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**Exercise – 2A**

1. Find the zeros of the polynomial  $f(x) = x^2 + 7x + 12$  and verify the relation between its zeroes and coefficients.

**Sol:**

$$x^2 + 7x + 12 = 0$$

$$\Rightarrow x^2 + 4x + 3x + 12 = 0$$

$$\Rightarrow x(x+4) + 3(x+4) = 0$$

$$\Rightarrow (x+4)(x+3) = 0$$

$$\Rightarrow (x+4) = 0 \text{ or } (x+3) = 0$$

$$\Rightarrow x = -4 \text{ or } x = -3$$

$$\text{Sum of zeroes} = -4 + (-3) = \frac{-7}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = (-4)(-3) = \frac{12}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

2. Find the zeroes of the polynomial  $f(x) = x^2 - 2x - 8$  and verify the relation between its zeroes and coefficients.

**Sol:**

$$x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x-4) + 2(x-4) = 0$$

$$\Rightarrow (x-4)(x+2) = 0$$

$$\Rightarrow (x-4) = 0 \text{ or } (x+2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

$$\text{Sum of zeroes} = 4 + (-2) = 2 = \frac{2}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = (4)(-2) = \frac{-8}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

3. Find the zeroes of the quadratic polynomial  $f(x) = x^2 + 3x - 10$  and verify the relation between its zeroes and coefficients.

**Sol:**

We have:

$$f(x) = x^2 + 3x - 10$$

$$= x^2 + 5x - 2x - 10$$

$$= x(x+5) - 2(x+5)$$

$$= (x-2)(x+5)$$

$$\therefore f(x) = 0 \Rightarrow (x-2)(x+5) = 0$$

$$\Rightarrow x - 2 = 0 \text{ or } x + 5 = 0$$

$$\Rightarrow x = 2 \text{ or } x = -5.$$

So, the zeroes of  $f(x)$  are 2 and  $-5$ .

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$$\text{Sum of zeroes} = 2 + (-5) = -3 = \frac{-3}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = 2 \times (-5) = -10 = \frac{-10}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

4. Find the zeroes of the quadratic polynomial  $f(x) = 4x^2 - 4x - 3$  and verify the relation between its zeroes and coefficients.

**Sol:**

We have:

$$\begin{aligned} f(x) &= 4x^2 - 4x - 3 \\ &= 4x^2 - (6x - 2x) - 3 \\ &= 4x^2 - 6x + 2x - 3 \\ &= 2x(2x - 3) + 1(2x - 3) \\ &= (2x + 1)(2x - 3) \end{aligned}$$

$$\begin{aligned} \therefore f(x) = 0 &\Rightarrow (2x + 1)(2x - 3) = 0 \\ &\Rightarrow 2x + 1 = 0 \text{ or } 2x - 3 = 0 \\ &\Rightarrow x = \frac{-1}{2} \text{ or } x = \frac{3}{2} \end{aligned}$$

So, the zeroes of  $f(x)$  are  $\frac{-1}{2}$  and  $\frac{3}{2}$ .

$$\text{Sum of zeroes} = \left(\frac{-1}{2}\right) + \left(\frac{3}{2}\right) = \frac{-1+3}{2} = \frac{2}{2} = 1 = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \left(\frac{-1}{2}\right) \times \left(\frac{3}{2}\right) = \frac{-3}{4} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

5. Find the zeroes of the quadratic polynomial  $f(x) = 5x^2 - 4 - 8x$  and verify the relationship between the zeroes and coefficients of the given polynomial.

**Sol:**

We have:

$$\begin{aligned} f(x) &= 5x^2 - 4 - 8x \\ &= 5x^2 - 8x - 4 \\ &= 5x^2 - (10x - 2x) - 4 \\ &= 5x^2 - 10x + 2x - 4 \\ &= 5x(x - 2) + 2(x - 2) \\ &= (5x + 2)(x - 2) \end{aligned}$$

$$\begin{aligned} \therefore f(x) = 0 &\Rightarrow (5x + 2)(x - 2) = 0 \\ &\Rightarrow 5x + 2 = 0 \text{ or } x - 2 = 0 \\ &\Rightarrow x = \frac{-2}{5} \text{ or } x = 2 \end{aligned}$$

So, the zeroes of  $f(x)$  are  $\frac{-2}{5}$  and 2.

$$\text{Sum of zeroes} = \left(\frac{-2}{5}\right) + 2 = \frac{-2+10}{5} = \frac{8}{5} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \left(\frac{-2}{5}\right) \times 2 = \frac{-4}{5} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

6. Find the zeroes of the polynomial  $f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$  and verify the relation between its zeroes and coefficients.

**Sol:**

$$2\sqrt{3}x^2 - 5x + \sqrt{3}$$

$$\Rightarrow 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$$

$$\Rightarrow 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$$

$$\Rightarrow (\sqrt{3}x - 1) \text{ or } (2x - \sqrt{3}) = 0$$

$$\Rightarrow (\sqrt{3}x - 1) = 0 \text{ or } (2x - \sqrt{3}) = 0$$

$$\Rightarrow x = \frac{1}{\sqrt{3}} \text{ or } x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \text{ or } x = \frac{\sqrt{3}}{2}$$

$$\text{Sum of zeroes} = \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{6} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

7. Find the zeroes of the quadratic polynomial  $2x^2 - 11x + 15$  and verify the relation between the zeroes and the coefficients.

**Sol:**

$$f(x) = 2x^2 - 11x + 15$$

$$= 2x^2 - (6x + 5x) + 15$$

$$= 2x^2 - 6x - 5x + 15$$

$$= 2x(x - 3) - 5(x - 3)$$

$$= (2x - 5)(x - 3)$$

$$\therefore f(x) = 0 \Rightarrow (2x - 5)(x - 3) = 0$$

$$\Rightarrow 2x - 5 = 0 \text{ or } x - 3 = 0$$

$$\Rightarrow x = \frac{5}{2} \text{ or } x = 3$$

So, the zeroes of  $f(x)$  are  $\frac{5}{2}$  and 3.

$$\text{Sum of zeroes} = \frac{5}{2} + 3 = \frac{5 + 6}{2} = \frac{11}{2} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{5}{2} \times 3 = \frac{-15}{2} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

8. Find the zeroes of the quadratic polynomial  $4x^2 - 4x + 1$  and verify the relation between the zeroes and the coefficients.

**Sol:**

$$4x^2 - 4x + 1 = 0$$

$$\Rightarrow (2x)^2 - 2(2x)(1) + (1)^2 = 0$$

$$\Rightarrow (2x - 1)^2 = 0 \quad [\because a^2 - 2ab + b^2 = (a-b)^2]$$

$$\Rightarrow (2x - 1)^2 = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = \frac{1}{2}$$

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

9. Find the zeroes of the quadratic polynomial  $(x^2 - 5)$  and verify the relation between the zeroes and the coefficients.

**Sol:**

We have:

$$f(x) = x^2 - 5$$

It can be written as  $x^2 + 0x - 5$ .

$$= (x^2 - (\sqrt{5})^2)$$

$$= (x + \sqrt{5})(x - \sqrt{5})$$

$$\therefore f(x) = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0$$

$$\Rightarrow x + \sqrt{5} = 0 \text{ or } x - \sqrt{5} = 0$$

$$\Rightarrow x = -\sqrt{5} \text{ or } x = \sqrt{5}$$

So, the zeroes of  $f(x)$  are  $-\sqrt{5}$  and  $\sqrt{5}$ .

Here, the coefficient of  $x$  is 0 and the coefficient of  $x^2$  is 1.

$$\text{Sum of zeroes} = -\sqrt{5} + \sqrt{5} = \frac{0}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = -\sqrt{5} \times \sqrt{5} = \frac{-5}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

10. Find the zeroes of the quadratic polynomial  $(8x^2 - 4)$  and verify the relation between the zeroes and the coefficients.

**Sol:**

We have:

$$f(x) = 8x^2 - 4$$

It can be written as  $8x^2 + 0x - 4$

$$= 4 \{ (\sqrt{2}x)^2 - (1)^2 \}$$

$$= 4 (\sqrt{2}x + 1)(\sqrt{2}x - 1)$$

$$\therefore f(x) = 0 \Rightarrow (\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$$

$$\Rightarrow (\sqrt{2}x + 1) = 0 \text{ or } \sqrt{2}x - 1 = 0$$

$$\Rightarrow x = \frac{-1}{\sqrt{2}} \text{ or } x = \frac{1}{\sqrt{2}}$$

So, the zeroes of  $f(x)$  are  $\frac{-1}{\sqrt{2}}$  and  $\frac{1}{\sqrt{2}}$

Here the coefficient of  $x$  is 0 and the coefficient of  $x^2$  is  $\sqrt{2}$

$$\text{Sum of zeroes} = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1+1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{-1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{-1 \times 1}{2} = \frac{-1}{2} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

11. Find the zeroes of the quadratic polynomial  $(5y^2 + 10y)$  and verify the relation between the zeroes and the coefficients.

**Sol:**

We have,

$$f(u) = 5u^2 + 10u$$

It can be written as  $5u(u+2)$

$$\therefore f(u) = 0 \Rightarrow 5u = 0 \text{ or } u + 2 = 0$$

$$\Rightarrow u = 0 \text{ or } u = -2$$

So, the zeroes of  $f(u)$  are  $-2$  and  $0$ .

$$\text{Sum of the zeroes} = -2 + 0 = -2 = \frac{-2 \times 5}{1 \times 5} = \frac{-10}{5} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } u^2)}$$

$$\text{Product of zeroes} = -2 \times 0 = 0 = \frac{0 \times 5}{1 \times 5} = \frac{-0}{5} = \frac{\text{constant term}}{(\text{coefficient of } u^2)}$$

12. Find the zeroes of the quadratic polynomial  $(3x^2 - x - 4)$  and verify the relation between the zeroes and the coefficients.

**Sol:**

$$3x^2 - x - 4 = 0$$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$$

$$\Rightarrow x(3x - 4) + 1(3x - 4) = 0$$

$$\Rightarrow (3x - 4)(x + 1) = 0$$

$$\Rightarrow (3x - 4) \text{ or } (x + 1) = 0$$

$$\Rightarrow x = \frac{4}{3} \text{ or } x = -1$$

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

13. Find the quadratic polynomial whose zeroes are 2 and -6. Verify the relation between the coefficients and the zeroes of the polynomial.

**Sol:**

Let  $\alpha = 2$  and  $\beta = -6$

Sum of the zeroes,  $(\alpha + \beta) = 2 + (-6) = -4$

Product of the zeroes,  $\alpha\beta = 2 \times (-6) = -12$

$$\therefore \text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-4)x - 12 \\ = x^2 + 4x - 12$$

$$\text{Sum of the zeroes} = -4 = \frac{-4}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = -12 = \frac{-12}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

14. Find the quadratic polynomial whose zeroes are  $\frac{2}{3}$  and  $\frac{-1}{4}$ . Verify the relation between the coefficients and the zeroes of the polynomial.

**Sol:**

$$\text{Let } \alpha = \frac{2}{3} \text{ and } \beta = \frac{-1}{4}.$$

$$\text{Sum of the zeroes} = (\alpha + \beta) = \frac{2}{3} + \left(\frac{-1}{4}\right) = \frac{8-3}{12} = \frac{5}{12}$$

$$\text{Product of the zeroes, } \alpha\beta = \frac{2}{3} \times \left(\frac{-1}{4}\right) = \frac{-2}{12} = \frac{-1}{6}$$

$$\therefore \text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{5}{12}x + \left(\frac{-1}{6}\right)$$

$$= x^2 - \frac{5}{12}x - \frac{1}{6}$$

$$\text{Sum of the zeroes} = \frac{5}{12} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{-1}{6} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

15. Find the quadratic polynomial, sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

**Sol:**

Let  $\alpha$  and  $\beta$  be the zeroes of the required polynomial  $f(x)$ .

$$\text{Then } (\alpha + \beta) = 8 \text{ and } \alpha\beta = 12$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow f(x) = x^2 - 8x + 12$$

$$\text{Hence, required polynomial } f(x) = x^2 - 8x + 12$$

$$\therefore f(x) = 0 \Rightarrow x^2 - 8x + 12 = 0$$

$$\Rightarrow x^2 - (6x + 2x) + 12 = 0$$

$$\Rightarrow x^2 - 6x - 2x + 12 = 0$$

$$\Rightarrow x(x - 6) - 2(x - 6) = 0$$

$$\Rightarrow (x - 2)(x - 6) = 0$$

$$\Rightarrow (x - 2) = 0 \text{ or } (x - 6) = 0$$

$$\Rightarrow x = 2 \text{ or } x = 6$$

So, the zeroes of  $f(x)$  are 2 and 6.

- 16.** Find the quadratic polynomial, sum of whose zeroes is 0 and their product is -1. Hence, find the zeroes of the polynomial.

**Sol:**

Let  $\alpha$  and  $\beta$  be the zeroes of the required polynomial  $f(x)$ .

$$\text{Then } (\alpha + \beta) = 0 \text{ and } \alpha\beta = -1$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow f(x) = x^2 - 0x + (-1)$$

$$\Rightarrow f(x) = x^2 - 1$$

Hence, required polynomial  $f(x) = x^2 - 1$ .

$$\therefore f(x) = 0 \Rightarrow x^2 - 1 = 0$$

$$\Rightarrow (x + 1)(x - 1) = 0$$

$$\Rightarrow (x + 1) = 0 \text{ or } (x - 1) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 1$$

So, the zeroes of  $f(x)$  are -1 and 1.

- 17.** Find the quadratic polynomial, sum of whose zeroes is  $(\frac{5}{2})$  and their product is 1. Hence, find the zeroes of the polynomial.

**Sol:**

Let  $\alpha$  and  $\beta$  be the zeroes of the required polynomial  $f(x)$ .

$$\text{Then } (\alpha + \beta) = \frac{5}{2} \text{ and } \alpha\beta = 1$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow f(x) = x^2 - \frac{5}{2}x + 1$$

$$\Rightarrow f(x) = 2x^2 - 5x + 2$$

Hence, the required polynomial is  $f(x) = 2x^2 - 5x + 2$

$$\therefore f(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$$

$$\Rightarrow 2x^2 - (4x + x) + 2 = 0$$

$$\Rightarrow 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (2x - 1)(x - 2) = 0$$

$$\Rightarrow (2x - 1) = 0 \text{ or } (x - 2) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2$$

So, the zeros of  $f(x)$  are  $\frac{1}{2}$  and 2.

18. Find the quadratic polynomial, sum of whose zeroes is  $\sqrt{2}$  and their product is  $(\frac{1}{3})$ .

**Sol:**

We can find the quadratic equation if we know the sum of the roots and product of the roots by using the formula

$$x^2 - (\text{Sum of the roots})x + \text{Product of roots} = 0$$

$$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3} = 0$$

$$\Rightarrow 3x^2 - 3\sqrt{2}x + 1 = 0$$

19. If  $x = \frac{2}{3}$  and  $x = -3$  are the roots of the quadratic equation  $ax^2 + 7x + b = 0$  then find the value of  $a$  and  $b$ .

**Sol:**

Given:  $ax^2 + 7x + b = 0$

Since,  $x = \frac{2}{3}$  is the root of the above quadratic equation

Hence, it will satisfy the above equation.

Therefore, we will get

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\Rightarrow \frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\Rightarrow 4a + 42 + 9b = 0$$

$$\Rightarrow 4a + 9b = -42 \quad \dots(1)$$

Since,  $x = -3$  is the root of the above quadratic equation

Hence, It will satisfy the above equation.

Therefore, we will get

$$a(-3)^2 + 7(-3) + b = 0$$

$$\Rightarrow 9a - 21 + b = 0$$

$$\Rightarrow 9a + b = 21 \quad \dots(2)$$

From (1) and (2), we get

$$a = 3, b = -6$$

20. If  $(x+a)$  is a factor of the polynomial  $2x^2 + 2ax + 5x + 10$ , find the value of  $a$ .

**Sol:**

Given:  $(x + a)$  is a factor of  $2x^2 + 2ax + 5x + 10$



$$\begin{aligned}\Rightarrow (x + 3)(3x + 9) &= 0 \\ \Rightarrow (x + 3) = 0 \text{ or } (3x + 9) &= 0 \\ \Rightarrow x = -3 \text{ or } x &= -3\end{aligned}$$

**Exercise – 2B**

1. Verify that 3, -2, 1 are the zeros of the cubic polynomial  $p(x) = (x^3 - 2x^2 - 5x + 6)$  and verify the relation between its zeros and coefficients.

**Sol:**

The given polynomial is  $p(x) = (x^3 - 2x^2 - 5x + 6)$

$$\therefore p(3) = (3^3 - 2 \times 3^2 - 5 \times 3 + 6) = (27 - 18 - 15 + 6) = 0$$

$$p(-2) = [(-2)^3 - 2 \times (-2)^2 - 5 \times (-2) + 6] = (-8 - 8 + 10 + 6) = 0$$

$$p(1) = (1^3 - 2 \times 1^2 - 5 \times 1 + 6) = (1 - 2 - 5 + 6) = 0$$

$\therefore$  3, -2 and 1 are the zeroes of  $p(x)$ ,

Let  $\alpha = 3$ ,  $\beta = -2$  and  $\gamma = 1$ . Then we have:

$$(\alpha + \beta + \gamma) = (3 - 2 + 1) = 2 = \frac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)}$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = (-6 - 2 + 3) = \frac{-5}{1} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = \{3 \times (-2) \times 1\} = \frac{-6}{1} = \frac{-(\text{constant term})}{(\text{coefficient of } x^3)}$$

2. Verify that 5, -2 and  $\frac{1}{3}$  are the zeroes of the cubic polynomial  $p(x) = (3x^3 - 10x^2 - 27x + 10)$  and verify the relation between its zeroes and coefficients.

**Sol:**

$p(x) = (3x^3 - 10x^2 - 27x + 10)$

$$p(5) = (3 \times 5^3 - 10 \times 5^2 - 27 \times 5 + 10) = (375 - 250 - 135 + 10) = 0$$

$$p(-2) = [3 \times (-2)^3 - 10 \times (-2)^2 - 27 \times (-2) + 10] = (-24 - 40 + 54 + 10) = 0$$

$$\begin{aligned}p\left(\frac{1}{3}\right) &= \left\{3 \times \left(\frac{1}{3}\right)^3 - 10 \times \left(\frac{1}{3}\right)^2 - 27 \times \frac{1}{3} + 10\right\} = \left(3 \times \frac{1}{27} - 10 \times \frac{1}{9} - 9 + 10\right) \\ &= \left(\frac{1}{9} - \frac{10}{9} + 1\right) = \left(\frac{1-10+9}{9}\right) = \left(\frac{0}{9}\right) = 0\end{aligned}$$

$\therefore$  5, -2 and  $\frac{1}{3}$  are the zeroes of  $p(x)$ .

Let  $\alpha = 5$ ,  $\beta = -2$  and  $\gamma = \frac{1}{3}$ . Then we have:

$$(\alpha + \beta + \gamma) = \left(5 - 2 + \frac{1}{3}\right) = \frac{10}{3} = \frac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)}$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left(-10 - \frac{2}{3} + \frac{5}{3}\right) = \frac{-27}{3} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = \left\{5 \times (-2) \times \frac{1}{3}\right\} = \frac{-10}{3} = \frac{-(\text{constant term})}{(\text{coefficient of } x^3)}$$



7. If  $f(x) = x^4 - 3x^2 + 4x + 5$  is divided by  $g(x) = x^2 - x + 1$

**Sol:**

$$\begin{array}{r}
 \phantom{x^2 - x + 1} \overline{x^2 + x - 3} \\
 x^2 - x + 1 \overline{) \begin{array}{r} x^4 + 0x^3 - 3x^2 + 4x + 5 \\ x^4 - x^3 + x^2 \\ \hline \phantom{x^4} - 4x^2 + 4x + 5 \\ \phantom{x^4} x^3 - x^2 + x \\ \hline \phantom{x^4} \phantom{x^3} - 3x^2 + 3x + 5 \\ \phantom{x^4} \phantom{x^3} - 3x^2 + 3x - 3 \\ \hline \phantom{x^4} \phantom{x^3} \phantom{- 3x^2} + 6x + 8 \\ \hline \phantom{x^4} \phantom{x^3} \phantom{- 3x^2} \phantom{+ 6x} + 8 \\ \hline \phantom{x^4} \phantom{x^3} \phantom{- 3x^2} \phantom{+ 6x} \phantom{+ 8} 8 \end{array}
 \end{array}$$

Quotient  $q(x) = x^2 + x - 3$

Remainder  $r(x) = 8$

8. If  $f(x) = x^4 - 5x + 6$  is divided by  $g(x) = 2 - x^2$ .

**Sol:**

We can write

$f(x)$  as  $x^4 + 0x^3 + 0x^2 - 5x + 6$  and  $g(x)$  as  $-x^2 + 2$

$$\begin{array}{r}
 \phantom{-x^2 + 2} \overline{-x^2 - 2} \\
 -x^2 + 2 \overline{) \begin{array}{r} x^4 + 0x^3 + 0x^2 - 5x + 6 \\ x^4 \phantom{+ 0x^3} - 2x^2 \\ \hline \phantom{x^4} \phantom{+ 0x^3} \phantom{+ 0x^2} - 5x + 6 \\ \phantom{x^4} \phantom{+ 0x^3} \phantom{+ 0x^2} - 4 \\ \hline \phantom{x^4} \phantom{+ 0x^3} \phantom{+ 0x^2} - 5x + 10 \end{array}
 \end{array}$$

Quotient  $q(x) = -x^2 - 2$

Remainder  $r(x) = -5x + 10$

9. By actual division, show that  $x^2 - 3$  is a factor of  $2x^4 + 3x^3 - 2x^2 - 9x - 12$ .

**Sol:**

Let  $f(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$  and  $g(x)$  as  $x^2 - 3$

$$\begin{array}{r}
 x^2 - 3 \quad \overline{) 2x^4 + 3x^3 - 2x^2 - 9x - 12} \\
 \underline{2x^4 \phantom{+ 3x^3} - 6x^2} \phantom{- 9x - 12} \\
 - \phantom{2x^4} + \phantom{3x^3} \phantom{- 2x^2} \phantom{- 9x} - 12 \\
 \underline{3x^3 + 4x^2 - 9x - 12} \\
 3x^3 \phantom{+ 4x^2} - 9x \phantom{- 12} \\
 \underline{- \phantom{3x^3} + \phantom{4x^2} \phantom{- 9x} \phantom{- 12}} \\
 4x^2 - 12 \\
 \underline{4x^2 - 12} \\
 - \phantom{4x^2} + \phantom{- 12} \\
 \underline{\phantom{- 4x^2} \phantom{+ 12}} \\
 x
 \end{array}$$

Quotient  $q(x) = 2x^2 + 3x + 4$

Remainder  $r(x) = 0$

Since, the remainder is 0.

Hence,  $x^2 - 3$  is a factor of  $2x^4 + 3x^3 - 2x^2 - 9x - 12$

10. On dividing  $3x^3 + x^2 + 2x + 5$  is divided by a polynomial  $g(x)$ , the quotient and remainder are  $(3x - 5)$  and  $(9x + 10)$  respectively. Find  $g(x)$ .

**Sol:**

By using division rule, we have

Dividend = Quotient  $\times$  Divisor + Remainder

$$\therefore 3x^3 + x^2 + 2x + 5 = (3x - 5)g(x) + 9x + 10$$

$$\Rightarrow 3x^3 + x^2 + 2x + 5 - 9x - 10 = (3x - 5)g(x)$$

$$\Rightarrow 3x^3 + x^2 - 7x - 5 = (3x - 5)g(x)$$

$$\Rightarrow g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$$

$$\begin{array}{r}
 3x - 5 \quad \overline{) 3x^3 + x^2 - 7x - 5} \\
 \underline{3x^3 - 5x^2} \phantom{- 7x - 5} \\
 - \phantom{3x^3} + \phantom{- 5x^2} \phantom{- 7x} - 5 \\
 \underline{6x^2 - 7x - 5} \\
 6x^2 - 10x \phantom{- 5} \\
 \underline{- \phantom{6x^2} + \phantom{- 10x} \phantom{- 5}} \\
 3x - 5 \\
 \underline{3x - 5} \\
 - \phantom{3x} + \phantom{- 5} \\
 \underline{\phantom{- 3x} \phantom{+ 5}} \\
 X
 \end{array}$$

$$\therefore g(x) = x^2 + 2x + 1$$



$$\begin{aligned}
 f(x) &= x^3 + 2x^2 - 11x - 12 \\
 &= (x + 1)(x^2 + x - 12) \\
 &= (x + 1)\{x^2 + 4x - 3x - 12\} \\
 &= (x + 1)\{x(x+4) - 3(x+4)\} \\
 &= (x + 1)(x - 3)(x + 4)
 \end{aligned}$$

$$\begin{aligned}
 \therefore f(x) = 0 &\Rightarrow (x + 1)(x - 3)(x + 4) = 0 \\
 &\Rightarrow (x + 1) = 0 \text{ or } (x - 3) = 0 \text{ or } (x + 4) = 0 \\
 &\Rightarrow x = -1 \text{ or } x = 3 \text{ or } x = -4
 \end{aligned}$$

Thus, all the zeroes are  $-1, 3$  and  $-4$ .

13. If  $1$  and  $-2$  are two zeroes of the polynomial  $(x^3 - 4x^2 - 7x + 10)$ , find its third zero.

**Sol:**

$$\text{Let } f(x) = x^3 - 4x^2 - 7x + 10$$

Since  $1$  and  $-2$  are the zeroes of  $f(x)$ , it follows that each one of  $(x-1)$  and  $(x+2)$  is a factor of  $f(x)$ .

Consequently,  $(x-1)(x+2) = (x^2 + x - 2)$  is a factor of  $f(x)$ .

On dividing  $f(x)$  by  $(x^2 + x - 2)$ , we get:

$$\begin{array}{r}
 x^2 + x - 2 \overline{) x^3 - 4x^2 - 7x + 10} \quad (x - 5 \\
 \underline{x^3 + x^2 - 2x} \phantom{+ 10} \\
 -5x^2 - 5x + 10 \\
 \underline{-5x^2 - 5x + 10} \\
 + \phantom{+} - \\
 \hline
 X
 \end{array}$$

$$\begin{aligned}
 f(x) = 0 &\Rightarrow (x^2 + x - 2)(x - 5) = 0 \\
 &\Rightarrow (x - 1)(x + 2)(x - 5) = 0 \\
 &\Rightarrow x = 1 \text{ or } x = -2 \text{ or } x = 5
 \end{aligned}$$

Hence, the third zero is  $5$ .

14. If  $3$  and  $-3$  are two zeroes of the polynomial  $(x^4 + x^3 - 11x^2 - 9x + 18)$ , find all the zeroes of the given polynomial.

**Sol:**

$$\text{Let } x^4 + x^3 - 11x^2 - 9x + 18$$

Since  $3$  and  $-3$  are the zeroes of  $f(x)$ , it follows that each one of  $(x + 3)$  and  $(x - 3)$  is a factor of  $f(x)$ .

Consequently,  $(x - 3)(x + 3) = (x^2 - 9)$  is a factor of  $f(x)$ .

On dividing  $f(x)$  by  $(x^2 - 9)$ , we get:

$$\begin{array}{r}
 x^2 - 9 \overline{) x^4 + x^3 - 11x^2 - 9x + 18} \left( x^2 + x - 2 \right. \\
 \underline{x^4 \quad \quad - 9x^2} \\
 \quad \quad \quad + \\
 \quad \quad \quad \underline{x^3 - 2x^2 - 9x + 18} \\
 \quad \quad \quad x^3 \quad \quad - 9x \\
 \quad \quad \quad \underline{-} \quad \quad \quad + \\
 \quad \quad \quad \quad \quad \quad -2x^2 + 18 \\
 \quad \quad \quad \quad \quad \quad -2x^2 + 18 \\
 \quad \quad \quad \quad \quad \quad \underline{+} \quad \quad \quad - \\
 \quad \quad \quad \quad \quad \quad \quad \quad \underline{x} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \underline{\quad}
 \end{array}$$

$$\begin{aligned}
 f(x) = 0 &\Rightarrow (x^2 + x - 2)(x^2 - 9) = 0 \\
 &\Rightarrow (x^2 + 2x - x - 2)(x - 3)(x + 3) \\
 &\Rightarrow (x - 1)(x + 2)(x - 3)(x + 3) = 0 \\
 &\Rightarrow x = 1 \text{ or } x = -2 \text{ or } x = 3 \text{ or } x = -3
 \end{aligned}$$

Hence, all the zeroes are 1, -2, 3 and -3.

15. If 2 and -2 are two zeroes of the polynomial  $(x^4 + x^3 - 34x^2 - 4x + 120)$ , find all the zeroes of the given polynomial.

**Sol:**

$$\text{Let } f(x) = x^4 + x^3 - 34x^2 - 4x + 120$$

Since 2 and -2 are the zeroes of  $f(x)$ , it follows that each one of  $(x - 2)$  and  $(x + 2)$  is a factor of  $f(x)$ .

Consequently,  $(x - 2)(x + 2) = (x^2 - 4)$  is a factor of  $f(x)$ .

On dividing  $f(x)$  by  $(x^2 - 4)$ , we get:

$$\begin{array}{r}
 x^2 - 4 \overline{) x^4 + x^3 - 34x^2 - 4x + 120} \left( x^2 + x - 2 \right. \\
 \underline{x^4 \quad \quad - 4x^2} \\
 \quad \quad \quad + \\
 \quad \quad \quad \underline{x^3 - 30x^2 - 4x + 120} \\
 \quad \quad \quad x^3 \quad \quad - 4x \\
 \quad \quad \quad \underline{-} \quad \quad \quad + \\
 \quad \quad \quad \quad \quad \quad -30x^2 + 120 \\
 \quad \quad \quad \quad \quad \quad -30x^2 + 120 \\
 \quad \quad \quad \quad \quad \quad \underline{+} \quad \quad \quad - \\
 \quad \quad \quad \quad \quad \quad \quad \quad \underline{x} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \underline{\quad}
 \end{array}$$

$$f(x) = 0$$

$$\Rightarrow (x^2 + x - 30)(x^2 - 4) = 0$$

$$\Rightarrow (x^2 + 6x - 5x - 30)(x - 2)(x + 2)$$

$$\Rightarrow [x(x + 6) - 5(x + 6)](x - 2)(x + 2)$$

$$\Rightarrow (x - 5)(x + 6)(x - 2)(x + 2) = 0$$

$$\Rightarrow x = 5 \text{ or } x = -6 \text{ or } x = 2 \text{ or } x = -2$$

Hence, all the zeroes are 2, -2, 5 and -6.

16. Find all the zeroes of  $(x^4 + x^3 - 23x^2 - 3x + 60)$ , if it is given that two of its zeroes are  $\sqrt{3}$  and  $-\sqrt{3}$ .

**Sol:**

$$\text{Let } f(x) = x^4 + x^3 - 23x^2 - 3x + 60$$

Since  $\sqrt{3}$  and  $-\sqrt{3}$  are the zeroes of  $f(x)$ , it follows that each one of  $(x - \sqrt{3})$  and  $(x + \sqrt{3})$  is a factor of  $f(x)$ .

Consequently,  $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$  is a factor of  $f(x)$ .

On dividing  $f(x)$  by  $(x^2 - 3)$ , we get:

$$\begin{array}{r}
 x^2 - 3 \overline{) x^4 + x^3 - 23x^2 - 3x + 60} \left( x^2 + x - 20 \right. \\
 \underline{x^4 \phantom{+ x^3} - 3x^2} \phantom{- 3x + 60} \\
 \phantom{x^4} + \phantom{- 3x^2} \phantom{- 3x + 60} \\
 \underline{\phantom{x^4} x^3 - 20x^2 - 3x + 60} \\
 \phantom{x^4} \phantom{+ x^3} - 3x \phantom{+ 60} \\
 \underline{\phantom{x^4} \phantom{+ x^3} + \phantom{60}} \\
 \phantom{x^4} \phantom{+ x^3} - 20x^2 + 60 \\
 \underline{\phantom{x^4} \phantom{+ x^3} - 20x^2 + 60} \\
 \phantom{x^4} \phantom{+ x^3} \phantom{- 20x^2} + \phantom{60} \\
 \underline{\phantom{x^4} \phantom{+ x^3} \phantom{- 20x^2} - \phantom{60}} \\
 \phantom{x^4} \phantom{+ x^3} \phantom{- 20x^2} \phantom{+ 60} x
 \end{array}$$

$$f(x) = 0$$

$$\Rightarrow (x^2 + x - 20)(x^2 - 3) = 0$$

$$\Rightarrow (x^2 + 5x - 4x - 20)(x^2 - 3)$$

$$\Rightarrow [x(x + 5) - 4(x + 5)](x^2 - 3)$$

$$\Rightarrow (x - 4)(x + 5)(x - \sqrt{3})(x + \sqrt{3}) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -5 \text{ or } x = \sqrt{3} \text{ or } x = -\sqrt{3}$$

Hence, all the zeroes are  $\sqrt{3}$ ,  $-\sqrt{3}$ , 4 and -5.

17. Find all the zeroes of  $(2x^4 - 3x^3 - 5x^2 + 9x - 3)$ , it is being given that two of its zeroes are  $\sqrt{3}$  and  $-\sqrt{3}$ .

**Sol:**

The given polynomial is  $f(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 3$

Since  $\sqrt{3}$  and  $-\sqrt{3}$  are the zeroes of  $f(x)$ , it follows that each one of  $(x - \sqrt{3})$  and  $(x + \sqrt{3})$  is a factor of  $f(x)$ .

Consequently,  $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$  is a factor of  $f(x)$ .

On dividing  $f(x)$  by  $(x^2 - 3)$ , we get:

$$\begin{array}{r}
 x^2 - 3 \overline{) 2x^4 - 3x^3 - 5x^2 + 9x - 3} \quad \left( 2x^2 - 3x + 1 \right. \\
 \underline{2x^4 \phantom{- 3x^3} - 6x^2} \phantom{+ 9x - 3} \\
 -3x^3 + x^2 + 9x - 3 \\
 \underline{-3x^3 \phantom{+ x^2} + 9x} \phantom{- 3} \\
 + \phantom{-3x^3 +} x^2 - 3 \\
 \phantom{+} \underline{x^2 - 3} \\
 \phantom{+} \phantom{x^2 - 3} - \phantom{+} \\
 \phantom{+} \phantom{x^2 - 3} \phantom{-} x
 \end{array}$$

$$f(x) = 0$$

$$\Rightarrow 2x^4 - 3x^3 - 5x^2 + 9x - 3 = 0$$

$$\Rightarrow (x^2 - 3)(2x^2 - 3x + 1) = 0$$

$$\Rightarrow (x^2 - 3)(2x^2 - 2x - x + 1) = 0$$

$$\Rightarrow (x - \sqrt{3})(x + \sqrt{3})(2x - 1)(x - 1) = 0$$

$$\Rightarrow x = \sqrt{3} \text{ or } x = -\sqrt{3} \text{ or } x = \frac{1}{2} \text{ or } x = 1$$

Hence, all the zeroes are  $\sqrt{3}$ ,  $-\sqrt{3}$ ,  $\frac{1}{2}$  and 1.

18. Obtain all other zeroes of  $(x^4 + 4x^3 - 2x^2 - 20x - 15)$  if two of its zeroes are  $\sqrt{5}$  and  $-\sqrt{5}$ .

**Sol:**

The given polynomial is  $f(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$ .

Since  $(x - \sqrt{5})$  and  $(x + \sqrt{5})$  are the zeroes of  $f(x)$  it follows that each one of  $(x - \sqrt{5})$  and  $(x + \sqrt{5})$  is a factor of  $f(x)$ .

Consequently,  $(x - \sqrt{5})(x + \sqrt{5}) = (x^2 - 5)$  is a factor of  $f(x)$ .

On dividing  $f(x)$  by  $(x^2 - 5)$ , we get:



$$\frac{\begin{array}{ccc} + & - & + \\ \hline & x & \\ \hline \end{array}}{\hline}$$

$$f(x) = 0$$

$$\Rightarrow 2x^4 - 11x^3 + 7x^2 + 13x - 7 = 0$$

$$\Rightarrow (x^2 - 6x + 7)(2x^2 + x - 7) = 0$$

$$\Rightarrow (x + 3 + \sqrt{2})(x + 3 - \sqrt{2})(2x - 1)(x + 1) = 0$$

$$\Rightarrow x = -3 - \sqrt{2} \text{ or } x = -3 + \sqrt{2} \text{ or } x = \frac{1}{2} \text{ or } x = -1$$

Hence, all the zeroes are  $(-3 - \sqrt{2})$ ,  $(-3 + \sqrt{2})$ ,  $\frac{1}{2}$  and  $-1$ .

### Exercise – 2C

1. If one zero of the polynomial  $x^2 - 4x + 1$  is  $(2 + \sqrt{3})$ , write the other zero.

**Sol:**

Let the other zeroes of  $x^2 - 4x + 1$  be  $a$ .

By using the relationship between the zeroes of the quadratic polynomial.

We have, sum of zeroes =  $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

$$\therefore 2 + \sqrt{3} + a = \frac{-(-4)}{1}$$

$$\Rightarrow a = 2 - \sqrt{3}$$

Hence, the other zeroes of  $x^2 - 4x + 1$  is  $2 - \sqrt{3}$ .

2. Find the zeroes of the polynomial  $x^2 + x - p(p + 1)$

**Sol:**

$$f(x) = x^2 + x - p(p + 1)$$

By adding and subtracting  $px$ , we get

$$f(x) = x^2 + px + x - px - p(p + 1)$$

$$= x^2 + (p + 1)x - px - p(p + 1)$$

$$= x[x + (p + 1)] - p[x + (p + 1)]$$

$$= [x + (p + 1)](x - p)$$

$$f(x) = 0$$

$$\Rightarrow [x + (p + 1)](x - p) = 0$$

$$\Rightarrow [x + (p + 1)] = 0 \text{ or } (x - p) = 0$$

$$\Rightarrow x = -(p + 1) \text{ or } x = p$$

So, the zeroes of  $f(x)$  are  $-(p + 1)$  and  $p$ .

3. Find the zeroes of the polynomial  $x^2 - 3x - m(m + 3)$

**Sol:**

$$f(x) = x^2 - 3x - m(m + 3)$$

By adding and subtracting  $mx$ , we get

$$\begin{aligned} f(x) &= x^2 - mx - 3x + mx - m(m + 3) \\ &= x[x - (m + 3)] + m[x - (m + 3)] \\ &= [x - (m + 3)](x + m) \end{aligned}$$

$$f(x) = 0 \Rightarrow [x - (m + 3)](x + m) = 0$$

$$\Rightarrow [x - (m + 3)] = 0 \text{ or } (x + m) = 0$$

$$\Rightarrow x = m + 3 \text{ or } x = -m$$

So, the zeroes of  $f(x)$  are  $-m$  and  $+3$ .

4. Find  $\alpha, \beta$  are the zeros of polynomial  $\alpha + \beta = 6$  and  $\alpha\beta = 4$  then write the polynomial.

**Sol:**

If the zeroes of the quadratic polynomial are  $\alpha$  and  $\beta$  then the quadratic polynomial can be found as  $x^2 - (\alpha + \beta)x + \alpha\beta$  .....(1)

Substituting the values in (1), we get

$$x^2 - 6x + 4$$

5. If one zero of the quadratic polynomial  $kx^2 + 3x + k$  is 2, then find the value of  $k$ .

**Sol:**

Given:  $x = 2$  is one zero of the quadratic polynomial  $kx^2 + 3x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$$k(2)^2 + 3(2) + k = 0$$

$$\Rightarrow 4k + 6 + k = 0$$

$$\Rightarrow 5k + 6 = 0$$

$$\Rightarrow k = -\frac{6}{5}$$

6. If 3 is a zero of the polynomial  $2x^2 + x + k$ , find the value of  $k$ .

**Sol:**

Given:  $x = 3$  is one zero of the polynomial  $2x^2 + x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$$2(3)^2 + 3 + k = 0$$

$$\Rightarrow 21 + k = 0$$

$$\Rightarrow k = -21$$

7. If  $-4$  is a zero of the polynomial  $x^2 - x - (2k + 2)$  is  $-4$ , then find the value of  $k$ .

**Sol:**

Given:  $x = -4$  is one zero of the polynomial  $x^2 - x - (2k + 2)$

Therefore, it will satisfy the above polynomial.

Now, we have

$$(-4)^2 - (-4) - (2k + 2) = 0$$

$$\Rightarrow 16 + 4 - 2k - 2 = 0$$

$$\Rightarrow 2k = -18$$

$$\Rightarrow k = 9$$

8. If  $1$  is a zero of the quadratic polynomial  $ax^2 - 3(a - 1)x - 1$  is  $1$ , then find the value of  $a$ .

**Sol:**

Given:  $x = 1$  is one zero of the polynomial  $ax^2 - 3(a - 1)x - 1$

Therefore, it will satisfy the above polynomial.

Now, we have

$$a(1)^2 - (a - 1)1 - 1 = 0$$

$$\Rightarrow a - 3a + 3 - 1 = 0$$

$$\Rightarrow -2a = -2$$

$$\Rightarrow a = 1$$

9. If  $-2$  is a zero of the polynomial  $3x^2 + 4x + 2k$  then find the value of  $k$ .

**Sol:**

Given:  $x = -2$  is one zero of the polynomial  $3x^2 + 4x + 2k$

Therefore, it will satisfy the above polynomial.

Now, we have

$$3(-2)^2 + 4(-2)1 + 2k = 0$$

$$\Rightarrow 12 - 8 + 2k = 0$$

$$\Rightarrow k = -2$$

10. Write the zeros of the polynomial  $f(x) = x^2 - x - 6$ .

**Sol:**

$$f(x) = x^2 - x - 6$$

$$= x^2 - 3x + 2x - 6$$

$$= x(x - 3) + 2(x - 3)$$

$$= (x - 3)(x + 2)$$

$$f(x) = 0 \Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow (x - 3) = 0 \text{ or } (x + 2) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -2$$

So, the zeroes of  $f(x)$  are 3 and  $-2$ .

11. If the sum of the zeros of the quadratic polynomial  $kx^2 - 3x + 5$  is 1 write the value of  $k$ .

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\Rightarrow 1 = \frac{-(-3)}{k}$$

$$\Rightarrow k = 3$$

12. If the product of the zero of the polynomial  $(x^2 - 4x + k)$  is 3. Find the value of  $k$ .

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\Rightarrow 3 = \frac{k}{1}$$

$$\Rightarrow k = 3$$

13. If  $(x + a)$  is a factor of  $(2x^2 + 2ax + 5x + 10)$ , then find the value of  $a$ .

**Sol:**

Given:  $(x + a)$  is a factor of  $2x^2 + 2ax + 5x + 10$

We have

$$x + a = 0$$

$$\Rightarrow x = -a$$

Since,  $(x + a)$  is a factor of  $2x^2 + 2ax + 5x + 10$

Hence, It will satisfy the above polynomial

$$\therefore 2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$

$$\Rightarrow -5a + 10 = 0$$

$$\Rightarrow a = 2$$

14. If  $(a-b)$ ,  $a$  and  $(a + b)$  are zeros of the polynomial  $2x^3 - 6x^2 + 5x - 7$  write the value of  $a$ .

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$$

$$\Rightarrow a - b + a + a + b = \frac{-(-6)}{2}$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

15. If  $x^3 + x^2 - ax + b$  is divisible by  $(x^2 - x)$ , write the value of  $a$  and  $b$ .

**Sol:**

Equating  $x^2 - x$  to 0 to find the zeroes, we will get

$$x(x - 1) = 0$$

$$\Rightarrow x = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1$$

Since,  $x^3 + x^2 - ax + b$  is divisible by  $x^2 - x$ .

Hence, the zeroes of  $x^2 - x$  will satisfy  $x^3 + x^2 - ax + b$

$$\therefore (0)^3 + 0^2 - a(0) + b = 0$$

$$\Rightarrow b = 0$$

And

$$(1)^3 + 1^2 - a(1) + 0 = 0 \quad [ \because b = 0 ]$$

$$\Rightarrow a = 2$$

16. If  $\alpha$  and  $\beta$  be the zeroes of the polynomial  $2x^2 - 7x + k$  write the value of  $(\alpha + \beta + \alpha\beta)$ .

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\therefore \alpha + \beta = \frac{-7}{2} \text{ and } \alpha\beta = \frac{5}{2}$$

$$\text{Now, } \alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = -1$$

17. State Division Algorithm for Polynomials.

**Sol:**

“If  $f(x)$  and  $g(x)$  are two polynomials such that degree of  $f(x)$  is greater than degree of  $g(x)$  where  $g(x) \neq 0$ , there exists unique polynomials  $q(x)$  and  $r(x)$  such that

$$f(x) = g(x) \times q(x) + r(x),$$

where  $r(x) = 0$  or degree of  $r(x) <$  degree of  $g(x)$ .

18. Find the sum of the zeros and the product of zeros of a quadratic polynomial, are  $-\frac{1}{2}$  and  $-3$  respectively. Write the polynomial.

**Sol:**

We can find the quadratic polynomial if we know the sum of the roots and product of the roots by using the formula

$$x^2 - (\text{sum of the zeroes})x + \text{product of zeroes}$$

$$\Rightarrow x^2 - \left(-\frac{1}{2}\right)x + (-3)$$

$$\Rightarrow x^2 + \frac{1}{2}x - 3$$

Hence, the required polynomial is  $x^2 + \frac{1}{2}x - 3$ .

19. Find the zeroes of the quadratic polynomial  $f(x) = 6x^2 - 3$ .

**Sol:**

To find the zeroes of the quadratic polynomial we will equate  $f(x)$  to 0

$$\therefore f(x) = 0$$

$$\Rightarrow 6x^2 - 3 = 0$$

$$\Rightarrow 3(2x^2 - 1) = 0$$

$$\Rightarrow 2x^2 - 1 = 0$$

$$\Rightarrow 2x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

Hence, the zeroes of the quadratic polynomial  $f(x) = 6x^2 - 3$  are  $\frac{1}{\sqrt{2}}$ ,  $-\frac{1}{\sqrt{2}}$ .

20. Find the zeroes of the quadratic polynomial  $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ .

**Sol:**

To find the zeroes of the quadratic polynomial we will equate  $f(x)$  to 0

$$\therefore f(x) = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$\Rightarrow (\sqrt{3}x + 2) = 0 \text{ or } (4x - \sqrt{3}) = 0$$

$$\Rightarrow x = -\frac{2}{\sqrt{3}} \text{ or } x = \frac{\sqrt{3}}{4}$$

Hence, the zeroes of the quadratic polynomial  $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$  are  $-\frac{2}{\sqrt{3}}$  or  $\frac{\sqrt{3}}{4}$

21. If  $\alpha, \beta$  are the zeroes of the polynomial  $f(x) = x^2 - 5x + k$  such that  $\alpha - \beta = 1$ , find the value of  $k = ?$

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\therefore \alpha + \beta = \frac{-(-5)}{1} \text{ and } \alpha\beta = \frac{k}{1}$$

$$\Rightarrow \alpha + \beta = 5 \text{ and } \alpha\beta = \frac{k}{1}$$

Solving  $\alpha - \beta = 1$  and  $\alpha + \beta = 5$ , we will get

$$\alpha = 3 \text{ and } \beta = 2$$

Substituting these values in  $\alpha\beta = \frac{k}{1}$ , we will get

$$k = 6$$

22. If  $\alpha$  and  $\beta$  are the zeros of the polynomial  $f(x) = 6x^2 + x - 2$  find the value of  $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\therefore \alpha + \beta = \frac{-1}{6} \text{ and } \alpha\beta = -\frac{2}{6}$$

$$\text{Now, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{-1}{6}\right)^2 - 2\left(-\frac{2}{6}\right)}{-\frac{2}{6}}$$

$$= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}}$$

$$= -\frac{25}{12}$$

23. If  $\alpha, \beta$  are the zeroes of the polynomial  $f(x) = 5x^2 - 7x + 1$ , then  $\frac{1}{\alpha} + \frac{1}{\beta} = ?$

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes =  $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$  and Product of zeroes =  $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$$\therefore \alpha + \beta = \frac{-(-7)}{5} \text{ and } \alpha\beta = \frac{1}{5}$$

$$\Rightarrow \alpha + \beta = \frac{7}{5} \text{ and } \alpha\beta = \frac{1}{5}$$

$$\begin{aligned} \text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{\frac{7}{5}}{\frac{1}{5}} \\ &= 7 \end{aligned}$$

24. If  $\alpha, \beta$  are the zeroes of the polynomial  $f(x) = x^2 + x - 2$ , then  $\left(\frac{\alpha}{\beta} - \frac{\alpha}{\beta}\right)$ .

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes =  $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$  and Product of zeroes =  $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$$\therefore \alpha + \beta = \frac{-1}{1} \text{ and } \alpha\beta = \frac{-2}{1}$$

$$\Rightarrow \alpha + \beta = -1 \text{ and } \alpha\beta = -2$$

$$\begin{aligned} \text{Now, } \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 &= \left(\frac{\beta - \alpha}{\alpha\beta}\right)^2 \\ &= \frac{(\alpha + \beta)^2 - 4\alpha\beta}{(\alpha\beta)^2} \quad [\because (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta] \end{aligned}$$

$$= \frac{(-1)^2 - 4(-2)}{(-2)^2} \quad [\because \alpha + \beta = -1 \text{ and } \alpha\beta = -2]$$

$$= \frac{(-1)^2 - 4(-2)}{4}$$

$$= \frac{9}{4}$$

$$\therefore \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \pm \frac{3}{2}$$

25. If the zeroes of the polynomial  $f(x) = x^3 - 3x^2 + x + 1$  are  $(a - b)$ ,  $a$  and  $(a + b)$ , find the values of  $a$  and  $b$ .

**Sol:**

By using the relationship between the zeroes of the quadratic polynomial.

We have, Sum of zeroes =  $\frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$

$$\therefore a - b + a + a + b = \frac{-(-3)}{1}$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

$$\text{Now, Product of zeroes} = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

$$\therefore (a - b)(a)(a + b) = \frac{-1}{1}$$

$$\Rightarrow (1 - b)(1)(1 + b) = -1 \quad [\because a = 1]$$

$$\Rightarrow 1 - b^2 = -1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

### Exercise – MCQ

1. Which of the following is a polynomial?

(a)  $x^2 - 5x + 6\sqrt{x} + 3$

(b)  $x^{3/2} - x + x^{1/2} + 1$

(c)  $\sqrt{x} + \frac{1}{\sqrt{x}}$

(d) None of these

**Sol:**

(d) none of these

A polynomial in  $x$  of degree  $n$  is an expression of the form  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ , where  $a_n \neq 0$ .

2. Which of the following is not a polynomial?

(a)  $\sqrt{3}x^2 - 2\sqrt{3}x + 5$

(b)  $9x^2 - 4x + \sqrt{2}$

(c)  $\frac{3}{2}x^3 + 6x^2 - \frac{1}{\sqrt{2}}x - 8$

(d)  $x + \frac{3}{x}$

**Sol:**

(d)  $x + \frac{3}{x}$  is not a polynomial.

It is because in the second term, the degree of  $x$  is  $-1$  and an expression with a negative degree is not a polynomial.

3. The Zeroes of the polynomial  $x^2 - 2x - 3$  are

(a)  $-3, 1$

(b)  $-3, -1$

(c)  $3, -1$

(d)  $3, 1$

**Sol:**

(c)  $3, -1$

$$\text{Let } f(x) = x^2 - 2x - 3 = 0$$

$$= x^2 - 3x + x - 3 = 0$$

$$= x(x - 3) + 1(x - 3) = 0$$

$$= (x - 3)(x + 1) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -1$$

4. The zeroes of the polynomial  $x^2 - \sqrt{2}x - 12$  are  
 (a)  $\sqrt{2}, -\sqrt{2}$     (b)  $3\sqrt{2}, -2\sqrt{2}$     (c)  $-3\sqrt{2}, 2\sqrt{2}$     (d)  $3\sqrt{2}, 2\sqrt{2}$

**Sol:**

(b)  $3\sqrt{2}, -2\sqrt{2}$

Let  $f(x) = x^2 - \sqrt{2}x - 12 = 0$

$$\Rightarrow x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12 = 0$$

$$\Rightarrow x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2}) = 0$$

$$\Rightarrow (x - 3\sqrt{2})(x + 2\sqrt{2}) = 0$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = -2\sqrt{2}$$

5. The zeroes of the polynomial  $4x^2 + 5\sqrt{2}x - 3$  are  
 (a)  $-3\sqrt{2}, \sqrt{2}$     (b)  $-3\sqrt{2}, \frac{\sqrt{2}}{2}$     (c)  $\frac{-3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$     (d) none of these

**Sol:**

(c)  $-\frac{3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$

Let  $f(x) = 4x^2 + 5\sqrt{2}x - 3 = 0$

$$\Rightarrow 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 = 0$$

$$\Rightarrow 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3) = 0$$

$$\Rightarrow (\sqrt{2}x + 3)(2\sqrt{2}x - 1) = 0$$

$$\Rightarrow x = -\frac{3}{\sqrt{2}} \text{ or } x = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow x = -\frac{3}{\sqrt{2}} \text{ or } x = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

6. The zeros of the polynomial  $x^2 + \frac{1}{6}x - 2$  are  
 (a)  $-3, 4$     (b)  $\frac{-3}{2}, \frac{4}{3}$     (c)  $\frac{-4}{3}, \frac{3}{2}$     (d) none of these

**Sol:**

(b)  $\frac{-3}{2}, \frac{4}{3}$

Let  $f(x) = x^2 + \frac{1}{6}x - 2 = 0$

$$\Rightarrow 6x^2 + x - 12 = 0$$

$$\Rightarrow 6x^2 + 9x - 8x - 12 = 0$$

$$\Rightarrow 3x(2x + 3) - 4(2x + 3) = 0$$

$$\Rightarrow (2x + 3)(3x - 4) = 0$$

$$\therefore x = \frac{-3}{2} \text{ or } x = \frac{4}{3}$$

7. The zeros of the polynomial  $7x^2 - \frac{11}{3}x - \frac{2}{3}$  are  
 (a)  $\frac{2}{3}, \frac{-1}{7}$       (b)  $\frac{2}{7}, \frac{-1}{3}$       (c)  $\frac{-2}{3}, \frac{1}{7}$       (d) none of these

**Sol:**

(a)  $\frac{2}{3}, \frac{-1}{7}$

Let  $f(x) = 7x^2 - \frac{11}{3}x - \frac{2}{3} = 0$

$\Rightarrow 21x^2 - 11x - 2 = 0$

$\Rightarrow 21x^2 - 14x + 3x - 2 = 0$

$\Rightarrow 7x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow (3x - 2)(7x + 1) = 0$

$\Rightarrow x = \frac{2}{3}$  or  $x = \frac{-1}{7}$

8. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively.  
 The quadratic polynomial is

(a)  $x^2 - 3x + 10$       (b)  $x^2 + 3x - 10$       (c)  $x^2 - 3x - 10$       (d)  $x^2 + 3x + 10$

**Sol:**

(c)  $x^2 - 3x - 10$

Given: Sum of zeroes,  $\alpha + \beta = 3$

Also, product of zeroes,  $\alpha\beta = -10$

$\therefore$  Required polynomial  $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$

9. A quadratic polynomial whose zeroes are 5 and -3, is  
 (a)  $x^2 + 2x - 15$       (b)  $x^2 - 2x + 15$       (c)  $x^2 - 2x - 15$       (d) none of these

**Sol:**

(c)  $x^2 - 2x - 15$

Here, the zeroes are 5 and -3.

Let  $\alpha = 5$  and  $\beta = -3$

So, sum of the zeroes,  $\alpha + \beta = 5 + (-3) = 2$

Also, product of the zeroes,  $\alpha\beta = 5 \times (-3) = -15$

The polynomial will be  $x^2 - (\alpha + \beta)x + \alpha\beta$

$\therefore$  The required polynomial is  $x^2 - 2x - 15$ .

10. A quadratic polynomial whose zeroes are  $\frac{3}{5}$  and  $\frac{-1}{2}$ , is  
 (a)  $10x^2 + x + 3$       (b)  $10x^2 + x - 3$       (c)  $10x^2 - x + 3$       (d)  $x^2 - \frac{1}{10}x - \frac{3}{10}$

**Sol:**



14. If one zero of the quadratic polynomial  $kx^2 + 3x + k$  is 2, then the value of k is  
 (a)  $\frac{5}{6}$       (b)  $\frac{-5}{6}$       (c)  $\frac{6}{5}$       (d)  $\frac{-6}{5}$

**Sol:**

(d)  $\frac{-6}{5}$

Since 2 is a zero of  $kx^2 + 3x + k$ , we have:

$$k \times (2)^2 + 3(2) + k = 0$$

$$\Rightarrow 4k + k + 6 = 0$$

$$\Rightarrow 5k = -6$$

$$\Rightarrow k = \frac{-6}{5}$$

15. If one zero of the quadratic polynomial  $(k - 1)x^2 - kx + 1$  is -4, then the value of k is  
 (a)  $\frac{-5}{4}$       (b)  $\frac{5}{4}$       (c)  $\frac{-4}{3}$       (d)  $\frac{4}{3}$

**Sol:**

(b)  $\frac{5}{4}$

Since -4 is a zero of  $(k - 1)x^2 - kx + 1$ , we have:

$$(k - 1) \times (-4)^2 - k \times (-4) + 1 = 0$$

$$\Rightarrow 16k - 16 - 4k + 1 = 0$$

$$\Rightarrow 12k - 15 = 0$$

$$5$$

$$\Rightarrow k = \frac{-15}{12}$$

$$4$$

$$\Rightarrow k = \frac{5}{4}$$

16. If -2 and 3 are the zeroes of the quadratic polynomial  $x^2 + (a + 1)x + b$ , then  
 (a)  $a = -2, b = 6$       (b)  $a = 2, b = -6$   
 (c)  $a = -2, b = -6$       (d)  $a = 2, b = 6$

**Sol:**

(c)  $a = -2, b = -6$

Given: -2 and 3 are the zeroes of  $x^2 + (a + 1)x + b$ .

$$\text{Now, } (-2)^2 + (a + 1) \times (-2) + b = 0 \Rightarrow 4 - 2a - 2 + b = 0$$

$$\Rightarrow b - 2a = -2 \quad \dots(1)$$

$$\text{Also, } 3^2 + (a + 1) \times 3 + b = 0 \Rightarrow 9 + 3a + 3 + b = 0$$

$$\Rightarrow b + 3a = -12 \quad \dots(2)$$

On subtracting (1) from (2), we get  $a = -2$

$$\therefore b = -2 - 4 = -6 \quad [\text{From (1)}]$$

17. If one zero of  $3x^2 - 8x + k$  be the reciprocal of the other, then  $k = ?$

(a) 3                      (b) -3                      (c)  $\frac{1}{3}$                       (d)  $\frac{-1}{3}$

**Sol:**

(a)  $k = 3$

Let  $\alpha$  and  $\frac{1}{\alpha}$  be the zeroes of  $3x^2 - 8x + k$ .

Then the product of zeroes =  $\frac{k}{3}$

$$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{3}$$

$$\Rightarrow 1 = \frac{k}{3}$$

$$\Rightarrow k = 3$$

18. If the sum of the zeroes of the quadratic polynomial  $kx^2 + 2x + 3k$  is equal to the product of its zeroes, then  $k = ?$

(a)  $\frac{1}{3}$                       (b)  $\frac{-1}{3}$                       (c)  $\frac{2}{3}$                       (d)  $\frac{-2}{3}$

**Sol:**

(d)  $\frac{-2}{3}$

Let  $\alpha$  and  $\beta$  be the zeroes of  $kx^2 + 2x + 3k$ .

Then  $\alpha + \beta = \frac{-2}{k}$  and  $\alpha\beta = 3$

$$\Rightarrow \alpha + \beta = \alpha\beta$$

$$\Rightarrow \frac{-2}{k} = 3$$

$$\Rightarrow k = \frac{-2}{3}$$

19. If  $\alpha, \beta$  are the zeroes of the polynomial  $x^2 + 6x + 2$ , then  $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = ?$

(a) 3                      (b) -3                      (c) 12                      (d) -12

**Sol:**

(b) -3

Since  $\alpha$  and  $\beta$  be the zeroes of  $x^2 + 6x + 2$ , we have:

$$\alpha + \beta = -6 \text{ and } \alpha\beta = 2$$

$$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{-6}{2} = -3$$

20. If  $\alpha, \beta, \gamma$  be the zeroes of the polynomial  $x^3 - 6x^2 - x + 30$ , then  $(\alpha\beta + \beta\gamma + \gamma\alpha) = ?$

(a) -1                      (b) 1                      (c) -5                      (d) 30

**Sol:**

(a) -1

It is given that  $\alpha$ ,  $\beta$  and  $\gamma$  are the zeroes of  $x^3 - 6x^2 - x + 30$ .

$$\therefore (\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{co-efficient of } x}{\text{co-efficient of } x^3} = \frac{-1}{1} = -1$$

**21.** If  $\alpha$ ,  $\beta$ ,  $\gamma$  be the zeroes of the polynomial  $2x^3 + x^2 - 13x + 6$ , then  $\alpha\beta\gamma = ?$ (a) -3                      (b) 3                      (c)  $\frac{-1}{2}$                       (d)  $\frac{-13}{2}$ **Sol:**

(a) -3

Since,  $\alpha$ ,  $\beta$  and  $\gamma$  are the zeroes of  $2x^3 + x^2 - 13x + 6$ , we have:

$$\alpha\beta\gamma = \frac{-(\text{constant term})}{\text{co-efficient of } x^3} = \frac{-6}{2} = -3$$

**22.** If  $\alpha$ ,  $\beta$ ,  $\gamma$  be the zeroes of the polynomial  $p(x)$  such that  $(\alpha + \beta + \gamma) = 3$ ,  $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$  and  $\alpha\beta\gamma = -24$ , then  $p(x) = ?$ (a)  $x^3 + 3x^2 - 10x + 24$                       (b)  $x^3 + 3x^2 + 10x - 24$   
(c)  $x^3 - 3x^2 - 10x + 24$                       (d) none of these**Sol:**(c)  $x^3 - 3x^2 - 10x + 24$ Given:  $\alpha$ ,  $\beta$  and  $\gamma$  are the zeroes of polynomial  $p(x)$ .Also,  $(\alpha + \beta + \gamma) = 3$ ,  $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$  and  $\alpha\beta\gamma = -24$ 

$$\begin{aligned} \therefore p(x) &= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma \\ &= x^3 - 3x^2 - 10x + 24 \end{aligned}$$

**23.** If two of the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$  are 0, then the third zero is(a)  $\frac{-b}{a}$                       (b)  $\frac{b}{a}$                       (c)  $\frac{c}{a}$                       (d)  $\frac{-d}{a}$ **Sol:**(a)  $\frac{-b}{a}$ Let  $\alpha$ , 0 and 0 be the zeroes of  $ax^3 + bx^2 + cx + d = 0$ Then the sum of zeroes =  $\frac{-b}{a}$ 

$$\Rightarrow \alpha + 0 + 0 = \frac{-b}{a}$$

$$\Rightarrow \alpha = \frac{-b}{a}$$

Hence, the third zero is  $\frac{-b}{a}$ .

24. If one of the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$  is 0, then the product of the other two zeroes is

(a)  $\frac{-c}{a}$       (b)  $\frac{c}{a}$       (c) 0      (d)  $\frac{-b}{a}$

**Sol:**

(b)  $\frac{c}{a}$

Let  $\alpha$ ,  $\beta$  and 0 be the zeroes of  $ax^3 + bx^2 + cx + d$ .

Then, sum of the products of zeroes taking two at a time is given by

$$(\alpha\beta + \beta \times 0 + \alpha \times 0) = \frac{c}{a}$$

$$\Rightarrow \alpha\beta = \frac{c}{a}$$

$\therefore$  The product of the other two zeroes is  $\frac{c}{a}$ .

25. If one of the zeroes of the cubic polynomial  $x^3 + ax^2 + bx + c$  is -1, then the product of the other two zeroes is

(a)  $a - b - 1$       (b)  $b - a - 1$       (c)  $1 - a + b$       (d)  $1 + a - b$

**Sol:**

(c)  $1 - a + b$

Since -1 is a zero of  $x^3 + ax^2 + bx + c$ , we have:

$$(-1)^3 + a \times (-1)^2 + b \times (-1) + c = 0$$

$$\Rightarrow a - b + c + 1 = 0$$

$$\Rightarrow c = 1 - a + b$$

Also, product of all zeroes is given by

$$\alpha\beta \times (-1) = -c$$

$$\Rightarrow \alpha\beta = c$$

$$\Rightarrow \alpha\beta = 1 - a + b$$

26. If  $\alpha$ ,  $\beta$  be the zeroes of the polynomial  $2x^2 + 5x + k$  such that  $(\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$ , then  $k = ?$

(a) 3      (b) -3      (c) -2      (d) 2

**Sol:**

(d) 2

Since  $\alpha$  and  $\beta$  are the zeroes of  $2x^2 + 5x + k$ , we have:

$$\alpha + \beta = \frac{-5}{2} \text{ and } \alpha\beta = \frac{k}{2}$$

$$\text{Also, it is given that } \alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}.$$

$$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\begin{aligned} \Rightarrow \left(\frac{-5}{2}\right)^2 - \frac{k}{2} &= \frac{21}{4} \\ \Rightarrow \frac{25}{4} - \frac{k}{2} &= \frac{21}{4} \\ \Rightarrow \frac{k}{2} &= \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1 \\ \Rightarrow k &= 2 \end{aligned}$$

27. On dividing a polynomial  $p(x)$  by a non-zero polynomial  $q(x)$ , let  $g(x)$  be the quotient and  $r(x)$  be the remainder, then  $p(x) = q(x) \cdot g(x) + r(x)$ , where
- $r(x) = 0$  always
  - $\deg r(x) < \deg g(x)$  always
  - either  $r(x) = 0$  or  $\deg r(x) < \deg g(x)$
  - $r(x) = g(x)$

**Sol:**

- (c) either  $r(x) = 0$  or  $\deg r(x) < \deg g(x)$

By division algorithm on polynomials, either  $r(x) = 0$  or  $\deg r(x) < \deg g(x)$ .

28. Which of the following is a true statement?

- $x^2 + 5x - 3$  is a linear polynomial.
- $x^2 + 4x - 1$  is a binomial
- $x + 1$  is a monomial
- $5x^2$  is a monomial

**Sol:**

- (d)  $5x^2$  is a monomial.

$5x^2$  consists of one term only. So, it is a monomial.

### Exercise – Formative Assessment

1. The zeroes of the polynomial  $P(x) = x^2 - 2x - 3$  are

- (a) -3, 1      (b) -3, -1      (c) 3, -1      (d) 3, 1

**Sol:**

- (c) 3, -1

Here,  $p(x) = x^2 - 2x - 3$

Let  $x^2 - 2x - 3 = 0$

$$\Rightarrow x^2 - (3 - 1)x - 3 = 0$$

$$\Rightarrow x^2 - 3x + x - 3 = 0$$

$$\Rightarrow x(x - 3) + 1(x - 3) = 0$$

$$\Rightarrow (x - 3)(x + 1) = 0$$

$$\Rightarrow x = 3, -1$$

2. If  $\alpha, \beta, \gamma$  be the zeroes of the polynomial  $x^3 - 6x^2 - x + 3$ , then the values of  $(\alpha\beta + \beta\gamma + \gamma\alpha) = ?$   
 (a) -1                      (b) 1                      (c) -5                      (d) 3

**Sol:**

(a) -1

Here,  $p(x) = x^3 - 6x^2 - x + 3$

Comparing the given polynomial with  $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$ , we get:  $(\alpha\beta + \beta\gamma + \gamma\alpha) = -1$

3. If  $\alpha, \beta$  are the zeros of  $kx^2 - 2x + 3k$  is equal  $\alpha + \beta = \alpha\beta$  then  $k = ?$   
 (a)  $\frac{1}{3}$                       (b)  $-\frac{1}{3}$                       (c)  $\frac{2}{3}$                       (d)  $-\frac{2}{3}$

**Sol:**

(c)  $\frac{2}{3}$

Here,  $p(x) = x^2 - 2x + 3k$

Comparing the given polynomial with  $ax^2 + bx + c$ , we get:

$a = 1, b = -2$  and  $c = 3k$

It is given that  $\alpha$  and  $\beta$  are the roots of the polynomial.

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \alpha + \beta = -\left(\frac{-2}{1}\right)$$

$$\Rightarrow \alpha + \beta = 2 \quad \dots(i)$$

$$\text{Also, } \alpha\beta = \frac{c}{a}$$

$$\Rightarrow \alpha\beta = \frac{3k}{1}$$

$$\Rightarrow \alpha\beta = 3k \quad \dots(ii)$$

Now,  $\alpha + \beta = \alpha\beta$

$$\Rightarrow 2 = 3k \quad [\text{Using (i) and (ii)}]$$

$$\Rightarrow k = \frac{2}{3}$$

4. It is given that the difference between the zeroes of  $4x^2 - 8kx + 9$  is 4 and  $k > 0$ . Then,  $k = ?$   
 (a)  $\frac{1}{2}$                       (b)  $\frac{3}{2}$                       (c)  $\frac{5}{2}$                       (d)  $\frac{7}{2}$

**Sol:**

(c)  $\frac{5}{2}$

Let the zeroes of the polynomial be  $\alpha$  and  $\alpha + 4$

Here,  $p(x) = 4x^2 - 8kx + 9$

Comparing the given polynomial with  $ax^2 + bx + c$ , we get:

$a = 4, b = -8k$  and  $c = 9$

Now, sum of the roots =  $\frac{-b}{a}$

$$\Rightarrow \alpha + \alpha + 4 = \frac{-(-8)}{4}$$

$$\Rightarrow 2\alpha + 4 = 2k$$

$$\Rightarrow \alpha + 2 = k$$

$$\Rightarrow \alpha = (k - 2) \quad \dots(i)$$

Also, product of the roots,  $\alpha\beta = \frac{c}{a}$

$$\Rightarrow \alpha(\alpha + 4) = \frac{9}{4}$$

$$\Rightarrow (k - 2)(k - 2 + 4) = \frac{9}{4}$$

$$\Rightarrow (k - 2)(k + 2) = \frac{9}{4}$$

$$\Rightarrow k^2 - 4 = \frac{9}{4}$$

$$\Rightarrow 4k^2 - 16 = 9$$

$$\Rightarrow 4k^2 = 25$$

$$\Rightarrow k^2 = \frac{25}{4}$$

$$\Rightarrow k = \frac{5}{2} \quad (\because k > 0)$$

5. Find the zeroes of the polynomial  $x^2 + 2x - 195$ .

**Sol:**

Here,  $p(x) = x^2 + 2x - 195$

Let  $p(x) = 0$

$$\Rightarrow x^2 + (15 - 13)x - 195 = 0$$

$$\Rightarrow x^2 + 15x - 13x - 195 = 0$$

$$\Rightarrow x(x + 15) - 13(x + 15) = 0$$

$$\Rightarrow (x + 15)(x - 13) = 0$$

$$\Rightarrow x = -15, 13$$

Hence, the zeroes are  $-15$  and  $13$ .

6. If one zero of the polynomial  $(a^2 + 9)x^2 - 13x + 6a$  is the reciprocal of the other, find the value of  $a$ .

**Sol:**

$$(a + 9)x^2 - 13x + 6a = 0$$

Here,  $A = (a^2 + 9)$ ,  $B = 13$  and  $C = 6a$

Let  $\alpha$  and  $\frac{1}{\alpha}$  be the two zeroes.

Then, product of the zeroes =  $\frac{C}{A}$

$$\Rightarrow \alpha \cdot \frac{1}{\alpha} = \frac{6a}{a^2 + 9}$$

$$\Rightarrow 1 = \frac{6a}{a^2 + 9}$$

$$\begin{aligned} \Rightarrow a^2 + 9 &= 6a \\ \Rightarrow a^2 - 6a + 9 &= 0 \\ \Rightarrow a^2 - 2 \times a \times 3 + 3^2 &= 0 \\ \Rightarrow (a - 3)^2 &= 0 \\ \Rightarrow a - 3 &= 0 \\ \Rightarrow a &= 3 \end{aligned}$$

7. Find a quadratic polynomial whose zeroes are 2 and -5.

**Sol:**

It is given that the two roots of the polynomial are 2 and -5.

Let  $\alpha = 2$  and  $\beta = -5$

Now, the sum of the zeroes,  $\alpha + \beta = 2 + (-5) = -3$

Product of the zeroes,  $\alpha\beta = 2 \times (-5) = -10$

$\therefore$  Required polynomial  $= x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - (-3)x + 10$$

$$= x^2 + 3x - 10$$

8. If the zeroes of the polynomial  $x^3 - 3x^2 + x + 1$  are  $(a - b)$ ,  $a$  and  $(a + b)$ , find the values of  $a$  and  $b$ .

**Sol:**

The given polynomial  $= x^3 - 3x^2 + x + 1$  and its roots are  $(a - b)$ ,  $a$  and  $(a + b)$ .

Comparing the given polynomial with  $Ax^3 + Bx^2 + Cx + D$ , we have:

$$A = 1, B = -3, C = 1 \text{ and } D = 1$$

$$\text{Now, } (a - b) + a + (a + b) = \frac{-B}{A}$$

$$\Rightarrow 3a = -\frac{-3}{1}$$

$$\Rightarrow a = 1$$

$$\text{Also, } (a - b) \times a \times (a + b) = \frac{-D}{A}$$

$$\Rightarrow a(a^2 - b^2) = \frac{-1}{1}$$

$$\Rightarrow 1(1^2 - b^2) = -1$$

$$\Rightarrow 1 - b^2 = -1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

$$\therefore a = 1 \text{ and } b = \pm\sqrt{2}$$

9. Verify that 2 is a zero of the polynomial  $x^3 + 4x^2 - 3x - 18$ .

**Sol:**

$$\text{Let } p(x) = x^3 + 4x^2 - 3x - 18$$

$$\text{Now, } p(2) = 2^3 + 4 \times 2^2 - 3 \times 2 - 18 = 0$$

$\therefore 2$  is a zero of  $p(x)$ .

10. Find the quadratic polynomial, the sum of whose zeroes is  $-5$  and their product is  $6$ .

**Sol:**

Given:

Sum of the zeroes =  $-5$

Product of the zeroes =  $6$

$$\begin{aligned}\therefore \text{Required polynomial} &= x^2 - (\text{sum of the zeroes})x + \text{product of the zeroes} \\ &= x^2 - (-5)x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

11. Find a cubic polynomial whose zeroes are  $3$ ,  $5$  and  $-2$ .

**Sol:**

Let  $\alpha$ ,  $\beta$  and  $\gamma$  are the zeroes of the required polynomial.

Then we have:

$$\alpha + \beta + \gamma = 3 + 5 + (-2) = 6$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times 5 + 5 \times (-2) + (-2) \times 3 = -1$$

$$\text{and } \alpha\beta\gamma = 3 \times 5 \times (-2) = -30$$

$$\begin{aligned}\text{Now, } p(x) &= x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma \\ &= x^3 - x^2 \times 6 + x \times (-1) - (-30) \\ &= x^3 - 6x^2 - x + 30\end{aligned}$$

So, the required polynomial is  $p(x) = x^3 - 6x^2 - x + 30$ .

12. Using remainder theorem, find the remainder when  $p(x) = x^3 + 3x^2 - 5x + 4$  is divided by  $(x - 2)$ .

**Sol:**

$$\text{Given: } p(x) = x^3 + 3x^2 - 5x + 4$$

$$\begin{aligned}\text{Now, } p(2) &= 2^3 + 3(2^2) - 5(2) + 4 \\ &= 8 + 12 - 10 + 4 \\ &= 14\end{aligned}$$

13. Show that  $(x + 2)$  is a factor of  $f(x) = x^3 + 4x^2 + x - 6$ .

**Sol:**

$$\text{Given: } f(x) = x^3 + 4x^2 + x - 6$$

$$\begin{aligned}\text{Now, } f(-2) &= (-2)^3 + 4(-2)^2 + (-2) - 6 \\ &= -8 + 16 - 2 - 6 \\ &= 0\end{aligned}$$

$\therefore (x + 2)$  is a factor of  $f(x) = x^3 + 4x^2 + x - 6$ .

14. If  $\alpha, \beta, \gamma$  are the zeroes of the polynomial  $p(x) = 6x^3 + 3x^2 - 5x + 1$ , find the value of  $\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$ .

**Sol:**

$$\begin{aligned}\text{Given: } p(x) &= 6x^3 + 3x^2 - 5x + 1 \\ &= 6x^3 - (-3)x^2 + (-5)x - 1\end{aligned}$$

Comparing the polynomial with  $x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$ , we get:

$$\alpha\beta + \beta\gamma + \gamma\alpha = -5$$

$$\text{and } \alpha\beta\gamma = -1$$

$$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$$

$$= \left(\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}\right)$$

$$= \left(\frac{-5}{-1}\right)$$

$$= 5$$

15. If  $\alpha, \beta$  are the zeroes of the polynomial  $f(x) = x^2 - 5x + k$  such that  $\alpha - \beta = 1$ , find the value of  $k$ .

**Sol:**

$$\text{Given: } x^2 - 5x + k$$

The co-efficients are  $a = 1$ ,  $b = -5$  and  $c = k$ .

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \alpha + \beta = \frac{-(-5)}{1}$$

$$\Rightarrow \alpha + \beta = 5 \quad \dots(1)$$

$$\text{Also, } \alpha - \beta = 1 \quad \dots(2)$$

From (1) and (2), we get:

$$2\alpha = 6$$

$$\Rightarrow \alpha = 3$$

Putting the value of  $\alpha$  in (1), we get  $\beta = 2$ .

$$\text{Now, } \alpha\beta = \frac{c}{a}$$

$$\Rightarrow 3 \times 2 = \frac{k}{1}$$

$$\therefore k = 6$$

16. Show that the polynomial  $f(x) = x^2 + 4x + 6$  has no zero.

**Sol:**

$$\text{Let } t = x^2$$

$$\text{So, } f(t) = t^2 + 4t + 6$$

Now, to find the zeroes, we will equate  $f(t) = 0$

$$\Rightarrow t^2 + 4t + 6 = 0$$

$$\begin{aligned}\text{Now, } t &= \frac{-4 \pm \sqrt{16-24}}{2} \\ &= \frac{-4 \pm \sqrt{-8}}{2} \\ &= -2 \pm \sqrt{-2}\end{aligned}$$

$$\text{i.e., } x^2 = -2 \pm \sqrt{-2}$$

$\Rightarrow x = \sqrt{-2 \pm \sqrt{-2}}$ , which is not a real number.

The zeroes of a polynomial should be real numbers.

$\therefore$  The given  $f(x)$  has no zeroes.

17. If one zero of the polynomial  $p(x) = x^3 - 6x^2 + 11x - 6$  is 3, find the other two zeroes.

**Sol:**

$p(x) = x^3 - 6x^2 + 11x - 6$  and its factor,  $x + 3$

Let us divide  $p(x)$  by  $(x - 3)$ .

$$\begin{aligned}\text{Here, } x^3 - 6x^2 + 11x - 6 &= (x - 3)(x^2 - 3x + 2) \\ &= (x - 3)[(x^2 - (2 + 1)x + 2)] \\ &= (x - 3)(x^2 - 2x - x + 2) \\ &= (x - 3)[x(x - 2) - 1(x - 2)] \\ &= (x - 3)(x - 1)(x - 2)\end{aligned}$$

$\therefore$  The other two zeroes are 1 and 2.

18. If two zeroes of the polynomial  $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$  are  $\sqrt{2}$  and  $-\sqrt{2}$ , find its other two zeroes.

**Sol:**

Given:  $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$  and the two zeroes,  $\sqrt{2}$  and  $-\sqrt{2}$

So, the polynomial is  $(x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$ .

Let us divide  $p(x)$  by  $(x^2 - 2)$

$$\begin{aligned}\text{Here, } 2x^4 - 3x^3 - 3x^2 + 6x - 2 &= (x^2 - 2)(2x^2 - 3x + 1) \\ &= (x^2 - 2)[(2x^2 - (2 + 1)x + 1)] \\ &= (x^2 - 2)(2x^2 - 2x - x + 1) \\ &= (x^2 - 2)[(2x(x - 1) - 1(x - 1))] \\ &= (x^2 - 2)(2x - 1)(x - 1)\end{aligned}$$

The other two zeroes are  $\frac{1}{2}$  and 1.

19. Find the quotient when  $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$  is divided by  $(x^2 + 3x + 1)$ .

**Sol:**

Given:  $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Dividing  $p(x)$  by  $(x^2 + 3x + 1)$ , we have:

$$\begin{array}{r}
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \quad \left( 3x^2 - 4x + 2 \right. \\
 \underline{3x^4 + 9x^3 + 3x^2} \phantom{+ 2x + 2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \phantom{+ 2} \\
 + \phantom{-} 2x^2 + 6x + 2 \\
 \underline{+ \phantom{-} 2x^2 + 6x + 2} \\
 - \phantom{-} - \phantom{-} - \\
 \underline{\phantom{-} - \phantom{-} -} \\
 x
 \end{array}$$

∴ The quotient is  $3x^2 - 4x + 2$

20. Use remainder theorem to find the value of  $k$ , it being given that when  $x^3 + 2x^2 + kx + 3$  is divided by  $(x - 3)$ , then the remainder is 21.

**Sol:**

Let  $p(x) = x^3 + 2x^2 + kx + 3$

Now,  $p(3) = (3)^3 + 2(3)^2 + 3k + 3$   
 $= 27 + 18 + 3k + 3$   
 $= 48 + 3k$

It is given that the remainder is 21

∴  $3k + 48 = 21$

$\Rightarrow 3k = -27$

$\Rightarrow k = -9$