

# Quadrilateral

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## EXERCISE 27 (A)

### Question 1.

Two angles of a quadrilateral are  $89^\circ$  and  $113^\circ$ . If the other two angles are equal; find the equal angles.

#### Solution:

Let the other angle =  $x^\circ$

According to given,

$$89^\circ + 113^\circ + x^\circ + x^\circ = 360^\circ$$

$$2x^\circ = 360^\circ - 202^\circ$$

$$2x^\circ = 158^\circ$$

$$x^\circ = \frac{158}{2} = 79^\circ$$

$\therefore$  other two angles =  $79^\circ$  each

### Question 2.

Two angles of a quadrilateral are  $68^\circ$  and  $76^\circ$ . If the other two angles are in the ratio 5 : 7; find the measure of each of them.

#### Solution:

Two angles are  $68^\circ$  and  $76^\circ$

Let other two angles be  $5x$  and  $7x$

$$\therefore 68^\circ + 76^\circ + 5x + 7x = 360^\circ$$

$$12x + 144^\circ = 360^\circ$$

$$12x = 360^\circ - 144^\circ$$

$$12x = 216^\circ$$

$$x = 18^\circ$$

angles are  $5x$  and  $7x$

i.e.  $5 \times 18^\circ$  and  $7 \times 18^\circ$  i.e.  $90^\circ$  and  $126^\circ$

**Question 3.**

Angles of a quadrilateral are  $(4x)^\circ$ ,  $5(x+2)^\circ$ ,  $(7x-20)^\circ$  and  $6(x+3)^\circ$ . Find

(i) the value of  $x$ .

(ii) each angle of the quadrilateral.

**Solution:**

Angles of quadrilateral are,

$(4x)^\circ$ ,  $5(x+2)^\circ$ ,  $(7x-20)^\circ$  and  $6(x+3)^\circ$ .

$$4x + 5(x+2) + (7x-20) + 6(x+3) = 360^\circ$$

$$4x + 5x + 10 + 7x - 20 + 6x + 18 = 360^\circ \quad 22x + 8 = 360^\circ$$

$$22x = 360^\circ - 8^\circ$$

$$22x = 352^\circ$$

$$x = 16^\circ$$

Hence angles are,

$$(4x)^\circ = (4 \times 16)^\circ = 64^\circ,$$

$$5(x+2)^\circ = 5(16+2)^\circ = 90^\circ,$$

$$(7x-20)^\circ = (7 \times 16 - 20)^\circ = 92^\circ$$

$$6(x+3)^\circ = 6(16+3) = 114^\circ$$

**Question 4.**

Use the information given in the following figure to find :

(i)  $x$

(ii)  $\angle B$  and  $\angle C$

**Solution:**

$$\because \angle A = 90^\circ \text{ (Given)}$$

$$\angle B = (2x + 4^\circ)$$

$$\angle C = (3x - 5^\circ)$$

$$\angle D = (8x - 15^\circ)$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$90^\circ + (2x + 4^\circ) + (3x - 5^\circ) + (8x - 15^\circ) = 360^\circ$$

$$90^\circ + 2x + 4^\circ + 3x - 5^\circ + 8x - 15^\circ = 360^\circ$$

$$\Rightarrow 74^\circ + 13x = 360^\circ$$

$$\Rightarrow 13x = 360^\circ - 74^\circ$$

$$\Rightarrow 13x = 286^\circ$$

$$\Rightarrow x = 22^\circ$$

$$\because \angle B = 2x + 4 = 2 \times 22^\circ + 4 = 48^\circ$$

$$\angle C = 3x - 5 = 3 \times 22^\circ - 5 = 61^\circ$$

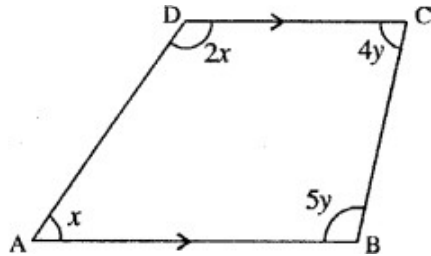
Hence (i)  $22^\circ$  (ii)  $\angle B = 48^\circ$ ,  $\angle C = 61^\circ$

**Question 5.**

In quadrilateral ABCD, side AB is parallel to side DC. If  $\angle A : \angle D = 1 : 2$  and  $\angle C : \angle B = 4 : 5$

- (i) Calculate each angle of the quadrilateral.
- (ii) Assign a special name to quadrilateral ABCD.

**Solution:**



$$\because \angle A : \angle D = 1 : 2$$

$$\text{Let } \angle A = x \text{ and } \angle D = 2x$$

$$\because \angle C : \angle B = 4 : 5 \text{ Let } \angle C = 4y \text{ and } \angle B = 5y$$

$$\because AB \parallel DC$$

$$\angle A + \angle D = 180^\circ \quad x + 2x = 180^\circ$$

$$3x = 180^\circ \quad x = 60^\circ$$

$$\therefore \angle A = 60^\circ$$

$$\angle D = 2x = 2 \times 60 = 120^\circ \text{ Again } \angle B + \angle C = 180^\circ$$

$$5y + 4y = 180^\circ$$

$$9y = 180^\circ$$

$$y = 20^\circ$$

$$\therefore \angle B = 5y = 5 \times 20 = 100^\circ$$

$$\angle C = 4y = 4 \times 20 = 80^\circ$$

$$\text{Hence } \angle A = 60^\circ ; \angle B = 100^\circ ; \angle C = 80^\circ \text{ and } \angle D = 120^\circ$$

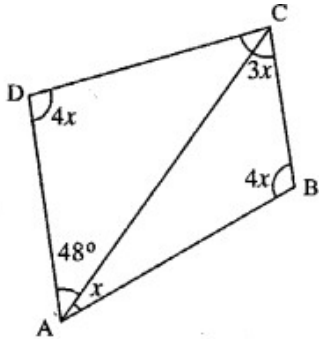
**Question 6.**

From the following figure find ;

- (i) x,
- (ii)  $\angle ABC$ ,
- (iii)  $\angle ACD$ .

**Solution:**

(i) In Quadrilateral ABCD,



$$x + 4x + 3x + 4x + 48^\circ = 360^\circ$$

$$12x = 360^\circ - 48^\circ$$

$$12x = 312$$

$$(ii) \angle ABC = 4x$$

$$4 \times 26 = 104^\circ$$

$$(iii) \angle ACD = 180^\circ - 4x - 48^\circ$$

$$= 180^\circ - 4 \times 26^\circ - 48^\circ$$

$$= 180^\circ - 104^\circ - 48^\circ$$

$$= 180^\circ - 152^\circ = 28^\circ$$

**Question 7.**

Given : In quadrilateral ABCD ;  $\angle C = 64^\circ$ ,  $\angle D = \angle C - 8^\circ$ ;  
 $\angle A = 5(a+2)^\circ$  and  $\angle B = 2(2a+7)^\circ$ .

Calculate  $\angle A$ .

**Solution:**

$$\therefore \angle C = 64^\circ \text{ (Given)}$$

$$\therefore \angle D = \angle C - 8^\circ$$

$$= 64^\circ - 8^\circ$$

$$= 56^\circ$$

$$\angle A = 5(a + 2)^\circ$$

$$\angle B = 2(2a + 7)^\circ$$

$$\text{Now } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$5(a + 2)^\circ + 2(2a + 7)^\circ + 64^\circ + 56^\circ = 360^\circ$$

$$5a + 10 + 4a + 14^\circ + 64^\circ + 56^\circ = 360^\circ$$

$$9a + 144^\circ = 360^\circ$$

$$9a = 360^\circ - 144^\circ$$

$$9a = 216^\circ$$

$$a = 24^\circ$$

$$\therefore \angle A = 5(a + 2)$$

$$= 5(24 + 2)$$

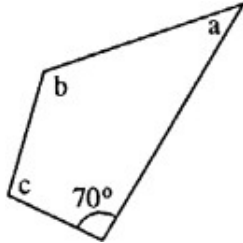
$$= 130^\circ$$

**Question 8.**

In the given figure :

$$\angle b = 2a + 15$$

and  $\angle c = 3a + 5$ ; find the values of  $b$  and  $c$ .



**Solution:**

$$\angle b = 2a + 15$$

$$\& \angle c = 3a + 5$$

$\therefore$  Sum of angles of quadrilateral =  $360^\circ$

$$70^\circ + a + 2a + 15 + 3a + 5 = 360^\circ$$

$$6a + 90^\circ = 360^\circ$$

$$6a = 270^\circ$$

$$a = 45^\circ$$

$$\therefore b = 2a + 15 = 2 \times 45 + 15 = 105^\circ$$

$$c = 3a + 5 = 3 \times 45 + 5 = 140^\circ$$

$$105^\circ \text{ and } 140^\circ$$

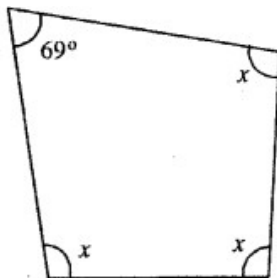
**Question 9.**

Three angles of a quadrilateral are equal. If the fourth angle is  $69^\circ$ ; find the measure of equal angles.

**Solution:**

Let each equal angle be

$$x^\circ \quad x + x + x + 69^\circ = 360^\circ$$



$$3x = 360^\circ - 69 \quad 3x = 291 \quad x = 97^\circ$$

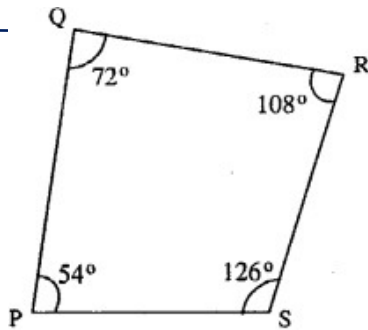
Each equal angle =  $97^\circ$

**Question 10.**

In quadrilateral PQRS,  $\angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$ .

Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other. Is PS also parallel to QR ?

**Solution:**



$$\therefore \angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$$

$$\text{Let } \angle P = 3x$$

$$\angle Q = 4x$$

$$\angle R = 6x \text{ \& } \angle S = 7x$$

$$\therefore \angle P + \angle Q + \angle R + \angle S = 360^\circ$$

$$3x + 4x + 6x + 7x = 360^\circ$$

$$20x = 360^\circ$$

$$x = 18^\circ$$

$$\therefore \angle P = 3x = 3 \times 18 = 54^\circ$$

$$\angle Q = 4x = 4 \times 18 = 72^\circ$$

$$\angle R = 6x = 6 \times 18 = 108^\circ$$

$$\angle S = 7x = 7 \times 18 = 126^\circ$$

$$\angle Q + \angle R = 72^\circ + 108^\circ = 180^\circ \text{ or } \angle P + \angle S = 54^\circ + 126^\circ = 180^\circ$$

Hence  $PQ \parallel RS$

$$\text{As } \angle P + \angle Q = 72^\circ + 54^\circ = 126^\circ$$

Which is  $\neq 180^\circ$ .

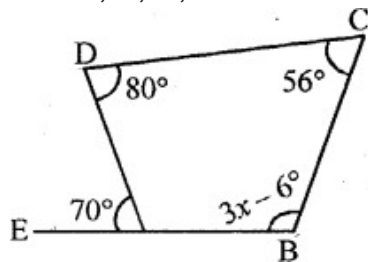
$\therefore PS$  and  $QR$  are not parallel.

### Question 11.

Use the information given in the following figure to find the value of  $x$ .

**Solution:**

Take  $A, B, C, D$  as the vertices of quadrilateral and  $BA$  is produced to  $E$  (say).



Since  $\angle EAD = 70^\circ$

$$\therefore \angle DAB = 180^\circ - 70^\circ = 110^\circ \text{ [}\because \text{EAB is a straight line and AD stands on it]}$$

$$\therefore \angle EAD + \angle DAB = 180^\circ$$

$$\therefore 110^\circ + 80^\circ + 56^\circ + 3x = 360^\circ$$

[ $\because$  sum of interior angles of a quadrilateral =  $360^\circ$ ]

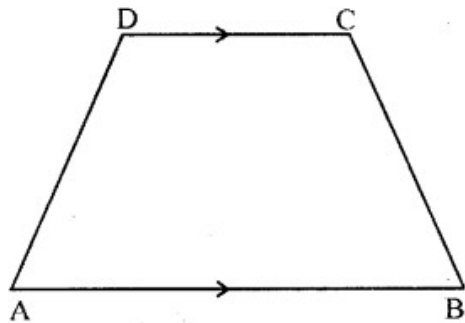
$$\therefore 3x = 360^\circ - 110^\circ - 80^\circ - 56^\circ + 6^\circ$$

$$3x = 360^\circ - 240^\circ = 120^\circ$$

$$\therefore x = 40^\circ$$

**Question 12.**

The following figure shows a quadrilateral in which sides AB and DC are parallel. If  $\angle A : \angle D = 4 : 5$ ,  $\angle B = (3x - 15)^\circ$  and  $\angle C = (4x + 20)^\circ$ , find each angle of the quadrilateral ABCD.



**Solution:**

$$\text{Let } \angle A = 4x$$

$$\angle D = 5x$$

$$\text{Since } \angle A + \angle D = 180^\circ [\because AB \parallel DC]$$

$$\therefore 4x + 5x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ \Rightarrow x = 20^\circ$$

$$\therefore \angle A = 4(20) = 80^\circ, \angle D = 5(20) = 100^\circ \text{ Again } \angle B + \angle C = 180^\circ [\because AB \parallel DC]$$

$$\therefore 3x - 15^\circ + 4x + 20^\circ = 180^\circ$$

$$7x = 180^\circ - 5^\circ$$

$$\Rightarrow 7x = 175^\circ \Rightarrow x = 25^\circ$$

$$\therefore \angle B = 75^\circ - 15^\circ = 60^\circ \text{ and } \angle C = 4(25) + 20 = 100^\circ + 20^\circ = 120^\circ$$