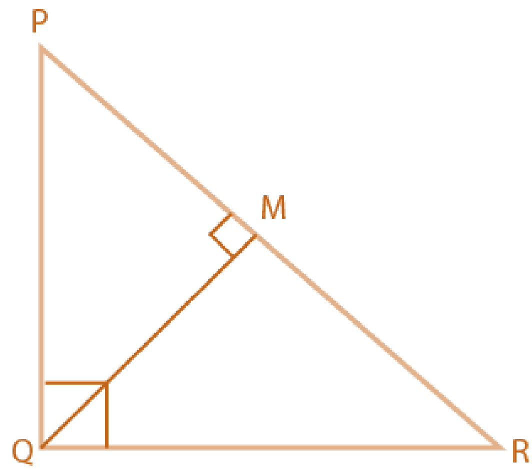
EXERCISE 6.3

1. In a ΔPQR, $PR^2 – PQ^2 = QR^2$ and M is a point on side PR such that QM ⊥ PR. Prove that $QM^2 = PM \times MR$.

Solution:

According to the question,



In ΔPQR,

$PR^2 = QR^2$ and QM⊥PR

Using Pythagoras theorem, we have,

$PR^2 = PQ^2 + QR^2$

ΔPQR is right angled triangle at Q.

From ΔQMR and ΔPMQ, we have,

∠M = ∠M

∠MQR = ∠QPM [= 90°-∠R]

So, using the AAA similarity criteria,

We have,

ΔQMR ~ ΔPMQ

Also, we know that,

Area of triangles = ½ × base × height

So, by property of area of similar triangles,

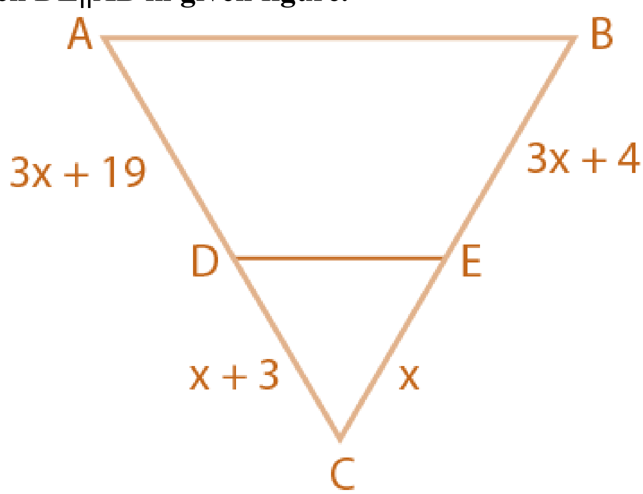
$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{\frac{1}{2} \times RM \times QM}{\frac{1}{2} \times PM \times QM}$$

$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$QM^2 = PM \times RM$
Hence proved.

2. Find the value of x for which DE||AB in given figure.

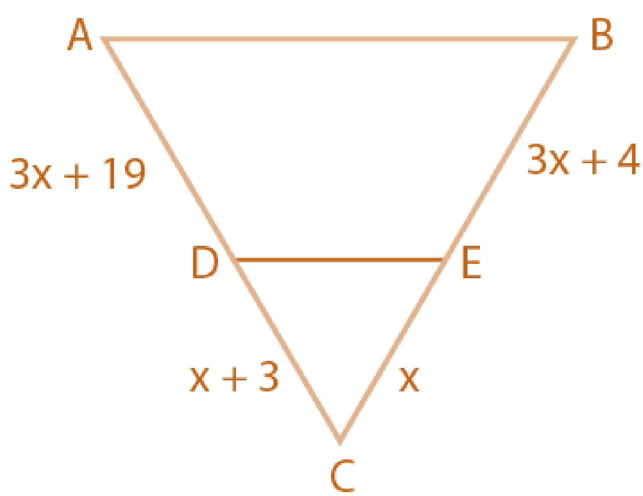


Solution:

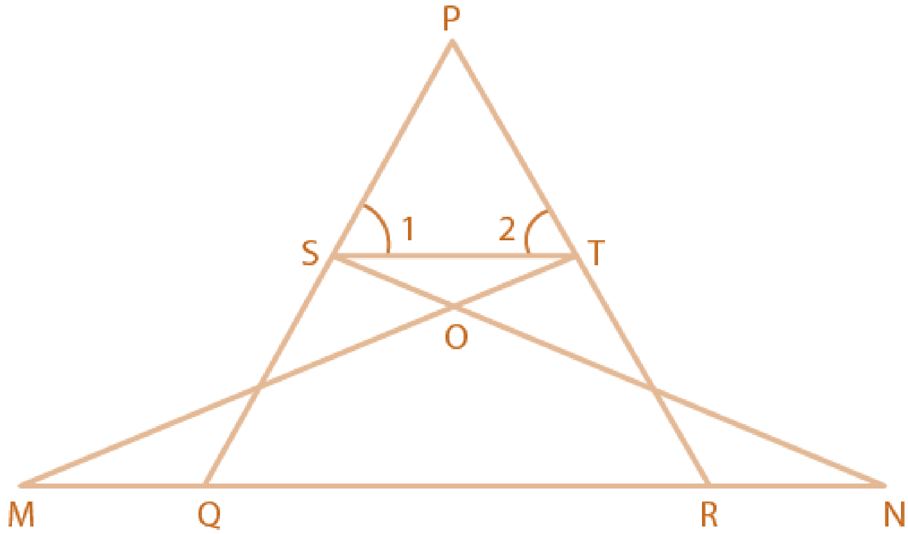
According to the question,
DE || AB
Using basic proportionality theorem,
CD/AD = CE/BE
∴ If a line is drawn parallel to one side of a triangle such that it intersects the other sides at distinct points, then, the other two sides are divided in the same ratio.
Hence, we can conclude that, the line drawn is equal to the third side of the triangle.

$$\frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$(x + 3)(3x + 4) = x(3x + 19)$
$3x^2 + 4x + 9x + 12 = 3x^2 + 19x$
$19x - 13x = 12$
$6x = 12$
$\therefore x = 12/6 = 2$



3. In figure, if ∠1 =∠2 and ΔNSQ = ΔMTR, then prove that ΔPTS ~ ΔPRQ.

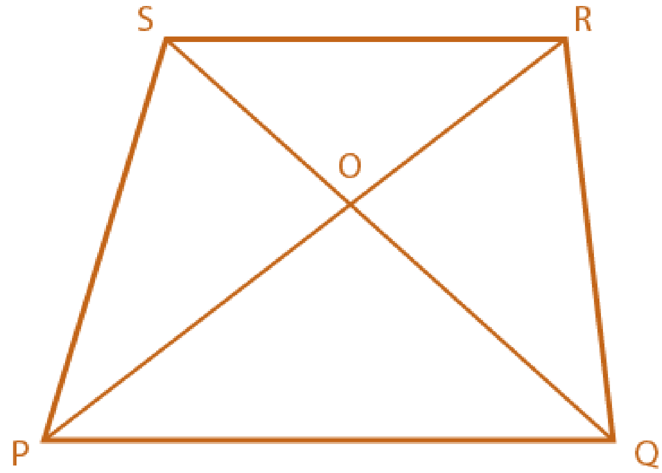


Solution:

According to the question,
Δ NSQ ≅ ΔMTR
∠1 = ∠2
Since,
ΔNSQ = ΔMTR
So,
SQ = TR ….(i)
Also,
∠1 = ∠2 ⇒ PT = PS….(ii)
[Since, sides opposite to equal angles are also equal]
From Equation (i) and (ii).
PS/SQ = PT/TR
⇒ ST || QR
By converse of basic proportionality theorem, If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.
∴ ∠1 = PQR
And
∠2 = ∠PRQ
In ΔPTS and ΔPRQ.
∠P = ∠P [Common angles]
∠1 = ∠PQR (proved)
∠2 = ∠PRQ (proved)
∴ ΔPTS - ΔPRQ
[By AAA similarity criteria]
Hence proved.

4. Diagonals of a trapezium PQRS intersect each other at the point 0, PQ || RS and PQ = 3 RS. Find the ratio of the areas of Δ POQ and Δ ROS.
Solution:

According to the question,
PQRS is a trapezium in which PQ∥RS and PQ = 3RS
PQ/RS = 3/1 = 3 …(i)



In ΔPOQ and ΔROS,
∠SOR = ∠QOP [vertically opposite angles]
∠SRP = ∠RPQ [alternate angles]
∴ ΔPOQ ~ ΔROS [by AAA similarity criterion]
By property of area of similar triangle,

$$\frac{Ar(\Delta POQ)}{ar(SOR)} = \frac{(PQ)^2}{(RS)^2} = \left(\frac{PQ}{RS}\right)^2 = \left(\frac{3}{1}\right)^2$$

$$\Rightarrow \frac{ar(\Delta POQ)}{ar(SOR)} = \frac{9}{1}$$

Therefore, the required ratio = 9:1.

5. In figure, if AB || DC and AC, PQ intersect each other at the point O. Prove that OA.CQ = 0C.AP.

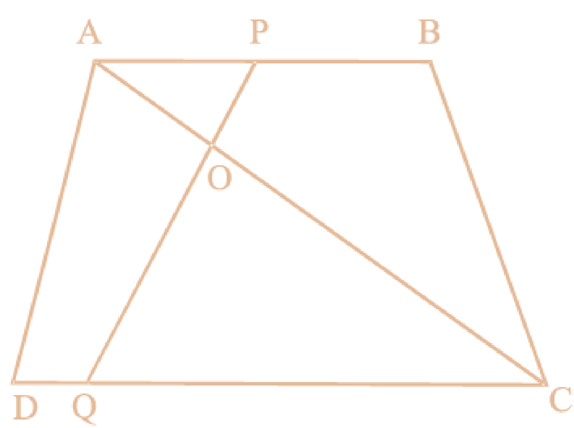


Fig. 6.10

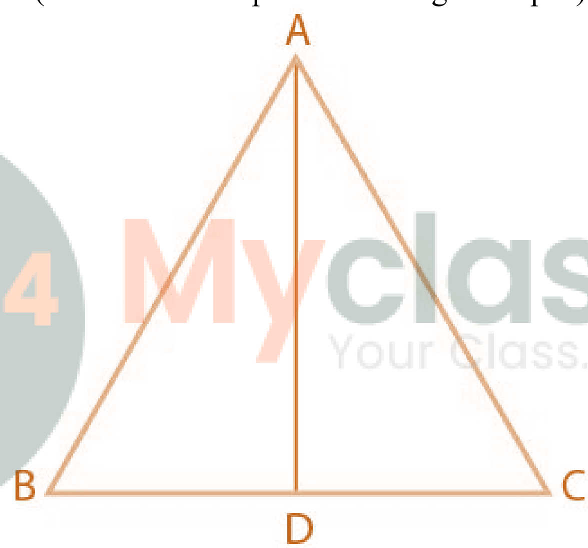
Solution:

According to the question,

AC and PQ intersect each other at the point O and AB||DC.
From ΔAOP and ΔCOQ,
∠AOP = ∠COQ [Since they are vertically opposite angles]
∠APO = ∠CQO [since, AB||DC and PQ is transversal, the angles are alternate angles]
∴ ΔAOP ~ ΔCOQ [using AAA similarity criterion]
Then, since, corresponding sides are proportional
We have,
OA/OC = AP/CQ
OA × CQ = OC × AP
Hence Proved.

6. Find the altitude of an equilateral triangle of side 8 cm.
Solution:

Let ABC be an equilateral triangle of side 8 cm
AB = BC = CA = 8 cm. (all sides of an equilateral triangle is equal)



Draw altitude AD which is perpendicular to BC.
Then, D is the mid-point of BC.
∴ BD = CD = ½
BC = 8/2 = 4 cm
Now, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$
⇒ $(8)^2 = AD^2 + (4)^2$
⇒ $64 = AD^2 + 16$
⇒ AD = √48 = 4√3 cm.
Hence, altitude of an equilateral triangle is 4√3 cm.

7. If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6, EF = 9 cm and FD = 12 cm, then find the perimeter of ΔABC.
Solution:

According to the question,
AB = 4 cm,

DE = 6 cm
EF = 9 cm
FD = 12 cm
Also,
ΔABC ~ ΔDEF
We have,

$$\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$$

By taking first two terms, we have

$$\frac{4}{6} = \frac{BC}{9}$$

$$BC = \frac{(4 \times 9)}{6} = 6\text{cm}$$

And by taking last two terms, we have,

$$\frac{BC}{9} = \frac{AC}{12}$$

$$\frac{6}{9} = \frac{AC}{12}$$

$$AC = \frac{6 \times 12}{9} = 8\text{ cm}$$

Now,
Perimeter of ΔABC = AB + BC + AC
= 4 + 6 + 8 = 18 cm
Thus, the perimeter of the triangle is 18 cm.