NCERT Solutions for Class-XII Maths

Chapter-6.4

NCERT Math Class 12

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

(i) $\sqrt{25.3}$ (ii) $\sqrt{49.5}$

(iii) $\sqrt{0.6}$ (iv) $(0.009)^{\frac{1}{3}}$

(v) $(0.999)^{\frac{1}{10}}$ (vi) $(15)^{\frac{1}{4}}$

(vii) $(26)^{\frac{1}{3}}$ (viii) $(255)^{\frac{1}{4}}$

(ix) $(82)^{\frac{1}{4}}$ (x) $(401)^{\frac{1}{2}}$

(xi) $(0.0037)^{\frac{1}{2}}$ (xii) $(26.57)^{\frac{1}{3}}$

(xiii) $(81.5)^{\frac{1}{4}}$ (xiv) $(3.968)^{\frac{3}{2}}$

(xv) $(32.15)^{\frac{1}{5}}$

1. **(i)** $\sqrt{25.3}$

Consider $y = \sqrt{x}$.

Let $x = 25$ and $\Delta x = 0.3$. Then, we get

$$\Delta y = \sqrt{x+\Delta x} - \sqrt{x}$$

$\Delta y = \sqrt{25.3} - \sqrt{25}$

$\Delta y = \sqrt{25.3} - 5$

$\Delta y = \sqrt{25.3} = \Delta y + 5$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(0.3)$$

$$= \frac{1}{2\sqrt{25}}(0.3)$$

$= 0.03$

Therefore, the approximate value of $\sqrt{25.3}$ is 5.03.

(ii) $\sqrt{49.5}$

Consider y = √x.

Let x = 49 and Δx = 0.5. Then, we get

$\Delta y = \sqrt{x+\Delta x} - \sqrt{x}$

= √49.5–√49

= √49.5 – 7

= √49.5 = Δy + 7

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(0.5)$$

$$= \frac{1}{2\sqrt{49}}(0.5)$$

= 0.035

Therefore, the approximate value of $\sqrt{49.5}$ is 7.035.

(iii) $\sqrt{0.6}$

Consider y = $\sqrt{x}$.

Let x = 1 and Δx = –0.4. Then, we get

$\Delta y = \sqrt{x+\Delta x} - \sqrt{x}$

= √0.6–1

= √0.6 = Δy + 1

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x)$$

$$= \frac{1}{2}(-0.4)$$

= –0.2

Therefore, the approximate value of $\sqrt{0.6}$ is 0.8.

(iv) $(0.009)^{\frac{1}{3}}$

Consider y = $(x)^{\frac{1}{3}}$

Let x = 0.008 and Δx = 0.001. Then, we get

$\Delta y = (x+\Delta x)^{1/3} - (x)^{1/3}$

$= (0.009)^{1/3} - (0.008)^{1/3} = (0.009)^{1/3} - 0.2$

$= (0.009)^{1/3} = \Delta y + 0.2$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x)$$

$$= \frac{1}{3 \times 0.04}(0.001)$$

= 0.008

Therefore, the approximate value of $(0.009)^{\frac{1}{3}}$ is 0.008.

(v) $(0.999)^{\frac{1}{10}}$

Consider y = $(x)^{\frac{1}{10}}$

Let x = 1 and Δx = –0.001. Then, we get

$\Delta y = (x+\Delta x)^{1/10} - (x)^{1/10}$

$\Delta y = (0.999)^{1/10} - 1$

$(0.999)^{1/10} = \Delta y + 1$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{10(x)^{\frac{9}{10}}}(\Delta x)$$

$$= \frac{1}{10}(-0.001)$$

= –0.0001

Therefore, the approximate value of $(0.999)^{\frac{1}{10}}$ is 0.9999.

(vi) $(15)^{\frac{1}{4}}$

Consider y = $(x)^{\frac{1}{4}}$

Let x = 16 and Δx = –1. Then, we get

$$\Delta y = (x + \Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}}$$

$$= (15)^{\frac{1}{4}} - (16)^{\frac{1}{4}}$$

$$= (15)^{\frac{1}{4}} - 2$$

$$\Rightarrow \quad (15)^{\frac{1}{4}} = \Delta y + 2$$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$= \frac{1}{4(16)^{\frac{3}{4}}}(-1)$$

$$= \frac{-1}{4\times 8}$$

$$= \frac{-1}{32}$$

$= -0.03125$

Therefore, the approximate value of $(15)^{\frac{1}{4}}$ is –0.03125.

(vii) $(26)^{\frac{1}{3}}$

Consider y = $(x)^{\frac{1}{3}}$

Let x = 27 and Δx = –1. Then, we get

$\Delta y = (x+\Delta x)^{1/3} - (x)^{1/3}$

$= (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}}$

$= (26)^{1/3} - 3$

$= (26)^{1/3} = \Delta y + 3$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x)$$

$$= \frac{1}{3(27)^{\frac{2}{3}}}(-1)$$

$$= \frac{-1}{27}$$

$= 0.0\overline{370}$

Therefore, the approximate value of $(26)^{\frac{1}{3}}$ is 2.9629.

(viii) $(225)^{\frac{1}{4}}$

Consider y = $(x)^{\frac{1}{4}}$

Let x = 256 and Δx = –1. Then, we get

$\Delta y = (x+\Delta x)^{1/4} - (x)^{1/4}$

$= (225)^{1/4} - (256)^{1/4}$

$= (225)^{1/4} - 4$

$= (225)^{1/4} = \Delta y + 4$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$= \frac{1}{4(256)^{\frac{3}{4}}}(-1)$$

$= -0.0039$

Therefore, the approximate value of $(255)^{1/4}$ is 3.9961.

(ix) $(82)^{\frac{1}{4}}$

Consider $y = (x)^{\frac{1}{4}}$

Let x = 81 and Δx = 1. Then, we get

$\Delta y = (x+\Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}}$

$= (82)^{\frac{1}{4}} - (81)^{\frac{1}{4}}$

$= (82)^{\frac{1}{4}} - 3$

$\Rightarrow \quad (81)^{\frac{1}{4}} = \Delta y + 3$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$= \frac{1}{4(81)^{\frac{3}{4}}}(1)$$

$= 0.009$

Therefore, the approximate value of $(82)^{\frac{1}{4}}$ is 3.009.

(x) $(401)^{\frac{1}{2}}$

Consider $y = (x)^{\frac{1}{2}}$

Let x = 400 and Δx = 1. Then, we get

$\Delta y = (x+\Delta x)^{\frac{1}{2}} - (x)^{\frac{1}{2}}$

$= (401)^{\frac{1}{2}} - (400)^{\frac{1}{2}}$

$= (401)^{\frac{1}{2}} - 20$

$\Rightarrow \quad (400)^{\frac{1}{2}} = \Delta y + 20$

Now, dy is approximately equal to Δy and is given by:

$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x)$

$= \frac{1}{2\times 20}(1)$

$= \frac{1}{40}$

= 0.025

Therefore, the approximate value of $(400)^{\frac{1}{2}}$ is 20.025.

(xi) $(0.0037)^{\frac{1}{2}}$

Consider $y = (x)^{\frac{1}{2}}$

Let x = 0.0036 and Δx = 0.0001. Then, we get

$\Delta y = (x+\Delta x)^{\frac{1}{2}} - (x)^{\frac{1}{2}}$

$= (0.0037)^{\frac{1}{2}} - (0.0036)^{\frac{1}{2}}$

$= (0.0037)^{\frac{1}{2}} - 0.06$

$\Rightarrow \quad (0.0037)^{\frac{1}{2}} = \Delta y + 0.06$

Now, dy is approximately equal to Δy and is given by:

$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x)$

$= \frac{1}{2\times 0.06}(0.0001)$

$= \frac{0.0001}{0.12}$

= 0.00083

Therefore, the approximate value of $(0.0037)^{\frac{1}{2}}$ is 0.06083.

(xii) $(26.57)^{\frac{1}{3}}$

Consider y = $(x)^{\frac{1}{3}}$

Let x = 27 and Δx = –0.43. Then, we get

Δy = $(x+\Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}}$

$= (26.57)^{\frac{1}{3}} - (27)^{\frac{1}{3}}$

$= (26.57)^{\frac{1}{3}} - 3$

$\Rightarrow \quad (26.57)^{\frac{1}{3}}$ = Δy + 3

Now, dy is approximately equal to Δy and is given by:

dy = $\left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x)$

$= \frac{1}{3(27)^{\frac{2}{3}}}(-0.43)$

$= \frac{1}{3(3)^{2}}(-0.43)$

$= \frac{1}{3(9)}(-0.43)$

$= \frac{-0.43}{27}$

= –0.015

Therefore, the approximate value of $(26.57)^{\frac{1}{3}}$ is 2.984.

(xiii) $(81.5)^{\frac{1}{4}}$

Consider y = $(x)^{\frac{1}{4}}$

Let x = 81 and Δx = 0.5. Then, we get

Δy = $(x+\Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}}$

$= (81.05)^{\frac{1}{4}} - (81)^{\frac{1}{4}}$

$= (81.05)^{\frac{1}{4}} - 3$

$\Rightarrow \quad (81.05)^{\frac{1}{4}}$ = Δy + 3

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$= \frac{1}{4(81)^{\frac{3}{4}}}(0.5)$$

$$= \frac{1}{4(3)^3}(0.5)$$

$$= \frac{1}{4(27)}(0.5)$$

= 0.0046

Therefore, the approximate value of $(81.05)^{\frac{1}{4}}$ is 3.0046.

(xiv) $(3.968)^{\frac{3}{2}}$

Consider y = $(x)^{\frac{3}{2}}$

Let x = 4 and Δx = –0.032. Then, we get

$$\Delta y = (x+\Delta x)^{\frac{3}{2}} - (x)^{\frac{3}{2}}$$

$$= (3.968)^{\frac{3}{2}} - (4)^{\frac{3}{2}}$$

$$= (3.968)^{\frac{3}{2}} - 8$$

$$\Rightarrow \quad (3.968)^{\frac{3}{2}} = \Delta y + 8$$

Now, dy is approximately equal to Δy and is given by:

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{3}{2}(x)^{\frac{1}{2}}(\Delta x)$$

$$= \frac{3}{2}(4)^{\frac{1}{2}}(-0.032)$$

$$= \frac{3}{2}(2)(-0.032)$$

= 3(–0.032)

= –0.096

Therefore, the approximate value of $(3.968)^{\frac{3}{2}}$ is 7.904.

(xv) $(32.15)^{\frac{1}{5}}$

Consider y = $(x)^{\frac{1}{5}}$

Let x = 32 and Δx = 0.15. Then, we get

$\Delta y = (x+\Delta x)^{\frac{1}{5}} - (x)^{\frac{1}{5}}$

$= (32.15)^{\frac{1}{5}} - (32)^{\frac{1}{5}}$

$= (32.15)^{\frac{1}{5}} - 2$

$\Rightarrow \quad (32.15)^{\frac{1}{5}} = \Delta y + 2$

Now, dy is approximately equal to Δy and is given by:

$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{5(x)^{\frac{4}{5}}}(\Delta x)$

$= \frac{1}{5(32)^{\frac{4}{5}}}(0.15)$

$= \frac{1}{5(2)^4}(0.15)$

$= \frac{1}{5(16)}(0.5)$

= 0.00187

Therefore, the approximate value of $(32.15)^{\frac{1}{5}}$ is 2.00187.

2. Find the approximate value of f (2.01), where f (x) = $4x^2 + 5x + 2$.

2. Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$f(2.01) = f(x+\Delta x) = 4(x+\Delta x)^2 + 5(x+\Delta x) + 2$

Now, $\Delta y = f(x+\Delta x) - f(x)$

$f(x+\Delta x) = f(x) + \Delta y$

$\approx f(x) + f'(x)\cdot\Delta x \qquad (\text{as } dx = \Delta x)$

$\Rightarrow f(2.01) \qquad \approx (4x^2+5x+2)+(8x+5)\Delta x$

$= \left[4(2)^2+5(2)+2\right]+\left[8(2)+5\right](0.01) \quad \left[\text{as } x=2, \Delta x = 0.01\right]$

$= (16+10+2)+(16+5)(0.01)$

$= 28+(21)(0.01)$

$= 28+0.21$

$= 28.21$

Hence, the approximate value of $f(2.01)$ is 28.21.

3. Find the approximate value of f (5.001), where f (x) = x^3 – 7x2 + 15.

3. Let x = 5 and Δx = 0.001. Then, we get,

$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\Rightarrow f(x + \Delta x) = f(x) + \Delta y$

$\sim f(x) + f'(x).\Delta x$ (as $dx = \Delta x$)

$\Rightarrow f(5.001) \sim (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x$

$= [(5)^2 - 7(5)^2 + 15] + [3(5)^2 - 14(5)](0.001)$

$= (125 - 175 + 15) + (75 - 70)(0.001)$

$= -35 + 0.005$

$= -34.995$

Therefore, the approximate value of f (5.001) is –34.995.

4. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

4. The volume of a cube (V) of side x is given by $V = x^3$.

Hence, the approximate change in the volume of the cube is $0.03x^3$ m^3.

$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$

$= (3x^2)\Delta x$

$= (3x^2)(0.01x)$ [as 1% of x is $0.01x$]

$= 0.03x^3$

Hence, the approximate change in the volume of the cube is $0.03x^3$ m^3.

5. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

5. It is given that the surface area of cube (S) of a side x

$\Rightarrow S = 6x^2$

$\Rightarrow dS = \left(\frac{dS}{dx}\right)\Delta x$

$= (12x)\Delta x$

$= (12x)(0.01x)$

$= 0.12x^2$

Therefore, the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1% is $0.12x^2$.

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

6. Let r be the radius of the sphere and Δr be the error in measuring the radius.
Then,
r = 7 m and Δr = 0.02 m
Now, the volume V of the sphere is given by,

$$V = \frac{4}{3}\pi r^3$$

$$\therefore \frac{dV}{dr} = 4\pi r^2$$

$$\therefore dV = \left(\frac{dV}{dr}\right)\Delta r$$

$$= \left(4\pi r^2\right)\Delta r$$

$$= 4\pi(7)^2\left(0.02\right)\text{m}^3 = 3.92\pi\text{m}^3$$

Hence, the approximate error in calculating the volume is $3.92\ \pi\ \text{m}^3$.

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

7. Let r be the radius of the sphere and Δr be the error in measuring the radius.
Now, it is given that r = 9m and Δr = 0.03m
We know that surface area of sphere (S) = $4\pi r^2$

Now, $\frac{dS}{dr} = 8\pi r$

$$\Rightarrow dS = \left(\frac{dS}{dr}\right)\Delta r$$

= (8πr)Δr
= 8π (9)(0.03)m^2
= 2.16π m^3

Therefore, the approximate error in calculating its surface area is 2.16π m^3.

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of f (3.02) is
A. 47.66 B. 57.66 C. 67.66 D. 77.66

8. Let $x = 3$ and $\Delta x = 0.02$. Then, we have:

$$f\left(3.02\right) = f\left(x + \Delta x\right) = 3(x + \Delta x)^2 + 15\left(x + \Delta x\right) + 5$$

Now, $\Delta y = f\left(x + \Delta x\right) - f\left(x\right)$

$\Rightarrow f(x+\Delta x)=f(x)+\Delta y$

$\approx f(x)+f'(x)\Delta x$

$\Rightarrow f(3.02)$

$\approx (3x^2+15x+5)+(6x+15)\Delta x$

$=\left[3(3)^2+15(3)+5\right]+\left[6(3)+15\right](0.02) \quad \text{As } dx=\Delta x)$

$=(27+45+5)+(18+15)(0.02) \quad \text{As } x=3, \Delta x=0.02]$

$=77+(33)(0.02)$

$=77+0.66$

$=77.66$

Hence, the approximate value of $f(3.02)$ is 77.66.

The correct answer is D.

9. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

(A) $0.06\ x^3\ m^3$ (B) $0.6\ x^3\ m^3$

(C) $0.09\ x^3\ m^3$ (D) $0.9\ x^3\ m^3$

9. It is given that the volume of a cube (V) of a side x

$\Rightarrow V = x^3$

$\Rightarrow dV = \left(\frac{dV}{dx}\right)\Delta x$

$= (3x^2)\Delta x$

$= (3x^2)(0.03x)$

$= 0.09x^3$

Therefore, the approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is $0.09x^3$.