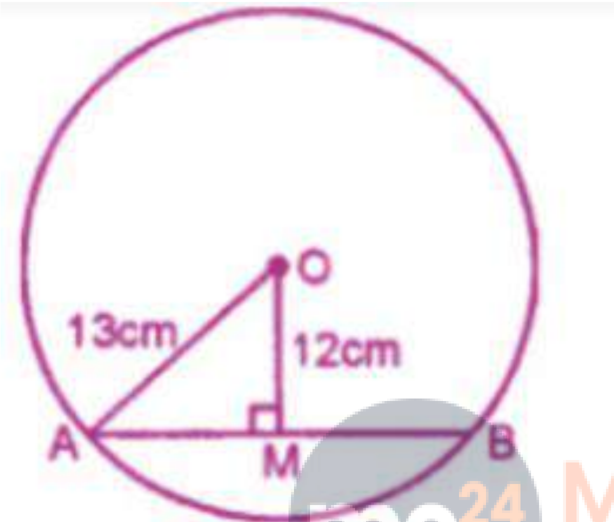
EXERCISE 15.1

1. Calculate the length of a chord which is at a distance of 12 cm from the centre of a circle of radius 13 cm.

Solution:

AB is chord of a circle with center O and OA is its radius OM ⊥ AB



Therefore, OA = 13 cm, OM = 12 cm

Now from right angled triangle OAM,

$OA^2 = OM^2 + AM^2$ by using Pythagoras theorem,

$13^2 = 12^2 + AM^2$

$AM^2 = 13^2 – 12^2$

$AM^2 = 169 – 144$

$AM^2 = 25$

$AM = 5^2$

We know that OM perpendicular to AB

Therefore, M is the midpoint of AB

AB = 2 AM

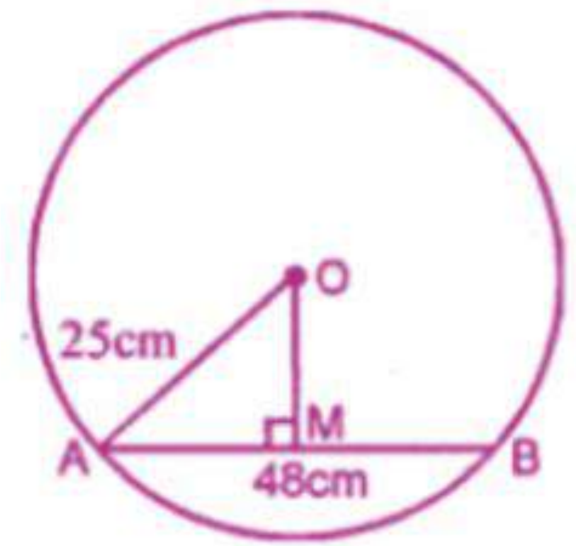
AB = 2 (5)

AB = 10 cm

2. A chord of length 48 cm is drawn in a circle of radius 25 cm. Calculate its distance from the center of the circle.

Solution:

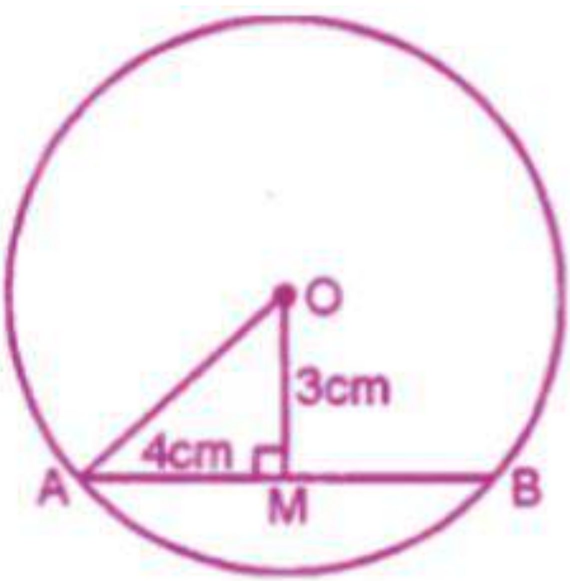
AB is the chord of the circle with centre O and radius OA
OM is perpendicular to AB



Therefore, AB = 48 cm
OA = 25 cm
OM ⊥ AB
M is the mid-point of AB
AM = 1/2 AB = ½ × 48 = 24 cm
Now right ΔOAM,
$OA^2 = OM^2 + AM^2$
(by Pythagoras Axiom)
$(25)^2 = OM^2 + (24)^2$
$OM^2 = (25)^2 – (24)^2 = 625 – 576$
$= 49 = (7)^2$
OM = 7 cm

3. A chord of length 8 cm is at a distance of 3 cm from the centre of the circle. Calculate the radius of the circle.

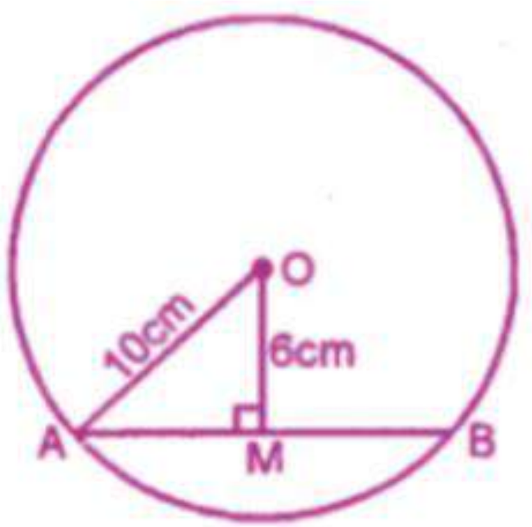
Solution:
AB is the chord of a circle with center O
And radius OA and OM ⊥ AB



AB = 8 cm
OM = 3 cm
OM ⊥ AB
M is the mid-point of AB
AM = ½ AB = ½ × 8 = 4 cm.
Now in right ΔOAM
$OA^2 = OM^2 + AM^2$
(By Pythagoras Axiom)
$= (3)^2 + (4)^2 = 9 + 16 = 25$
$= (5)^2$
OA = 5 cm.

4. Calculate the length of the chord which is at a distance of 6 cm from the centre of a circle of diameter 20 cm.

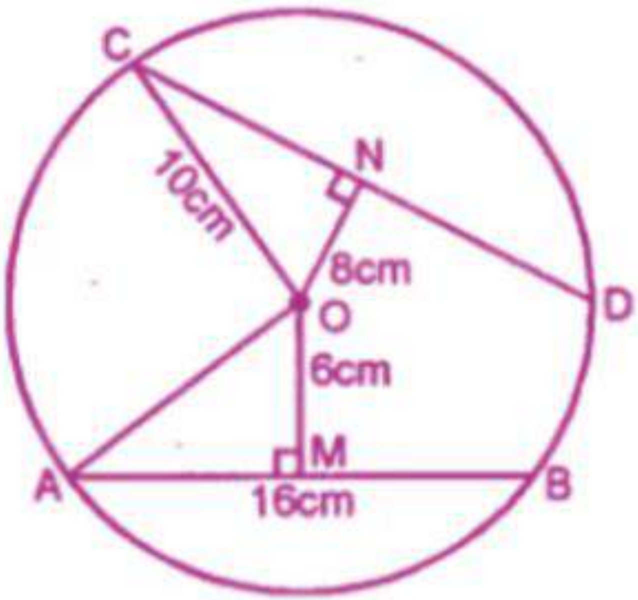
Solution:
AB is the chord of the circle with centre O
And radius OA and OM ⊥ AB



Diameter of the circle = 20 cm
Radius = 20/2 = 10 cm
OA = 10 cm, OM = 6 cm
Now in right ΔOAM,
$OA^2 = AM^2 + OM^2$
(By Pythagoras Axiom)
$(10)^2 = AM^2 + (6)^2$
$AM^2 = 10^2 – 6^2$
$AM^2 = 100 – 36 = 64 = (8)^2$
AM = 8 cm
OM ⊥ AB
M is the mid-point of AB.
AB = 2 AM = 2 × 8 = 16 cm.

5. A chord of length 16 cm is at a distance of 6 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 8 cm from the centre.

Solution:
AB is a chord a circle with centre O and
OA is the radius of the circle and OM ⊥ AB

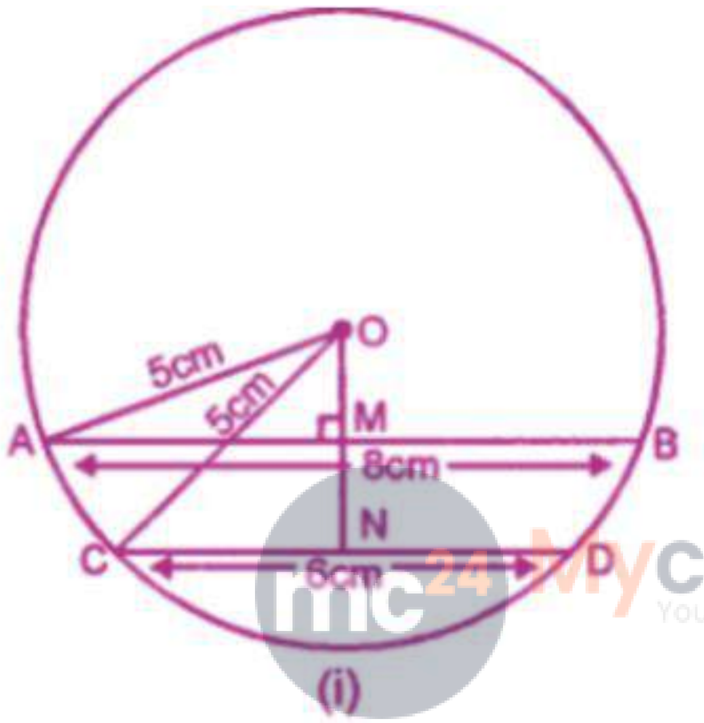


AB = 16 cm, OM = 6 cm
OM ⊥ AB
AM = ½ AB = ½ × 16 = 8 cm
Now in right ΔOAM
$OA^2 = AM^2 + OM^2$
(By Pythagoras Axiom)
$= (8)^2 + (6)^2$
$64 + 36 = 100 = (10)^2$
Now CD is another chord of the same circle
ON ⊥ CD and OC is the radius.
In right ΔONC
$OC^2 = ON^2 + NC^2$
(By Pythagoras Axioms)
$(10)^2 = (8)^2 + (NC)^2$
$100 = 64 + NC^2$
$NC^2 = 100 – 64 = 36 = (6)^2$
NC = 6
But ON ⊥ AB
N is the mid-point of CD
CD = 2 NC = 2 × 6 = 12 cm

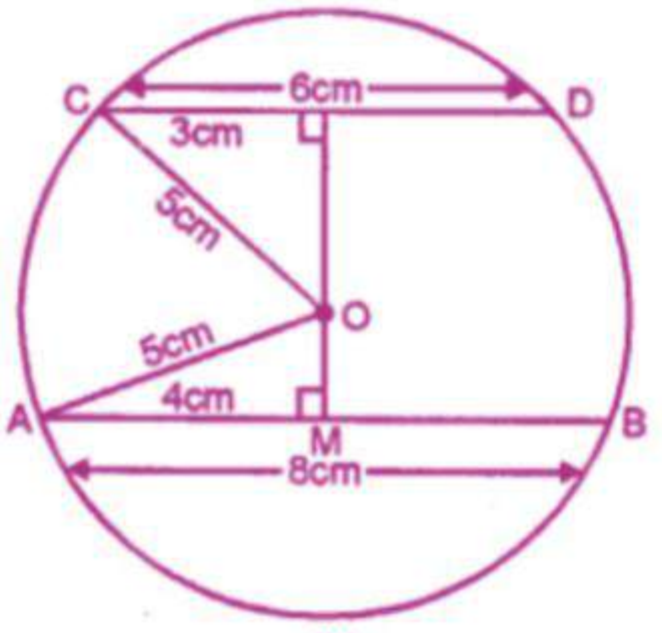
6. In a circle of radius 5 cm, AB and CD are two parallel chords of length 8 cm and 6 cm respectively. Calculate the distance between the chords if they are on :

(i) the same side of the centre.
(ii) the opposite sides of the centre

Solution:
Two chords AB and CD of a Circle with centre O and radius OA or OC



OA = OC = 5 cm
AB = 8 cm
CD = 6 cm
OM and ON are perpendiculars from O to AB and CD respectively.
M and N are the Mid-points of AB and
CD respectively
In figure (i) chord are on the same side
And in figure (ii) chord are on the opposite
Sides of the centre



(ii)

In right ΔOAM

$OA^2 = AM^2 + OM^2$

(By Pythagoras Axiom)

$(5)^2 = (4)^2 + OM^2$

AM = ½ AB

$25 = 16 + OM^2$

$OM^2 = 25 - 16 = 9 = (3)^2$

OM = 3 cm

Again in right ΔOCN,

$OC^2 = CN^2 + ON^2$

$(5)^2 = (3)^2 + ON^2$

(CN = ½ CD)

$25 = 9 + ON^2$

$ON^2 = 25 - 9 = 16 = (4)^2$

ON = 4

In fig (i), distance MN = ON – OM

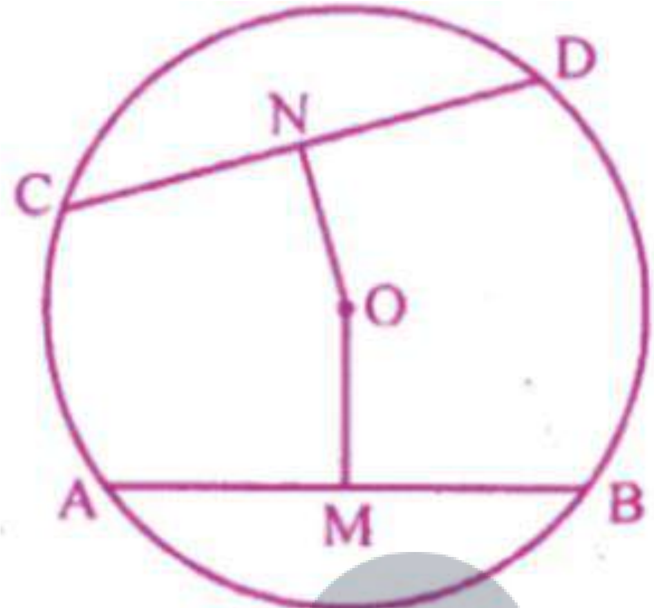
= 4 – 3 = 1cm.

In fig (ii)

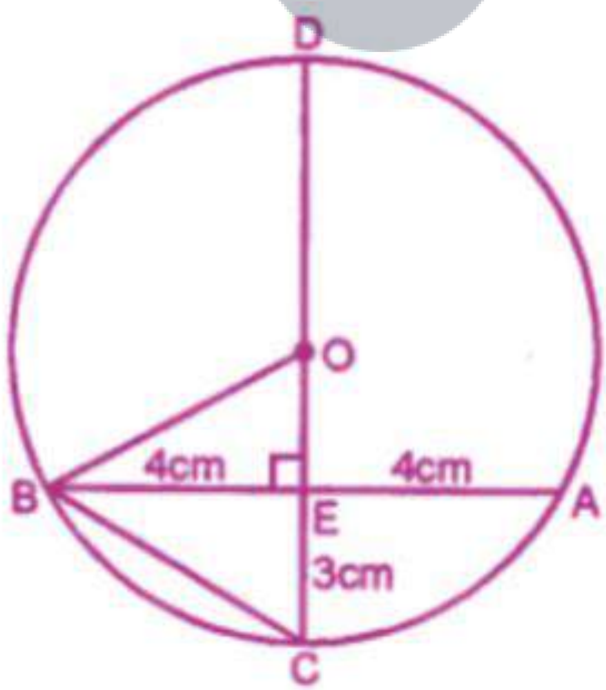
MN = OM + ON = 3 + 4 = 7 Cm

7. (a) In the figure given below, O is the centre of the circle. AB and CD are two chords

of the circle, OM is perpendicular to AB and ON is perpendicular to CD. AB = 24 cm, OM = 5 cm, ON = 12 cm. Find the:
(i) radius of the circle.
(ii) length of chord CD.



(b) In the figure (ii) given below, CD is the diameter which meets the chord AB in E such that AE = BE = 4 cm. If CE = 3 cm, find the radius of the circle.



Solution:

(a)Given : AB = 24 cm, OM = 5cm, ON = 12cm

OM ⊥ AB

M is midpoint of AB

AM = 12 cm

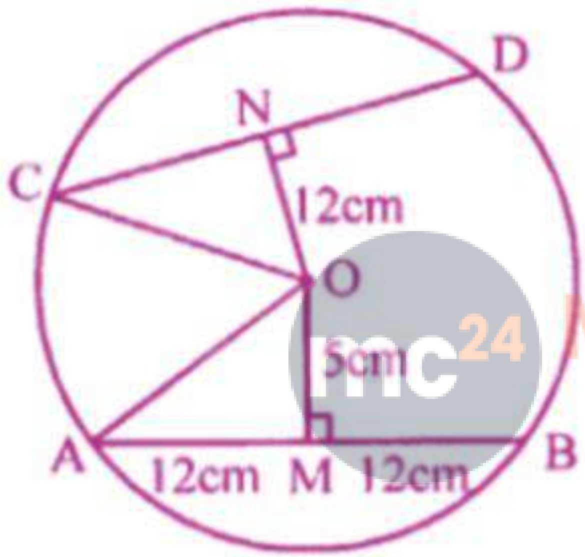
(i) Radius of circle OA = $\sqrt{OM^2 + AM^2}$

(ii) Again $OC^2 = ON^2 + CN^2$

$13^2 = 12^2 + CN^2$

$CN = \sqrt{(13^2 – 12^2)} = \sqrt{(169 – 144)} = \sqrt{25}$

CN = 5 cm



As ON ⊥ CD, N is mid-Point of CD

CD = 2CN = 2 × 5 = 10 cm

(b) AB = 8 cm, EC = 3 cm

Let radius OB = OC = r

OE = (r-3) Cm.

Now in right ΔOBE

$OB^2 = BE^2 + OE^2$

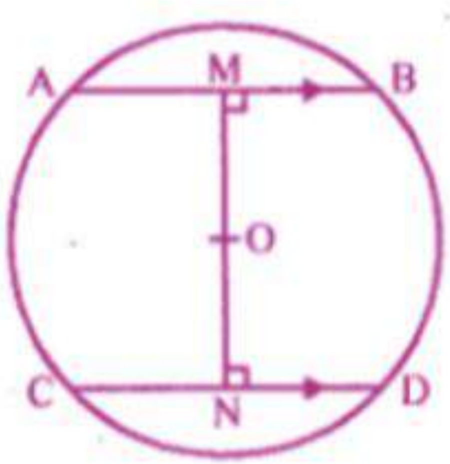
$r^2 = (4)^2 + (r – 3)^2$

$r^2 = 16 + r^2 – 6r + 9$

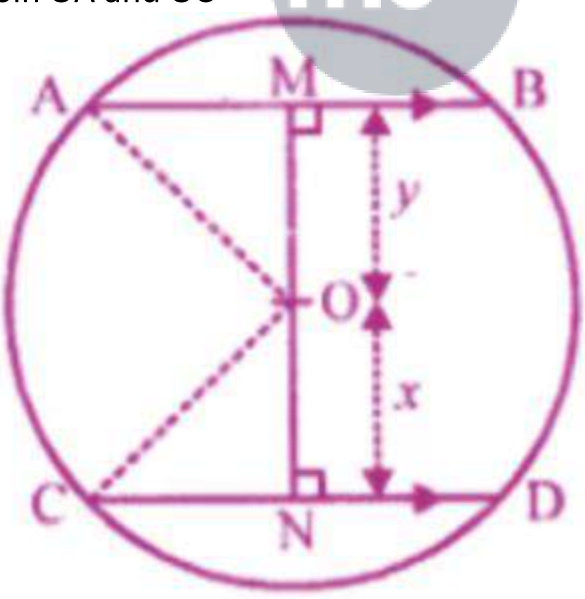
6r = 25 (r = 25/6 = 4 1/6 cm

8. In the adjoining figure, AB and CD ate two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length

24 cm and 18 cm respectively.



Solution:
In the figure, chords AB ∥ CD
O is the centre of the circle
Radius of the Circle = 15 cm
Length of AB = 24 cm and CD = 18 cm
Join OA and OC

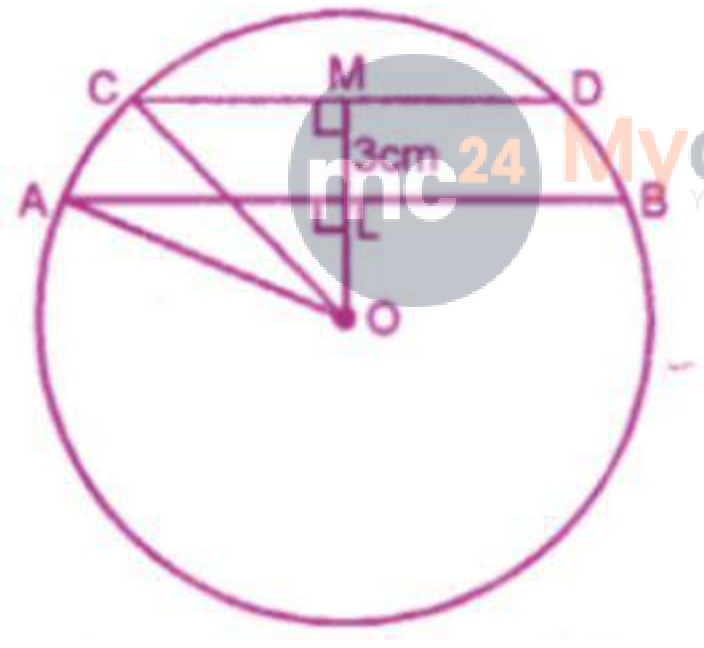


AB = 24 cm and OM ⊥ AB
AM = MB = 24/2 = 12 cm
Similarly ON ⊥ CD
CN = ND = 18/2 = 9 cm

Similarly In right Δ CNO
$OC^2 = CN^2 + ON^2$ $(15)^2 = (9)^2 + ON^2$
$225 = 81 + ON^2$
$ON^2 = 225 – 81 = 144 = (12)^2$
ON = 12 cm
Now MN = OM + ON = 9 + 12 + 21 cm

9. AB and CD are two parallel chords of a circle of lengths 10 cm and 4 cm respectively. If the chords lie on the same side of the centre and the distance between them is 3 cm, find the diameter of the circle.

Solution :
AB and CD are two parallel chords and AB = 10 cm, CD = 4 cm and distance between AB and CD = 3 cm



Let radius of circle OA = OC = r
OM ⊥ CD which intersects AB in L.
Let OL =x, then OM = x + 3
Now right ΔOLA
$OA^2 = AL^2 + OL^2$
$r^2 = (5)^2 + x^2 = 25 + x^2$
(l is mid- point of AB)

Again in right ΔOCM

$OC^2 = CM^2 + OM^2$

$r^2 = (2)^2 + (x + 3)^2$

(M is mid-point of CD)

$r^2 = 4 + (x + 3)^2$

(M is mid-Point of CD)

$r^2 = 4 + (x + 3)^2$

from (i) and (ii)

$25 + x^2 = 4 + (x + 3)^2$

$25 + x^2 = 4 + x^2 + 9 + 6x$

$6x = 25 – 13 = 12$

$x = 12/6 = 2$ cm

Substituting the value of x in (i)

$r^2 = 25 + x^2 = 25 + (2)^2 = 25 + 4$

$r^2 = 29$

$r = \sqrt{29}$cm

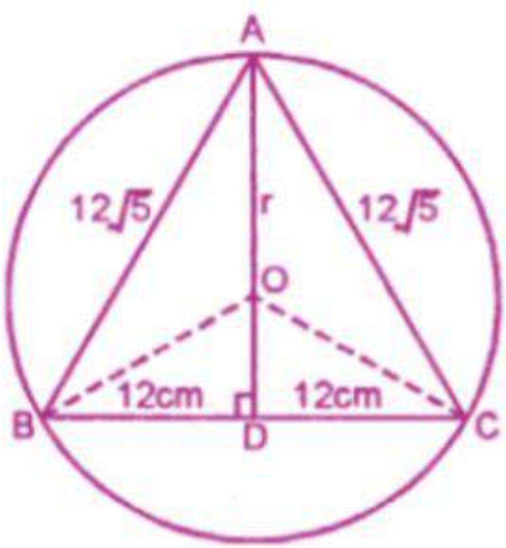
Diameter of the circle = 2 r

= 2 × √29 cm = 2 √29 cm

10. ABC is an isosceles triangle inscribed in a circle. If AB = AC = 12√5 cm and BC = 24 cm, find the radius of the circle.

Solution :

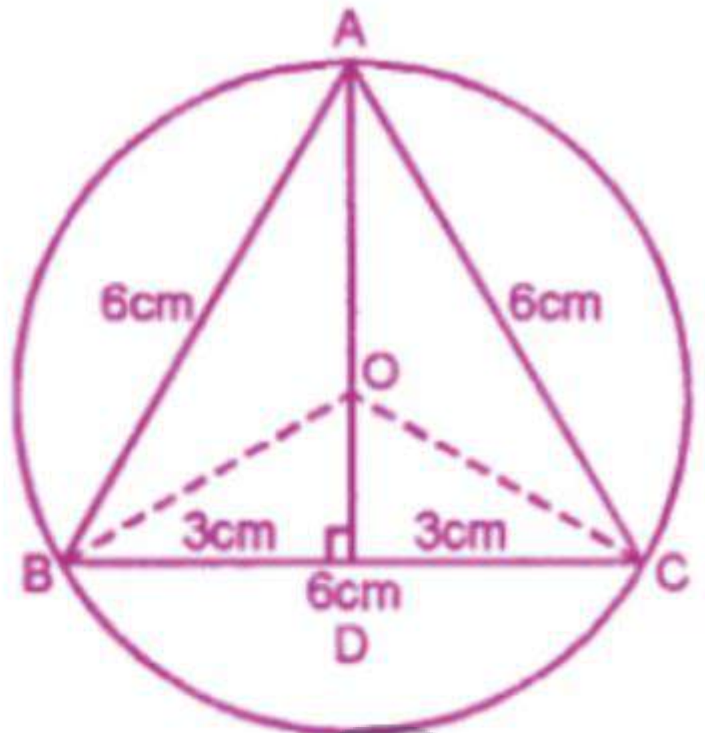
AB = AC 12√5 and BC = 24 cm.



Join OB and OC and OA
Draw AD ⊥ BC which will pass through
Centre O
OD bisect BC in D
BD = DC = 12 cm
In right ΔABD
$AB^2 = AD^2 + BD^2$
$(12\sqrt{5})^2 = AD^2 + BD^2$
$(12\sqrt{5})^2 = AD^2 + (12)^2$
$144 \times 5 = AD^2 + 144$
$720 - 144 = AD^2$
$AD^2 = 576$ (AD = √576 = 24
Let radius of the circle = OA = OB = OC = r
OD = AD – AO = 24 – r
Now in right ΔOBD,
$OB^2 = BD^2 + OD^2$
$r^2 = (12)^2 + (24 - r)^2$
$r^2 = 144 + 576 + r^2 - 48r$
48r = 720
48r = 720
r = 720/48 = 48r
48r = 720
r = 720/48 = 15cm.
Radius = 15 cm.

11. An equilateral triangle of side 6 cm is inscribed in a circle. Find the radius of the circle.

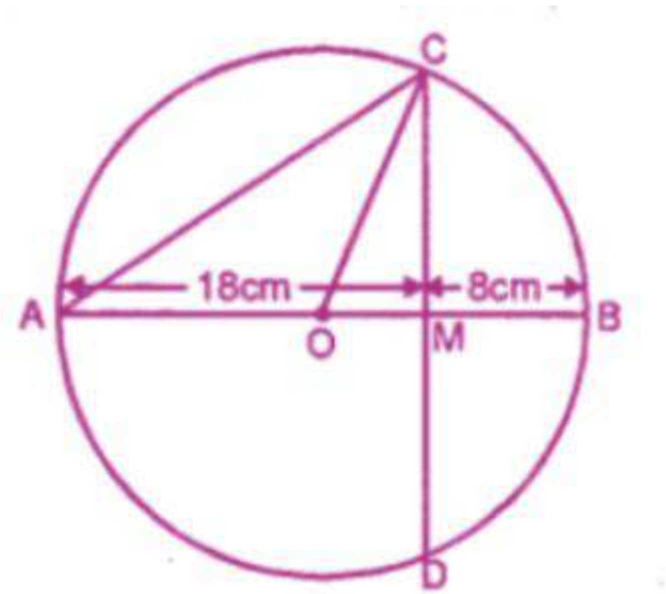
Solution :
ABC is an equilateral triangle inscribed in a
Circle with centre O. Join OB and OC,
From A, Draw AD ⊥ BC which will pass
Through the centre O of the circle.



Each side of ΔABC = 6 cm.
AD = √3/2 a= √3/2 × 6 = 3 √3 cm.
OD = AD – AO = 3√3 – r
Now in right ΔOBD
$OB^2 = BD^2 + OD^2$
$r^2 = (3)^2 + (3\sqrt{3}-r)^2$
$r^2 = 9 + 27 + r^2 - 6\sqrt{3}r$
(D is mid-point of BC)
6√3r = 36
R = 36/6√3 = 6/√3 × √3/√3 = 6√3/3 = 2√3 cm
Radius = 2√3 cm

12. AB is a diameter of a circle. M is a point in AB such that AM = 18 cm and MB = 8 cm. Find the length of the shortest chord through M.

Solution:



AM = 18 cm and MB = 8 cm

AB = AM + MB = 18 + 8 = 26 cm

Radius of the circle = 26/2 = 13 cm

Let CD is the shortest chord drawn through M.

CD ⊥ AB

Join OC

OM = AM – AO = 18 – 13 = 5 cm

OC = OA = 13 cm

Now in right ΔOMC

$OC^2 = OM^2 + MC^2$

$(13)^2 = (5)^2 + MC^2$ ($MC^2 = 13^2 – 5^2$

$MC^2 = 169 – 25 = 144 = (12)^2$

MC = 12

M is Mid-Point of CD

CD = 2 × MC = 2 × 12 = 24 cm

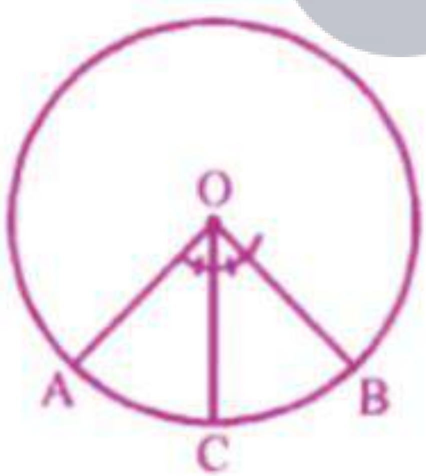
EXERCISE 15.2

1. If arcs APB and CQD of a circle are congruent, then find the ratio of AB: CD.

Solution:
arc APB = arc CQD (given)
AB = CD
(if two arcs are congruent, then their
Corresponding chords are equal)
Ratio of AB and CD = AB / CD = AB /AB = 1/1
AB : CD = 1 : 1

2. A and B are points on a circle with centre O. C is a point on the circle such that OC bisects ∠AOB, prove that OC bisects the arc AB.

Solution:
Given : in a given circle with centre O,A
And B are Two points on the circle. C i
another point on the circle such that
∠AOC = ∠BOC

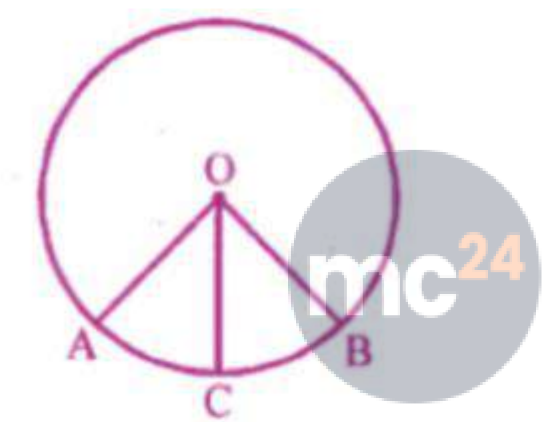


To prove : arc AC = arc BC
Proof : OC is the bisector of ∠AOB
Or ∠AOC = ∠BOC
But these are the angle subtended by the
arc AC and BC

arc AC = arc BC.

3. Prove that the angle subtended at the centre of a circle is bisected by the radius passing through the mid-point of the arc.

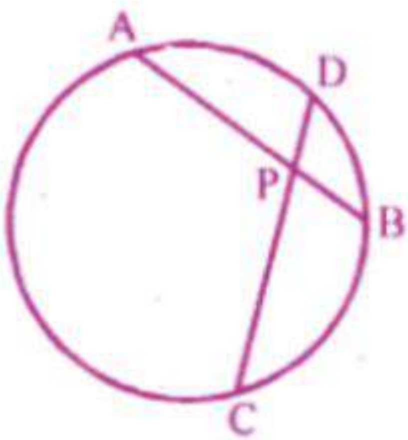
Solution :
Given : AB is the arc of the circle with
Centre O and C is the mid-Point od arc AB.
To prove : OC bisects the ∠AOB
I,e ∠AOC = ∠BOC
Proof : C is the mid-point of arc AB.
arc AC = arc BC



But arc AC and arc BC subtend ∠AOC and
∠BOC at the centre
∠AOC = ∠BOC
Hence OC Bisects the ∠AOB.

4. In the given figure, two chords AB and CD of a circle intersect at P. If AB = CD, prove that arc AD = arc CB.

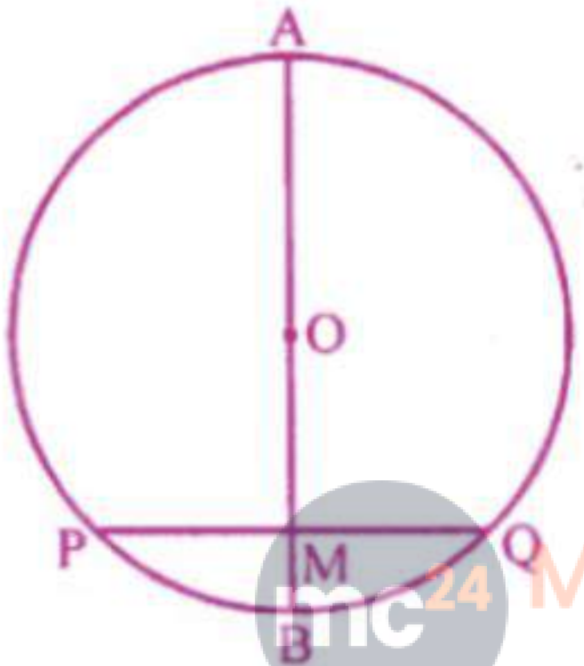
Solution :



Given: two chord AB and CD of a Circle
Intersect at P and AB = CD
To prove : arc AD = arc CB
Proof : AB = CD (given)
minor arc AB = minor arc CD
subtracting arc BD from both sides
arc AB = arc BD = arc CD – arc BD
arc AD = arc CD

CHAPTER TEST

1. In the given figure, a chord PQ of a circle with centre O and radius 15 cm is bisected at M by a diameter AB. If OM = 9 cm, find the lengths of : (i) PQ (ii) AP (iii) BP

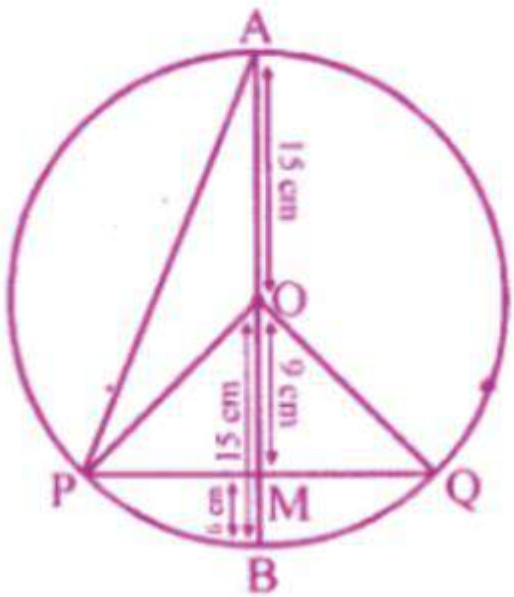


Solution:

Given, radius = 15 cm

OA = OB = OP = OQ = 15 cm

Also, OM = 9 cm

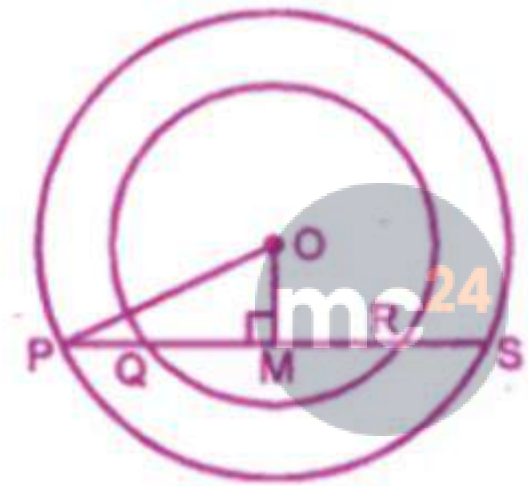


MB = OB – OM = 15 – 9 = 6 cm
AM = OA + OM =15 + 9 cm = 24 cm
In ΔOMP, By using Pythagoras Theorem,
$OP^2 = OM^2 + PM^2$
$15^2 = 9^2 + PM^2$
$PM^2 = 255 – 81$
PM = √144 = 12 cm
Also, In ΔOMQ
By using Pythagoras Theorem
$OQ^2 = OM^2 + QM^2$
$15^2 = OM^2 + QM^2$
$15^2 = 9^2 + QM^2$ ($QM^2 = 225 – 81$)
QM = √144 = 12 cm
PQ = PM + QM
(As radius is bisected at M)
PQ = 12 + 12 cm = 24 cm
(ii) Now in ΔAPM
$AP^2 = AM^2 + OM^2$
$AP^2 =24^2 + 12^2$
$AP^2 = 576 + 144$
AP = √720 = 12 √5 cm
(iii) Now in ΔBMP
$BP^2 = BM^2 + PM^2$

$BP^2 = 6^2 + 12^2$
$BP^2 = 36 + 144$
$BP = \sqrt{180} = 6\sqrt{5}$ cm

2. The radii of two concentric circles are 17 cm and 10 cm ; a line PQRS cuts the larger circle at P and S and the smaller circle at Q and R. If QR = 12 cm, calculate PQ.

Solution :
A line PQRS intersects the outer circle at P
And S and inner circle at Q and R radius of
Outer circle OP = 17 cm and radius of inner
Circle OQ = 10 cm



QR = 12 cm
From O, draw OM ⊥ PS
QM = ½ QR = ½ × 12 = 6 cm
In right ΔOQM
$OQ^2 = OM^2 + QM^2$
$(10)^2 = OM^2 + (6)^2$
$OM^2 = 10^2 – 6^2$
$= 100 – 36 = 64 = (8)^2$
OM = 8 cm
Now in right ΔOPM
$OP^2 \; OM^2 + PM^2$
$(17)^2 = OM^2 + PM^2$
$PM^2 = (17)^2 – (8)^2$

$= 289 - 64 = 225 = (15)^2$
PM = 15 cm
PQ = PM – QM = 15 – 6 = 9 cm