EXERCISE 22.11

Question. 1

Solution:

From the question it is given that,

Initial radius of balloon = 1 unit

After 3 seconds radius of balloon = 2 units

So, let us assume A be the surface area of balloon,

$(dA/dt) \propto t$

Then,

$dA/dt = \lambda t$

$d(4\pi r^2)/dt = \lambda t$

$8\pi r\,(dr/dt) = \lambda t$

Integrating on both side we get,

$8\pi \int r dr = \lambda \int t\, dt$

$8\pi (r^2/2) = (\lambda t^2/2) + c$

$4\pi r^2 = (\lambda t^2/2) + c$ …. … [equation (i)]

From question, r = 1 unit when t = 0,

$4\pi (1)^2 = 0 + c$

$4\pi = c$

From equation (i) $c = 4\pi r^2 - (\lambda t^2/2)$

Then, $4\pi r^2 = (\lambda t^2/2) + 4\pi$ … [equation (ii)]

And also form question, given r = 2 units when t = 3 sec

$4\pi(2)^2 = (\lambda(3)^2/2) + 4\pi$

$16\pi = (9/2)\lambda + 4\pi$

$(9/2)\lambda = 12\pi$

By cross multiplication we get,

$\lambda = (24/9)\pi$

$\lambda = (8/3)\pi$

So, now equation (ii) becomes,

$4\pi r^2 = (8\pi/6) t^2 + 4\pi$

$4\pi (r^2 - 1) = (4/3)\pi t^2$

By cross multiplication,

$r^2 - 1 = (1/3) t^2$

$r^2 = 1 + (1/3)t^2$

$r = \sqrt{(1 + (1/3)t^2)}$

Question. 2

Solution:

From the question it is given that,

Population grows at the rate of 5% per year.

So, let us assume population after time t be p and initial population be P_o,

(dp/dt) = 5% × P

Then,

dP/dt = P/20

By cross multiplication we get,

20 (dP/P) = dt

Integrating on both side we get,

20 ∫(dP/P) = ∫ dt

20 log P = t + c …. … [equation (i)]

From question, P = P_o unit when t = 0,

20 log (P_o) = 0 + c

20log(P/P_o) = c

Then, equation (i) becomes,

20 log (P) = t + 20 log (P_o)

By cross multiplication we get,

20 log (P/P_o) = t

Let time is t, when P = $2P_o$,

Then, 20 log (2P/$2P_o$) = t_1

t_1 = 20 log 2

Therefore, time period required is 20 log 2 years.

Question. 3

Solution:

From the question it is given that,

The present population is 1,00,000

Then, the population of a city doubled in the past 25 years,

So, let us assume P be the surface area of balloon,

(dP/dt) ∝ P

Then,

dP/dt = λP

dP/dt = λ dt

Integrating on both side we get,

∫ dP/dt = λ ∫ dt

Log P = λt + c … [equation (i)]

From question, $P = P_o$ t when t = 0,

log (P_o) = 0 + c

c = log (P_o)

Then, equation (i) becomes,

log (P) = λt + log (P_o)

log (P/P_o) = λt … [equation (ii)]

And also form question, given $P = 2P_o$ when t = 25

log $(2P_o/P_o)$ = 25λ

log 2 = 25λ

By cross multiplication we get,

λ = log2/25

So, now equation (ii) becomes,

log (P/P_o) = (log2/25)t

let us assume that t_1 be the time to become population 5,00,000 from 1,00,000,

Then, log (5,00,000/1,00,000) = (log2/25) t_1

By cross multiplication we get,

t_1 = 25 log 5/log 2

= 25(1.609)/(0.6931)

= 58

Therefore, the required time is 58 years.

Question. 4

Solution:

From the question it is given that,

The bacteria count is 1,00,000

The number is increased by 10% in 2 hours.

So, let us assume C be the surface area of balloon,

(dC/dt) ∝ C

Then,

dC/dt = λC

dC/dt = λ dt

Integrating on both side we get,

∫ dC/dt = λ ∫ dt

Log C = λt + log k … [equation (i)]

From question, t = 0 when c = 1,00,000,

log (1,00,000) = λ × 0 + log k … [equation (ii)]

log (1,00,000) = log k … [equation (iii)]
And also form question, given t = 2, c = 1,00,000 + 1,00,000 × (10/100) = 110000
So, from equation (i) we have,
log 110000 = λ × 2 + log K … [equation (iv)]
Now, subtracting equation (ii) from equation (iv), we have,
Log 110000 – log 100000 = 2 λ
Then, log 11 × 10000 – log 10 × 10000 = 2 λ
Log ((11 × 10000)/(10 × 10000)) = 2λ
Log (11/10) = 2λ
So, λ = ½ log (11/10) … [equation (v)]
Now we need to find the time 't' in which the count reaches 200000.
Then, substituting the values of λ and k from equation (iii) and (v) in equation (i),
Log 200000 = ½ log (11/10)t + log 100000
½ log (11/10)t = log 200000 – log 100000
½ log (11/10)t = log (200000/100000)
½ log(11/10)t = log 2
Therefore, the required time t = 2log2/log(11/10) hours.

Question. 5
Solution:
From the question it is given that,
The interest is compounded continuously at 6% per annum
So, let us assume P be the principal,
(dP/dt) = Pr/100
Then,
dP/dt = (r/100) dt
Integrating on both side we get,
∫ dP/P = ∫ r/100 dt
Log P = (rt/100) + c … [equation (i)]
Let us assume P_o be the initial principal at t = 0
log (P_o) = 0 + C
C = log (P_o)
Now, substitute the value of C in equation (i)
log (P) = rt/100 + log (P_o)
log (P/P_o) = rt/100
Now in case 1:
P_o = 1000, t = 10 years and r = 6

log (P/1000) = (6 × 10)/100

log P – log 1000 = 0.6

log P = $\log e^{0.6}$ + log 1000

Taking log common in both terms,

log P = log ($e^{0.6}$ + 1000)

log P = log (1.822 + 1000)

log P = log 1822

Then,

P = ₹ 1822

₹ 1000 will be ₹ 1822 after 10 years,

Now in case 2:

Let us assume t_1 ne the time to double ₹ 1000,

P = 2000, P_o = 1000, r = 6%

log (P/P_o) = rt/100

log (2000/1000) = $6t_1/100$

(100 log2)/6 = t_1

(100 × 0.6931)/6 = t_1

t_1 = 11.55 years

Therefore, it will take 12 years approximately to double