

### EXERCISE 6.3

1. Find the area of the triangle with vertices at the points:

- (i) (3, 8), (-4, 2) and (5, -1)
- (ii) (2, 7), (1, 1) and (10, 8)
- (iii) (-1, -8), (-2, -3) and (3, 2)
- (iv) (0, 0), (6, 0) and (4, 3)

**Solution:**

(i) Given (3, 8), (-4, 2) and (5, -1) are the vertices of the triangle.

We know that, if vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$= \frac{1}{2} \left[ 3 \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8 \begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix} \right]$$

$$= \frac{1}{2} [3(3) - 8(-9) + 1(-6)]$$

$$= \frac{1}{2} [9 + 72 - 6]$$

$$= \frac{75}{2} \text{ Square units}$$

Thus area of triangle is  $\frac{75}{2}$  square units

(ii) Given (2, 7), (1, 1) and (10, 8) are the vertices of the triangle.

We know that if vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the

triangle is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$= \frac{1}{2} [2 \begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7 \begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix}]$$

$$= \frac{1}{2} [2(-7) - 7(-9) + 1(-2)]$$

$$= \frac{1}{2} [-14 + 63 - 2]$$

$$= \frac{47}{2} \text{ Square units}$$

Thus area of triangle is  $\frac{47}{2}$  square units

(iii) Given  $(-1, -8)$ ,  $(-2, -3)$  and  $(3, 2)$  are the vertices of the triangle.

We know that if vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$\begin{aligned}
 &= \frac{1}{2} \left[ -1 \begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} - 8 \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix} \right] \\
 &= \frac{1}{2} [-1(-5) - 8(-5) + 1(5)] \\
 &= \frac{1}{2} [5 - 40 + 5] \\
 &= \frac{-30}{2} \text{ Square units}
 \end{aligned}$$

As we know area cannot be negative. Therefore, 15 square unit is the area  
Thus area of triangle is 15 square units

(iv) Given  $(-1, -8)$ ,  $(-2, -3)$  and  $(3, 2)$  are the vertices of the triangle.

We know that if vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$\begin{aligned}
 &= \frac{1}{2} \left[ 0 \begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} - 0 \begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} + 1 \begin{vmatrix} 6 & 0 \\ 4 & 3 \end{vmatrix} \right] \\
 &= \frac{1}{2} [0 - 0 + 1(18)] \\
 &= \frac{1}{2} [18]
 \end{aligned}$$

= 9 square units

Thus area of triangle is 9 square units

**2. Using the determinants show that the following points are collinear:**

(i)  $(5, 5)$ ,  $(-5, 1)$  and  $(10, 7)$

- (ii) (1, -1), (2, 1) and (10, 8)  
 (iii) (3, -2), (8, 8) and (5, 2)  
 (iv) (2, 3), (-1, -2) and (5, 8)

**Solution:**

(i) Given (5, 5), (-5, 1) and (10, 7)

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2} \begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix}$$

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Expanding along  $R_1$

$$= \frac{1}{2} \left[ 5 \begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} - 5 \begin{vmatrix} -5 & 1 \\ 10 & 1 \end{vmatrix} + 1 \begin{vmatrix} -5 & 1 \\ 10 & 7 \end{vmatrix} \right]$$

$$= \frac{1}{2} [5(-6) - 5(-15) + 1(-45)]$$

$$= \frac{1}{2} [-35 + 75 - 45]$$

$$= 0$$

Since, Area of triangle is zero

Hence, points are collinear

(ii) Given (1, -1), (2, 1) and (10, 8)

We have the condition that three points to be collinear, the area of the triangle formed

by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2} \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$= \frac{1}{2} [1 \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix}]$$

$$= \frac{1}{2} [1 - 5 + 2 - 4 + 10 - 4]$$

$$= \frac{1}{2} [0]$$

$$= 0$$

Since, Area of triangle is zero.

Hence, points are collinear.

(iii) Given  $(3, -2)$ ,  $(8, 8)$  and  $(5, 2)$

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2} \begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$= \frac{1}{2} \left[ 3 \begin{vmatrix} 8 & 1 \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 8 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 8 & 8 \\ 5 & 2 \end{vmatrix} \right]$$

$$= \frac{1}{2} [3(6) + 2(3) + 1(-24)]$$

$$= \frac{1}{2} [0]$$

$$= 0$$

Since, Area of triangle is zero

Hence, points are collinear.

(iv) Given (2, 3), (-1, -2) and (5, 8)

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2} \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$\begin{aligned}
 &= \frac{1}{2} \left[ 2 \begin{vmatrix} -2 & 1 \\ 8 & 1 \end{vmatrix} - 3 \begin{vmatrix} -1 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} -1 & -2 \\ 5 & 8 \end{vmatrix} \right] \\
 &= \frac{1}{2} [2(-10) - 3(-1 - 5) + 1(-8 + 10)] \\
 &= \frac{1}{2} [-20 + 18 + 2] \\
 &= 0
 \end{aligned}$$

Since, Area of triangle is zero

Hence, points are collinear.

**3. If the points (a, 0), (0, b) and (1, 1) are collinear, prove that a + b = ab**

**Solution:**

Given (a, 0), (0, b) and (1, 1) are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus

$$\frac{1}{2} \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\Rightarrow 0 = \frac{1}{2} \left[ a \begin{vmatrix} b & 1 \\ 1 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & b \\ 1 & 1 \end{vmatrix} \right]$$

$$\Rightarrow \frac{1}{2} [a(b - 1) - 0(-1) + 1(-b)] = 0$$

$$\Rightarrow \frac{1}{2} [ab - a - b] = 0$$

$$\Rightarrow a + b = ab$$

Hence Proved

**4. Using the determinants prove that the points (a, b), (a', b') and (a - a', b - b) are**

collinear if  $a b' = a' b$ .

**Solution:**

Given  $(a, b)$ ,  $(a', b')$  and  $(a - a', b - b)$  are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus

$$\frac{1}{2} \begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a - a' & b - b' & 1 \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\begin{aligned} \Rightarrow 0 &= \frac{1}{2} \left[ a \begin{vmatrix} b' & 1 \\ b - b' & 1 \end{vmatrix} - b \begin{vmatrix} a' & 1 \\ a - a' & 1 \end{vmatrix} + 1 \begin{vmatrix} a' & b' \\ a - a' & b - b' \end{vmatrix} \right] \\ \Rightarrow \frac{1}{2} [a(b' - b + b') - b(a' - a + a') + 1(a'b - a'b' - ab' + a'b')] &= 0 \\ \Rightarrow \frac{1}{2} [a'b - ab + ab' - a'b + ab + a'b + a'b - a'b' - ab' + a'b'] &= 0 \end{aligned}$$

$$\Rightarrow ab' - a'b = 0$$

$$\Rightarrow a b' = a' b$$

Hence, the proof.

**5. Find the value of  $\lambda$  so that the points  $(1, -5)$ ,  $(-4, 5)$  and  $(\lambda, 7)$  are collinear.**

**Solution:**

Given  $(1, -5)$ ,  $(-4, 5)$  and  $(\lambda, 7)$  are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\Rightarrow \frac{1}{2} \left[ 1 \begin{vmatrix} 5 & 1 \\ 7 & 1 \end{vmatrix} + 5 \begin{vmatrix} -4 & 1 \\ \lambda & 1 \end{vmatrix} + 1 \begin{vmatrix} -4 & 5 \\ \lambda & 7 \end{vmatrix} \right] = 0$$

$$\Rightarrow \frac{1}{2} [1(-2) + 5(-4 - \lambda) + 1(-28 - 5\lambda)] = 0$$

$$\Rightarrow \frac{1}{2} [-2 - 20 - 5\lambda - 28 - 5\lambda] = 0$$

$$\Rightarrow -50 - 10\lambda = 0$$

$$\Rightarrow \lambda = -5$$

6. Find the value of  $x$  if the area of  $\Delta$  is 35 square cms with vertices  $(x, 4)$ ,  $(2, -6)$  and  $(5, 4)$ .

**Solution:**

Given  $(x, 4)$ ,  $(2, -6)$  and  $(5, 4)$  are the vertices of a triangle.

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , then the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, by substituting given value in above formula

$$\Rightarrow 35 = \left| \frac{1}{2} \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix} \right|$$

Removing modulus

$$\Rightarrow \pm 2 \times 35 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow \left[ x \begin{vmatrix} -6 & 1 \\ 4 & 1 \end{vmatrix} - 4 \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -6 \\ 5 & 4 \end{vmatrix} \right] = \pm 70$$

$$\Rightarrow [x(-10) - 4(-3) + 1(8 - 30)] = \pm 70$$

$$\Rightarrow [-10x + 12 + 38] = \pm 70$$

$$\Rightarrow \pm 70 = -10x + 50$$

Taking positive sign, we get

$$\Rightarrow +70 = -10x + 50$$

$$\Rightarrow 10x = -20$$

$$\Rightarrow x = -2$$

Taking -negative sign, we get

$$\Rightarrow -70 = -10x + 50$$

$$\Rightarrow 10x = 120$$

$$\Rightarrow x = 12$$

Thus  $x = -2, 12$

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