

### EXERCISE 19.9

Evaluate the following integrals:

1.  $\int \frac{\log x}{x} dx$

**Solution:**

Assume  $\log x = t$

$$\Rightarrow d(\log x) = dt$$

$$\Rightarrow \frac{1}{x} dx = dt$$

Substituting  $t$  and  $dt$  in above equation we get

$$\Rightarrow \int t \cdot dt$$

$$\Rightarrow \frac{t^2}{2} + c$$

But  $t = \log(x)$

$$\Rightarrow \frac{\log^2 x}{2} + c$$

2.  $\int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx$

**Solution:**

Assume  $\log\left(1 + \frac{1}{x}\right) = t$

$$\Rightarrow d\left(\log\left(1 + \frac{1}{x}\right)\right) = dt$$

$$\Rightarrow \frac{1}{1 + \frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{x}{x+1} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{-1 \cdot dx}{x(x+1)} = dt$$

$$\Rightarrow \frac{dx}{x(x+1)} = -dt$$

∴ Substituting  $t$  and  $dt$  in the given equation we get

$$\Rightarrow \int -t \cdot dt$$

$$\Rightarrow -\int t \cdot dt$$

$$\Rightarrow \frac{-t^2}{2} + c$$

But  $\log\left(1 + \frac{1}{x}\right) = t$

$$\Rightarrow -\frac{1}{2} \left\{ \log\left(1 + \frac{1}{x}\right) \right\}^2 + c$$

3.  $\int \frac{(1 + \sqrt{x})^2}{\sqrt{x}} dx$

**Solution:**

Assume  $1 + \sqrt{x} = t$

$$\Rightarrow d(1 + \sqrt{x}) = dt$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

∴ Substituting  $t$  and  $dt$  in the given equation we get

$$\Rightarrow \int 2t^2 \cdot dt$$

$$\Rightarrow 2 \int t^2 \cdot dt$$

$$\Rightarrow \frac{2t^3}{3} + c$$

But  $1 + \sqrt{x} = t$

$$\Rightarrow \frac{2(1 + \sqrt{x})^3}{3} + c$$

4.  $\int \sqrt{1 + e^x} e^x dx$

**Solution:**

Assume  $1 + e^x = t$

$$\Rightarrow d(1 + e^x) = dt$$

$$\Rightarrow e^x dx = dt$$

$\therefore$  Substituting  $t$  and  $dt$  in given equation we get

$$\Rightarrow \int \sqrt{t} \cdot dt$$

$$\Rightarrow \int t^{1/2} \cdot dt$$

$$\Rightarrow \frac{2t^{3/2}}{3} + c$$

But  $1 + e^x = t$

$$\Rightarrow \frac{2(1 + e^x)^{3/2}}{3} + c$$

5.  $\int \sqrt[3]{\cos^2 x} \sin x dx$

**Solution:**

Assume  $\cos x = t$

$$\Rightarrow d(\cos x) = dt$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow dx = \frac{-dt}{\sin x}$$

∴ Substituting  $t$  and  $dt$  in the given equation we get

$$\Rightarrow \int \sqrt[3]{t^2} \sin x \cdot \frac{-dt}{\sin x}$$

$$\Rightarrow \int -t^{2/3} \cdot dt$$

$$\Rightarrow -\frac{3}{5} t^{5/3} + c$$

But  $\cos x = t$

$$\Rightarrow -\frac{3}{5} \cos^{5/3} x + c$$

6.  $\int \frac{e^x}{(1 + e^x)^2} dx$

**Solution:**

Assume  $1 + e^x = t$

$$\Rightarrow d(1 + e^x) = dt$$

$$\Rightarrow e^x dx = dt$$

∴ Substituting  $t$  and  $dt$  in given equation we get

$$\Rightarrow \int \frac{1}{t^2} dt$$

$$\Rightarrow \int t^{-2} \cdot dt$$

$$\Rightarrow \frac{-1}{t} + c$$

But  $1 + e^x = t$

$$\Rightarrow \frac{-1}{1 + e^x} + c$$

7.  $\int \cot^3 x \operatorname{cosec}^2 x dx$

**Solution:**

Assume  $\cot x = t$

$$\Rightarrow d(\cot x) = dt$$

$$\Rightarrow -\operatorname{cosec}^2 x \cdot dx = dt$$

$$\Rightarrow dx = \frac{-dt}{\operatorname{csc}^2 x}$$

$\therefore$  Substituting  $t$  and  $dt$  in the given equation we get

$$\Rightarrow \int t^3 \operatorname{csc}^2 x \cdot \frac{-dt}{\operatorname{csc}^2 x}$$

$$\Rightarrow \int -t^3 \cdot dt$$

$$\Rightarrow -\int t^3 \cdot dt$$

$$\Rightarrow \frac{-t^4}{4} + c$$

But  $t = \cot x$

$$\Rightarrow \frac{-\cot^4 x}{4} + c$$

8.  $\int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1-x^2}} dx$

**Solution:**

Assume  $\sin^{-1} x = t$

$$\Rightarrow d(\sin^{-1} x) = dt$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$$

$\therefore$  Substituting  $t$  and  $dt$  in the given equation we get

$$\Rightarrow \int e^{t^2} dt$$

$$\Rightarrow \int e^{2t} \cdot dt$$

$$\Rightarrow \frac{e^{2t}}{2} + c$$

But  $t = \sin^{-1}x$

$$\Rightarrow \frac{1}{2} \{ e^{\sin^{-1}x} \}^2 + c$$

9.  $\int \frac{1 + \sin x}{\sqrt{x - \cos x}} dx$

**Solution:**

Assume  $x - \cos x = t$

$$\Rightarrow d(x - \cos x) = dt$$

$$\Rightarrow (1 + \sin x) dx = dt$$

$\therefore$  Substituting  $t$  and  $dt$  in given equation we get

$$\Rightarrow \int \frac{1}{\sqrt{t}} dt$$

$$\Rightarrow \int t^{-1/2} dt$$

$$\Rightarrow 2t^{1/2} + c$$

But  $t = x - \cos x$ .

$$\Rightarrow 2(x - \cos x)^{1/2} + c.$$

10.  $\int \frac{1}{\sqrt{1-x^2}(\sin^{-1}x)^2} dx$

**Solution:**

Assume  $\sin^{-1}x = t$

$$\Rightarrow d(\sin^{-1}x) = dt$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$$

$\therefore$  Substituting  $t$  and  $dt$  in the given equation we get

$$\Rightarrow \int \frac{1}{t^2} dt$$

$$\Rightarrow \int t^{-2} . dt$$

On integrating the above equation we get

$$\Rightarrow \frac{t^{-1}}{-1} + c$$

But  $t = \sin^{-1}x$

$$\Rightarrow \frac{-1}{\sin^{-1}x} + c$$



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