Exercise 8(A)

1. Express each of the following in logarithmic form:
(i) $5^3 = 125$
(ii) $3^{-2} = 1/9$
(iii) $10^{-3} = 0.001$
(iv) $(81)^{3/4} = 27$
Solution:

We know that,
$a^b = c \Rightarrow \log_a c = b$

(i) $5^3 = 125$
$\log_5 125 = 3$

(ii) $3^{-2} = 1/9$
$\log_3 1/9 = -2$

(iii) $10^{-3} = 0.001$
$\log_{10} 0.001 = -3$

(iv) $(81)^{3/4} = 27$
$\log_{81} 27 = 3/4$

2. Express each of the following in exponential form:
(i) $\log_8 0.125 = -1$
(ii) $\log_{10} 0.01 = -2$
(iii) $\log_a A = x$
(iv) $\log_{10} 1 = 0$
Solution:

We know that,
$\log_a c = b \Rightarrow a^b = c$

(i) $\log_8 0.125 = -1$
$8^{-1} = 0.125$

(ii) $\log_{10} 0.01 = -2$
$8^{-1} = 0.125$

(iii) $\log_a A = x$
$a^x = A$

(iv) $\log_{10} 1 = 0$
$10^0 = 1$

3. Solve for x: $\log_{10} x = -2$.

Solution:

We have,
$\log_{10} x = -2$
$10^{-2} = x$ [As $\log_a c = b \Rightarrow a^b = c$]
$x = 10^{-2}$
$x = 1/10^2$
$x = 1/100$
Hence, $x = 0.01$

4. Find the logarithm of:
(i) 100 to the base 10
(ii) 0.1 to the base 10
(iii) 0.001 to the base 10
(iv) 32 to the base 4
(v) 0.125 to the base 2
(vi) 1/16 to the base 4
(vii) 27 to the base 9
(viii) 1/81 to the base 27
Solution;

(i) Let $\log_{10} 100 = x$
So, $10^x = 100$
$10^x = 10^2$
Then,
$x = 2$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_{10} 100 = 2$

(ii) Let $\log_{10} 0.1 = x$
So, $10^x = 0.1$
$10^x = 1/10$
$10^x = 10^{-1}$
Then,
$x = -1$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_{10} 0.1 = -1$

(iii) Let $\log_{10} 0.001 = x$
So, $10^x = 0.001$
$10^x = 1/1000$
$10^x = 1/10^3$
$10^x = 10^{-3}$
Then,
$x = -3$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_{10} 0.001 = -3$

(iv) Let $\log_4 32 = x$

So, $4^x = 32$
$4^x = 2 \times 2 \times 2 \times 2 \times 2$
$(2^2)^x = 2^5$
$2^{2x} = 2^5$
Then,
$2x = 5$ [If $a^m = a^n$; then $m = n$]
$x = 5/2$
Hence, $\log_4 32 = 5/2$

(v) Let $\log_2 0.125 = x$
So, $2^x = 0.125$
$2^x = 125/1000$
$2^x = 1/8$
$2^x = (½)^3$
$2^x = 2^{-3}$
Then,
$x = -3$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_2 0.125 = -3$

(vi) Let $\log_4 1/16 = x$
So, $4^x = 1/16$
$4^x = (¼)^{-2}$
$4^x = 4^{-2}$
Then,
$x = -2$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_4 1/16 = -2$

(vii) Let $\log_9 27 = x$
So, $9^x = 27$
$9^x = 3 \times 3 \times 3$
$(3^2)^x = 3^3$
$3^{2x} = 3^3$
Then,
$2x = 3$ [If $a^m = a^n$; then $m = n$]
$x = 3/2$
Hence, $\log_9 27 = 3/2$

(viii) Let $\log_{27} 1/81 = x$
So, $27^x = 1/81$
$27^x = 1/9^2$
$(3^3)^x = 1/(3^2)^2$
$3^{3x} = 1/3^4$
$3^{3x} = 3^{-4}$
Then,
$3x = -4$ [If $a^m = a^n$; then $m = n$]
$x = -4/3$

Hence, $\log_{27} 1/81 = -4/3$

5. State, true or false:
(i) If $\log_{10} x = a$, then $10^x = a$
(ii) If $x^y = z$, then $y = \log_z x$
(iii) $\log_2 8 = 3$ and $\log_8 2 = 1/3$
Solution:

(i) We have,
$\log_{10} x = a$
So, $10^a = x$
Thus, the statement $10^x = a$ is false

(ii) We have,
$x^y = z$
So, $\log_x z = y$
Thus, the statement $y = \log_z x$ is false

(iii) We have,
$\log_2 8 = 3$
So, $2^3 = 8$ … (1)
Now consider the equation,
$\log_8 2 = 1/3$
$8^{1/3} = 2$
$(2^3)^{1/3} = 2$ … (2)
Both equations (1) and (2) are correct
Thus, the given statements, $\log_2 8 = 3$ and $\log_8 2 = 1/3$ are true

6. Find x, if:
(i) $\log_3 x = 0$
(ii) $\log_x 2 = -1$
(iii) $\log_9 243 = x$
(iv) $\log_5 (x - 7) = 1$
(v) $\log_4 32 = x - 4$
(vi) $\log_7 (2x^2 - 1) = 2$
Solution:

(i) We have, $\log_3 x = 0$
So, $3^0 = x$
$1 = x$
Hence, $x = 1$

(ii) we have, $\log_x 2 = -1$
So, $x^{-1} = 2$
$1/x = 2$
Hence, $x = ½$

(iii) We have, $\log_9 243 = x$
$9^x = 243$
$(3^2)^x = 3^5$
$3^{2x} = 3^5$
On comparing the exponents, we get
$2x = 5$
$x = 5/2 = 2\frac{1}{2}$

(iv) We have, $\log_5 (x - 7) = 1$
So, $5^1 = x - 7$
$5 = x - 7$
$x = 5 + 7$
Hence, $x = 12$

(v) We have, $\log_4 32 = x - 4$
So, $4^{(x - 4)} = 32$
$(2^2)^{(x - 4)} = 2^5$
$2^{(2x - 8)} = 2^5$
On comparing the exponents, we get
$2x - 8 = 5$
$2x = 5 + 8$
Hence,
$x = 13/2 = 6\frac{1}{2}$

(vi) We have, $\log_7 (2x^2 - 1) = 2$
So, $(2x^2 - 1) = 7^2$
$2x^2 - 1 = 49$
$2x^2 = 49 + 1$
$2x^2 = 50$
$x^2 = 25$
Taking square root on both side, we get
$x = \pm 5$
Hence, $x = 5$ (Neglecting the negative value)

7. Evaluate:
(i) $\log_{10} 0.01$
(ii) $\log_2 (1 \div 8)$
(iii) $\log_5 1$
(iv) $\log_5 125$
(v) $\log_{16} 8$
(vi) $\log_{0.5} 16$
Solution:

(i) Let $\log_{10} 0.01 = x$
Then, $10^x = 0.01$

$10^x = 1/100 = 1/10^2$
So, $10^x = 10^{-2}$
On comparing the exponents, we get
$x = -2$
Hence, $\log_{10} 0.01 = -2$

(ii) Let $\log_2 (1 \div 8) = x$
Then, $2^x = 1/8$
$2^x = 1/2^3$
So, $2^x = 2^{-3}$
On comparing the exponents, we get
$x = -3$
Hence, $\log_{10} (1 \div 8) = -3$

(iii) Let $\log_5 1 = x$
Then, $5^x = 1$
$5^x = 5^0$
On comparing the exponents, we get
$x = 0$
Hence, $\log_5 1 = 0$

(iv) Let $\log_5 125 = x$
Then, $5^x = 125$
$5^x = (5 \times 5 \times 5) = 5^3$
So, $5^x = 5^3$
On comparing the exponents, we get
$x = 3$
Hence, $\log_5 125 = 3$

(v) Let $\log_{16} 8 = x$
Then, $16^x = 8$
$(2^4)^x = (2 \times 2 \times 2) = 2^3$
So, $2^{4x} = 2^3$
On comparing the exponents, we get
$4x = 3$
$x = 3/4$
Hence, $\log_{16} 8 = 3/4$

(vi) Let $\log_{0.5} 16 = x$
Then, $0.5^x = 16$
$(5/10)^x = (2 \times 2 \times 2 \times 2)$
$(1/2)^x = 2^4$
So, $2^{-x} = 2^4$
On comparing the exponents, we get
$-x = 4$
$\Rightarrow x = -4$

Hence, $\log_{0.5} 16 = -4$

8. If $\log_a m = n$, express a^{n-1} in terms in terms of a and m.
Solution:

We have, $\log_a m = n$
So,
$a^n = m$
Dividing by a on both sides, we get
$a^n/a = m/a$
$a^{n-1} = m/a$

9. Given $\log_2 x = m$ and $\log_5 y = n$
(i) Express 2^{m-3} in terms of x
(ii) Express 5^{3n+2} in terms of y
Solution:

Given, $\log_2 x = m$ and $\log_5 y = n$
So,
$2^m = x$ and $5^n = y$
(i) Taking, $2^m = x$
$2^m/2^3 = x/2^3$
$2^{m-3} = x/8$

(ii) Taking, $5^n = y$
Cubing on both sides, we have
$(5^n)^3 = y^3$
$5^{3n} = y^3$
Multiplying by 5^2 on both sides, we have
$5^{3n} \times 5^2 = y^3 \times 5^2$
$5^{3n+2} = 25y^3$

10. If $\log_2 x = a$ and $\log_3 y = a$, write 72^a in terms of x and y.
Solution:

Given, $\log_2 x = a$ and $\log_3 y = a$
So,
$2^a = x$ and $3^a = y$
Now, the prime factorization of 72 is
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
Hence,
$(72)^a = (2^3 \times 3^2)^a$
$= 2^{3a} \times 3^{2a}$
$= (2^a)^3 \times (3^a)^2$
$= x^3y^2$ [As $2^a = x$ and $3^a = y$]

11. Solve for x: log (x - 1) + log (x + 1) = $\log_2 1$
Solution:

We have,
log (x - 1) + log (x + 1) = $\log_2 1$
log (x - 1) + log (x + 1) = 0
log [(x - 1) (x + 1)] = 0
Then,
(x - 1) (x + 1) = 1 [As log 1 = 0]
$x^2 - 1 = 1$
$x^2 = 1 + 1$
$x^2 = 2$
$x = \pm\sqrt{2}$
The value $-\sqrt{2}$ is not a possible, since log of a negative number is not defined.
Hence, $x = \sqrt{2}$

12. If log (x^2 - 21) = 2, show that x = ± 11.
Solution:

Given, log (x^2 - 21) = 2
So,
$x^2 - 21 = 10^2$
$x^2 - 21 = 100$
$x^2 = 121$
Taking square root on both sides, we get
$x = \pm 11$