Chapter 5: Arithmetic Progressions

EXERCISE 5.2:

1. Fill in the blanks in the following table, given that a is the first term, d the commondifference and a_n the nth term of the AP:

	a	d	n	a_n
(i)	7	3	8	…
(ii)	-18	…	10	0
(iii)	…	-3	18	-5
(iv)	-18.9	2.5	…	3.6
(v)	3.5	0	105	….

Solution:(i) Given: a = 7, d = 3 and n = 8,

a_n = ?

We know:

$a_n = a + (n – 1)d$

Thus, $a_n = 7 + (8 – 1)3 = 7 + 21 = 28$

(ii) Given a = - 18, n = 10, a_n = 0, d = ?

We know that $a_n = a + (n – 1)d$

Thus, $0 = - 18 + (10 – 1)d$

$0 = - 18 + 9d$

Or, $9d = 18$

Or, $d = 18/9 = 2$

(iii) Given d = - 3, n = 18, a_n = - 5, a = ?

We know that, $a_n = a + (n – 1)d$

Or, $- 5 = a + (18 – 1) (- 3)$

Or, - 5 = a – 51

Or, a = - 5 + 51 = 46

(iv) Given a = - 18.9, d = 2.5, a_n = 3.6, n = ?

We know that, $a_n = a + (n – 1)d$

Or, 3.6 = – 18.9 + (n – 1)2.5

Or, 2.5(n – 1) = 3.6 + 18.9 = 22.5

n – 1 = 22.5/2.5 = 9

n = 9 + 1 = 10

(v) Given that a = 3.5, d = 0, n = 105, a_n= ?

We know:,

$a_n = a + (n – 1)d$

a_n = 3.5 + (104 – 1)0

a_n = 3.5 + 0

= 3.5

2. Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10, 7, 4, . . . , is

A. 97 B. 77

C. –77 D. – 87

(ii) 11th term of the AP: $-3, -\frac{1}{2}, 2, ..., \text{is}$

A. 28 B. 22

C. –38 D. – 48

Solution:(i)

Here, a = 10, d = – 3 and n = 30

We know:

$a_n = a + (n - 1)d$

Or, $a_{30} = 10 + (30 - 1)(- 3)$

$= 10 - 87 = - 77$

Hence,

Answer (C) – 77

(ii) Here, $a = - 3$, $d = 5/2$ and $n = 11$

We know that, $a_n = a + (n - 1)d$

Or, $a_{11} = - 3 + (11 - 1)\ 5/2$

$= - 3 + 10 \times (5/2) = - 3 + 25 = 22$

Hence,
Answer (B) 22

3. In the following APs, find the missing terms in the boxes :

(i) 2, ☐, 26

(ii) ☐, 13, ☐, 3

(iii) 5, ☐, ☐, $9\frac{1}{2}$

(iv) –4, ☐, ☐, ☐, ☐, 6

(v) ☐, 38, ☐, ☐, ☐, – 22

Solution:(i) We know: In AP, middle term is average of the other two terms

Hence, middle term = (2 + 26)/2 = 28/2 = 14

Thus, above AP can be written as 2, 14, 26

(ii) The middle term between 13 and 3 will be;

(13 + 3)/2 = 16/2 = 8

Now, $a_4 - a_3 = 3 - 8 = -5$

$a_3 - a_2 = 8 - 13 = -5$

Thus, $a_2 - a_1 = -5$

Or, $13 - a_1 = -5$

Or, $a_1 = 13 + 5 = 18$

Thus, above AP can be written as 18, 13, 8, 3

(iii) We have, $a = 5$ and $a_4 = 9½$

Now common difference:

$a_4 = a + 3d$

$\frac{19}{2} = 5 + 3d$

$3d = \frac{19}{2} - 5$

$3d = \frac{9}{2}$

$d = \frac{3}{2}$

Hence, using d, 2nd term and 3rd term can be calculated as:

$a_2 = a + d$

$= 5 + \frac{3}{2}$

$= \frac{13}{2}$

$a_3 = a + 2d$

$= 5 + 2 \times \frac{3}{2}$

$= 8$

Therefore, the A.P. can be written as:

$5, \frac{13}{2}, 8, \frac{19}{2}$

(iv) Here, a = - 4 and $a_6 = 6$

Common difference:

$a_6 = a + 5d$

$6 = -4 + 5d$

$5d = 6 + 4 = 10$

$d = 2$

The second, third, fourth and fifth terms of this AP are:

$a_2 = a + d = - 4 + 2 = - 2$

$a_3 = a + 2d = - 4 + 4 = 0$

$a_4 = a + 3d = - 4 + 6 = 2$

$a_5 = a + 4d = - 4 + 8 = 4$

Thus, the given AP can be written as: - 4, - 2, 0, 2, 4, 6

Let us take 38 as the first term and – 22 as the 5th term

Using this, common difference can be calculated as follows:

$a_5 = a + 4d$

$= 38 + 4d$

$4d = - 22 – 38 = - 60$

$d = - 15$

If 38 is the second term, then first term:

$a = a_2 - d = 38 + 15 = 53$

Third, fourth and fifth terms ar:

$a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$a_4 = a + 3d = 53 - 45 = 8$

$a_5 = a + 4d = 53 - 60 = -7$

So, the AP can be written as: 53, 38, 23, 8, - 7, - 22

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Solution: Given, $a = 3$, $d = a_2 - a_1 = 8 - 3 = 5$, $a_n = 78$, $n = ?$

We know that $a_n = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$(n - 1)5 = 78 - 3 = 75$

$n - 1 = 15$

$n = 15 + 1 = 16$

Thus, 78 is the 16th term of given AP.

5. Find the number of terms in each of the following APs :
   (i) 7, 13, 19, . . . , 205
   (ii) $18,\ 15\frac{1}{2}, 13, \ldots\ldots, -47$

Solution: (i) Here, $a = 7$, $d = 6$, $a_n = 205$, $n = ?$

We know that $a_n = a + (n - 1)d$

$205 = 7 + (n - 1)6$

$(n - 1)6 = 205 - 7 = 198$

$n - 1 = 33$

n = 34

Thus, 205 is the 34th of term of this AP.

(ii) Here, a = 18, d = 15.5 – 18 = - 2.5

We know that $a_n = a + (n – 1)d$

Or, - 47 = 18 + (n – 1)(- 2.5)

Or, (n – 1)(- 2.5) = - 47 – 18 = - 65

Or, n – 1 = 26

Or, n = 27

Thus, - 47 is the 27th term of this AP.

6. Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

Solution:Here, a = 11, d = 8 – 11 = - 3, a_n = - 150, n = ?

We know that $a_n = a + (n – 1)d$

Or, - 150 = 11 + (n – 1)(- 3)

Or, (n – 1)(-3) = - 150 – 11 = - 161

Or, n – 1 = 161/3

It is clear that 161 is not divisible by three and we shall get a fraction as a result. But number of term cannot be a fraction.

Hence, - 150 is not a term of the given AP.

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Solution:Given, $a_{11} = 38$ and $a_{16} = 73$

We know that $a_n = a + (n – 1)d$

Hence, $a_{11} = a + 10d = 38$

And, $a_{16} = a + 15d = 73$

Subtracting 11th term from 16th term, we get following:

$a + 15d – a – 10d = 73 – 38$

Or, $5d = 35$

Or, $d = 7$

Substituting the value of d in 11th term we get;

$a + 10 \times 7 = 38$

Or, $a + 70 = 38$

Or, $a = 38 – 70 = - 32$

Now 31st term can be calculated as follows:

$a_{31} = a + 30d$

$= - 32 + 30 \times 7$

$= - 32 + 210 = 178$

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29^{th}term.

Solution:Given, $a_3 = 12$ and $a_{50} = 106$

$a_3 = a + 2d = 12$

$a_{50} = a + 49d = 106$

Subtracting 3rd term from 50th term, we get;

$a + 49d – a – 2d = 106 – 12$

$47d = 94$

$d = 2$

Substituting the value of d in 12th term, we get;

a + 2 x 2 = 12

Or, a + 4 = 12

Or, a = 8

Now, 29th term can be calculated as follows:

a_{29} = a + 28d

= 8 + 28 x 2

= 8 + 56 = 64

9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP iszero?

Solution:Given, a_3 = 4 and a_9 = - 8

a_3 = a + 2d = 4

a_9 = a + 8d = - 8

Subtracting 3rd term from 9th term, we get;

a + 8d – a – 2d = - 8 – 4 = - 12

6d = - 12

d = - 2

Substituting the value of d in 3rd term, we get;

a + 2(-2) = 4

a – 4 = 4

a = 8

Now; 0 = a + (n – 1)d

0 = 8 + (n – 1)(- 2)

(n – 1)(- 2) = - 8

n – 1 = 4

n = 5

Thus, 5th term of this AP is zero.

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution:Tenth and seventeenth terms of this AP can be given as follows:

$a_{10} = a + 9d$

$a_{17} = a + 16d$

Subtracting 10th term from 17th term, we get;

a + 16d – a – 9d = 7

7d = 7

d = 1

11. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Solution:Here, a = 3, d = 15 – 3 = 12

54th term can be given as follows:

$a_{54} = a + 53d$

= 3 + 53 x 12

= 3 + 636 = 639

So, the required term = 639 + 132 = 771

771 = a + (n – 1)d

771 = 3 + (n -1)12

(n – 1)12 = 771 – 3 = 768

n – 1 = 64

n = 65

Thus, the required term is 65th term

12. Two APs have the same common difference. The difference between their 100th terms is100, what is the difference between their 1000th terms?

Solution:Since, both the APs have same common difference, thus in this case the difference between each corresponding terms will be 100.

13. How many three-digit numbers are divisible by 7?

Solution:Since, 100 is the smallest three digit number and it gives a reminder of 2 when divided by 7, therefore, 105 is the smallest three digit number which is divisible by 7.

Since, 999 is greatest three digit number, and it gives a reminder of 5, thus 999 – 5 = 994 will be the greatest three digit number which is divisible by 7.

Therefore, here we have,

First term (a) = 105,

The last term (a_n) = 994

The common difference = 7

We know that, $a_n = a + (n – 1)d$

Or, 994 = 105 + (n – 1)7

Or, (n – 1)7 = 994 – 105 = 889

Or, n – 1 = 127

Or, n = 128

Thus, there are 128 three digit numbers which are divisible by 7.

14. How many multiples of 4 lie between 10 and 250?

Solution:12 is the first number after 10 which is divisible by 4.

Since, 250 gives a remainder of 2 when divided by 4, thus 250 – 2 = 248 is the greatest number less than 250 which is divisible by 4.

Here, we have first term (a) = 12, last term (n) = 248 and common difference (d) = 4

Thus, number of terms (n) =?

We know that, $a_n = a + (n -1)d$

Or, 248 = 12 + (n – 1)4

Or, (n – 1)4 = 248 – 12 = 236

Or, n – 1 = 59

Or, n = 60

Thus, there are 60 numbers between 10 and 250 that are divisible by 4.

15. For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

Solution:In first AP: a = 63, d = 2

In second AP: a = 3, d = 7

As per question:

63 + (n – 1) 2 = 3 + (n – 1) 7

⇒ 63 – 3 + (n – 1) 2 = (n – 1) 7

⇒ 60 + 2n – 2 = 7n – 7

⇒ 2n + 58 = 7n – 7

⇒ 2n + 58 + 7 = 7n

⇒ 2n + 65 = 7n

⇒ 7n – 2n = 65

⇒ 5n = 65

⇒ n = 65/5 = 13

Thus, for the 13 value of n, nth term of given two APs will be equal

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution:Given $a_3 = 16$ and $a_7 – a_5 = 12$

$a_3 = a + 2d = 16$

$a_5 = a + 4d$

$a_7 = a + 6d$

As per question;

$a + 6d – a – 4d = 12$

Or, $2d = 12$

Or, $d = 6$

Substituting the value of d in third term, we get;

$a + 2 \times 12 = 16$

Or, $a + 24 = 16$

Or, $a = 16 – 24 = - 8$

Thus, the AP can be given as follows:

-8, 4, 16, 28, 40, …

17. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.

Solution:a = 3, d = 5

Now, $253 = a + (n + 1) d$

$\Rightarrow 253 = 3 + (n -1) \times 5$

$\Rightarrow 253 = 3 + 5n – 5 = – 2$

$\Rightarrow 5n = 253 + 2 = 255$

⇒ n = 255/5 = 51

Therefore, 20th term from the last term = 51 – 19 = 32

a_{32} = a + 31d

= 3 + 31 x 5

= 3 + 155 = 158

Thus, required term is 158

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is44. Find the first three terms of the AP.

Solution:Given, $a_8 + a_4 = 24$ and $a_{10} + a_6 = 44$

a_8 = a + 7d

a_4 = a + 3d

As per question;

a + 7d + a + 3d = 24

Or, 2a + 10d = 24

Or, a + 5d = 12 ……(1)

a_{10} = a + 9d

a + 9d + a + 5d = 44

2a + 14d = 44

a + 7d = 22 (2)

Subtracting equation (1) from equation (2);

a + 7d – a – 5d = 22 – 12

2d = 10

d = 5

19. SubbaRao started work in 1995 at an annual salary of Rs 5000 and received an incrementof Rs 200 each year. In which year did his income reach Rs 7000?

Solution:Here, a = 5000, d = 200 and a_n = 7000

We know, $a_n = a + (n - 1)d$

, 7000 = 5000 + (n – 1)200

(n -1)200 = 7000 – 5000

(n – 1)200 = 2000

n – 1 = 10

n = 11

Thus, 1995 + 10 = 2005

Hence, his salary reached at Rs. 7000 in 2005.

20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings byRs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Solution:Here, a = 5, d = 1.75 and a_n = 20.75

We know, $a_n = a + (n - 1)d$

20.75 = 5 + (n – 1)1.75

(n – 1)1.75 = 20.75 – 5

(n – 1)1.75 = 15.75

n – 1 = 15.75/1.75 = 9

n = 10

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