$$

$$\Rightarrow \quad = \frac{25}{36} + \frac{2}{3} \quad \text{[from (1)]}$$

$$\Rightarrow \quad = \frac{25 + 24}{36}$$

$$\Rightarrow \quad = \frac{49}{36}$$

$$\Rightarrow x + \frac{1}{3x} = \pm\sqrt{\frac{49}{36}}$$

$$\Rightarrow x + \frac{1}{3x} = \pm\frac{7}{6} \ldots.(2)$$



**Solution 5:**

(i)

$2(x^2 + 1\} = 5x$

$$\left(x^2 + 1\right) = \frac{5}{2}x$$

Dividing by x, we have

$$\frac{\left(x^2 + 1\right)}{x} = \frac{5}{2}$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = \frac{5}{2} \ldots.(1)$$

Now consider the expansion of $\left(x+\frac{1}{x}\right)^2$ :

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow \left(\frac{5}{2}\right)^2 = x^2+\frac{1}{x^2}+2 \text{ [from (1)]}$$

$$\Rightarrow \left(\frac{5}{2}\right)^2 - 2 = x^2+\frac{1}{x^2}$$

$$\Rightarrow \frac{25}{4} - 2 = x^2+\frac{1}{x^2}$$

$$\Rightarrow x^2+\frac{1}{x^2} = \frac{25-8}{4}$$

$$\Rightarrow x^2+\frac{1}{x^2} = \frac{17}{4} \ldots.(2)$$

Now consider the expansion of $\left(x-\frac{1}{x}\right)^2$ :

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = \frac{17}{4}-2 \text{ [from (2)]}$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = \frac{17-8}{4}$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \left(x-\frac{1}{x}\right) = \pm\frac{3}{2} \ldots..(3)$$

(ii)

We know that,

$$\left(x^3-\frac{1}{x^3}\right) = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$$

$$\therefore \left(x^3-\frac{1}{x^3}\right) = \left(\pm\frac{3}{2}\right)^3 + 3\left(\pm\frac{3}{2}\right) \text{ [from (3)]}$$

$$= \pm\frac{27}{8}+\frac{9}{2}$$

$$\Rightarrow \left(x^3-\frac{1}{x^3}\right) = \pm\frac{27+36}{8}$$

$$\Rightarrow \left(x^3-\frac{1}{x^3}\right) = \pm\frac{63}{8}$$

**Solution 6:**

$a^2 + b^2 = 34$, $ab = 12$

$(a + b)^2 = a^2 + b^2 + 2ab$

$= 34 + 2 \times 12 = 34 + 24 = 58$

$(a - b)^2 = a^2 + b^2 - 2ab$

$= 34 - 2 \times 12 = 34 - 24 = 10$

(i) $3(a + b)^2 + 5(a - b)^2$

$= 3 \times 58 + 5 \times 10 = 174 + 50$

$= 224$

(ii) $7(a - b)^2 - 2(a + b)^2$

$= 7 \times 10 - 2 \times 58 = 70 - 116 = -46$

**Solution 7:**

Given $3x - \frac{4}{x} = 4$;

We need to find $27x^3 - \frac{64}{x^3}$

Let us now consider the expansion of $\left(3x - \frac{4}{x}\right)^3$ :

$$\left(3x - \frac{4}{x}\right)^3 = 27x^3 - \frac{64}{x^3} - 3 \times 3x \times \frac{4}{x}\left(3x - \frac{4}{x}\right)$$

$$\Rightarrow (4)^3 = 27x^3 - \frac{64}{x^3} - 144 \quad \left[\text{Given}: 3x - \frac{4}{x} = 4\right]$$

$$\Rightarrow 64 + 144 = 27x^3 - \frac{64}{x^3}$$

$$\Rightarrow 27x^3 - \frac{64}{x^3} = 208$$

**Solution 8:**

Given that $x^2 + \frac{1}{x^2} = 7$

We need to find the value of $7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x}$

Consider the given equation:

$$x^2 + \frac{1}{x^2} - 2 = 7 - 2 \text{ [subtract 2 from both the sides]}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 5$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = \pm\sqrt{5} \ldots\ldots(1)$$

$$\therefore 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} = 7x^3 - \frac{7}{x^3} + 8x - \frac{8}{x}$$

$$= 7\left(x^3 - \frac{1}{x^3}\right) + 8\left(x - \frac{1}{x}\right) \ldots.(2)$$

Now consider the expansion of $\left(x - \frac{1}{x}\right)^3$:

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$\Rightarrow x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$\Rightarrow x^3 - \frac{1}{x^3} = \left(\sqrt{5}\right)^3 + 3\left(\sqrt{5}\right) \ldots.(3)$$

Now substitute the value of $x^3 - \frac{1}{x^3}$ in equation (2), we have

$$7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} = 7\left(x^3 - \frac{1}{x^3}\right) + 8\left(x - \frac{1}{x}\right)$$

$$\Rightarrow 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} = 7\left[\left(\sqrt{5}\right)^3 + 3\left(\sqrt{5}\right)\right] + 8\left[\sqrt{5}\right] \text{ [from (3)]}$$

$$\Rightarrow 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} = 7\left[5\left(\sqrt{5}\right) + 3\left(\sqrt{5}\right)\right] + 8\left[\sqrt{5}\right]$$

$$\Rightarrow 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} = 64\sqrt{5}$$

## Solution 9:

Given $x = \frac{1}{x-5}$;

By cross multiplication,

=> x (x - 5) = 1 => $x^2$ - 5x = 1 => $x^2$ - 1 = 5x
Dividing both sides by x,

$$\frac{x^2-1}{x} = 5$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 5....(1)$$

Now consider the expansion of $\left(x - \frac{1}{x}\right)^2$:

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow (5)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 25 + 2 = 27....(2)$$



Let us consider the expansion of $\left(x + \frac{1}{x}\right)^2$:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 27 + 2 \quad [\text{from } (2)]$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 29$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = \pm\sqrt{29}.....(3)$$

We know that

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= \left(\pm\sqrt{29}\right)(5) \quad [\text{from equations } (1) \text{ and } (3)]$$

$$\Rightarrow x^2 - \frac{1}{x^2} = \pm5\sqrt{29}$$

**Solution 10:**

Given $x = \frac{1}{5 - x}$;

By cross multiplication,

=> x (5 - x) = 1 => $x^2$ - 5x =-1 => $x^2$ + 1 = 5x
Dividing both sides by x,

$$\frac{x^2 + 1}{x} = 5$$
$$\Rightarrow \left(x + \frac{1}{x}\right) = 5 \ldots (1)$$

We know that

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$
$$= (5)^3 - 3(5) \quad \text{[from equation (1)]}$$
$$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 15 = 110$$



**Solution 11:**

Given that 3a + 5b + 4c = 0

3a + 5b = -4c

Cubing both sides,

$(3a + 5b)^3 = (-4c)^3$

$=> (3a)^3 + (5b)^3 + 3 \times 3a \times 5b\ (3a + 5b) = -64c^3$

$=> 27a^3 + 125b^3 + 45ab \times (-4c) = -64c^3$

$=> 27a^3 + 125b^3 - 180abc = -64c^3$

$=> 27a^3 + 125b^3 + 64c^3 = 180abc$

Hence proved.

**Solution 12:**

Let a, b be the two numbers

$\therefore a + b = 7$ and $a^3 + b^3 = 133$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$\Rightarrow (7)^3 = 133 + 3ab(7)$

$\Rightarrow 343 = 133 + 21ab \Rightarrow 21ab = 343 - 133 = 210$

$\Rightarrow 21ab = 210 \Rightarrow ab = 2l$

Now $a^2 + b^2 = (a + b)^2 - 2ab$

$= 7^2 - 2 \times 10 = 49 - 20 = 29$

**Solution 13:**

(i) $4x^2 + ax + 9 = (2x + 3)^2$

Comparing coefficients of x terms, we get

$ax = 12x$

so, $a = 12$

(ii) $4x^2 + ax + 9 = (2x - 3)^2$

Comparing coefficients of x terms, we get

$ax = -12x$

so, $a = -12$

(iii) $9x^2 + (7a - 5)x + 25 = (3x + 5)^2$

Comparing coefficients of x terms, we get

$(7a - 5)x = 30x$

$7a - 5 = 30$

$7a = 35$

$a = 5$

**Solution 14:**

Given

$$\frac{x^2+1}{x}=\frac{10}{3}$$

$$x+\frac{1}{x}=\frac{10}{3}$$

Squaring on both sides. we get

$$x^2+\frac{1}{x^2}+2=\frac{100}{9}$$

$$x^2+\frac{1}{x^2}=\frac{100-18}{9}=\frac{82}{9}$$

$$x-\frac{1}{x}=\sqrt{\left(x+\frac{1}{x}\right)^2-4}=\sqrt{\frac{100}{9}-4}=\sqrt{\frac{64}{9}}=\frac{8}{3}$$

$$\therefore\ x-\frac{1}{x}=\frac{8}{3}$$

Cubing both sides, we get

$$\left(x-\frac{1}{x}\right)^3=\frac{512}{27}$$

$$x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=\frac{512}{27}$$

$$x^3-\frac{1}{x^3}=\frac{512}{27}+8=\frac{512+216}{27}=\frac{728}{27}$$

**Solution 1:**

Given difference between two positive numbers is 4 and difference between their cubes is 316.

Let the positive numbers be a and b

$a - b = 4$

$a^3 - b^3 = 316$

Cubing both sides,

$(a - b)^3 = 64$

$a^3 - b^3 - 3ab(a - b) = 64$

Given $a^3 - b^3 = 316$

So $316 - 64 = 3ab(4)$

$252 = 12ab$

So $ab = 21$; product of numbers is 21

Squaring both sides, we get

$(a - b)^2 = 16$

$a^2 + b^2 - 2ab = 16$

$a^2 + b^2 = 16 + 42 = 58$

Sum of their squares is 58.