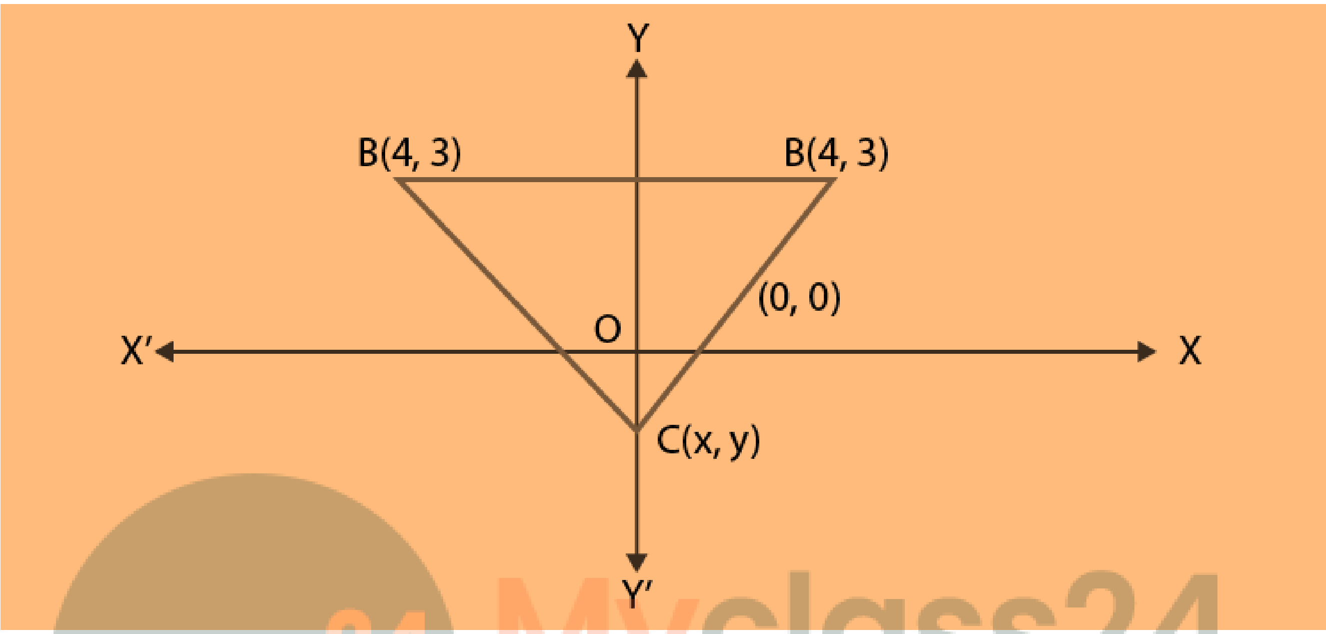
EXERCISE 7.4

If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

Solution:



Let the vertices be (x,y)

Distance between (x,y) & (4,3) is = $\sqrt{(x-4)^2 + (y-3)^2}$ ..... (1)

Distance between (x,y) & (-4,3) is = $\sqrt{(x+4)^2 + (y-3)^2}$. .... (2)

Distance between (4,3) &(-4,3) is =$\sqrt{(4+4)^2 + (3-3)^2} = \sqrt{(8)^2}=8$

According to the question,

Equation (1)=(2)

$(x-4)^2=(x+4)^2$

$x^2-8x+16=x^2+8x+16$

$16x=0$

$x=0$

Also, equation (1)=8

$(x-4)^2+(y-3)^2=64$. .... (3)

Substituting the value of x in (3)

Then $(0-4)^2+(y-3)^2=64$

$(y-3)^2=64-16$

$(y-3)^2=48$

$y-3=(\pm)4\sqrt{3}$

$y=3(\pm)\ 4\sqrt{3}$

Neglect $y = 3+4\sqrt{3}$ as if $y = 3+4\sqrt{3}$ then origin cannot interior of triangle

Therefore, the third vertex = $(0, 3-4\sqrt{3})$

1. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of △ ADE.

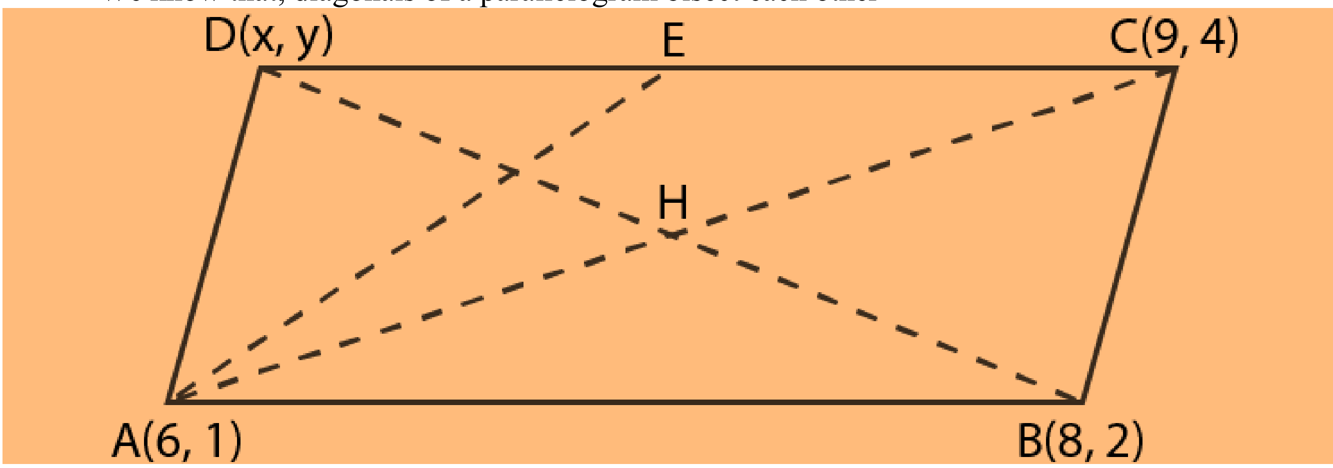
Solution:

According to the question,

The three vertices of a parallelogram ABCD are A (6, 1), B (8, 2) and C (9, 4)

Let the fourth vertex of parallelogram = (x, y),

We know that, diagonals of a parallelogram bisect each other



Since, mid - point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is given by,

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Mid - point of BD = Mid - point of AC

$$\left(\frac{8 + x}{2}, \frac{2 + y}{2}\right) = \left(\frac{6 + 9}{2}, \frac{1 + 4}{2}\right)$$

$$\left(\frac{8 + x}{2}, \frac{2 + y}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

So, we have,

$$\frac{8 + x}{2} = \frac{15}{2}$$

$8 + x = 15$

$x = 7$

And,

$$\frac{2 + y}{2} = \frac{5}{2}$$

$2 + y = 5 \rightarrow y = 3$

So, fourth vertex of a parallelogram is D (7, 3)

Now,

Mid - point of side

$$DC = \left(\frac{7 + 9}{2}, \frac{3 + 4}{2}\right)$$

$$E = \left(8, \frac{7}{2}\right)$$

∵ Area of ΔABC with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3);

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

∴ Area of ΔADE with vertices A (6, 1), D (7, 3) and E (8, (7/2))

$$\Delta = \frac{1}{2}\left[6\left(3-\frac{7}{2}\right) + 7\left(\frac{7}{2}-1\right) + 8(1-3)\right]$$
$$= \frac{1}{2}\left[6 \times \left(\frac{-1}{2}\right) + 7\left(\frac{5}{2}\right) + 8(-2)\right]$$
$$= \frac{1}{2}\left(\frac{35}{2} - 19\right)$$
$$= \frac{1}{2}\left(\frac{-3}{2}\right)$$

= - ¾ but area can't be negative

Hence, the required area of ΔADE is ¾ sq. units

2. The points A (x_1, y_1), B (x_2, y_2) and C $(x_3\ y_3)$ are the vertices of ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1

(iv) What are the coordinates of the centroid of the triangle ABC?

Solution:

According to the question,

The vertices of ΔABC = A, B and C

Coordinates of A, B and C = $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

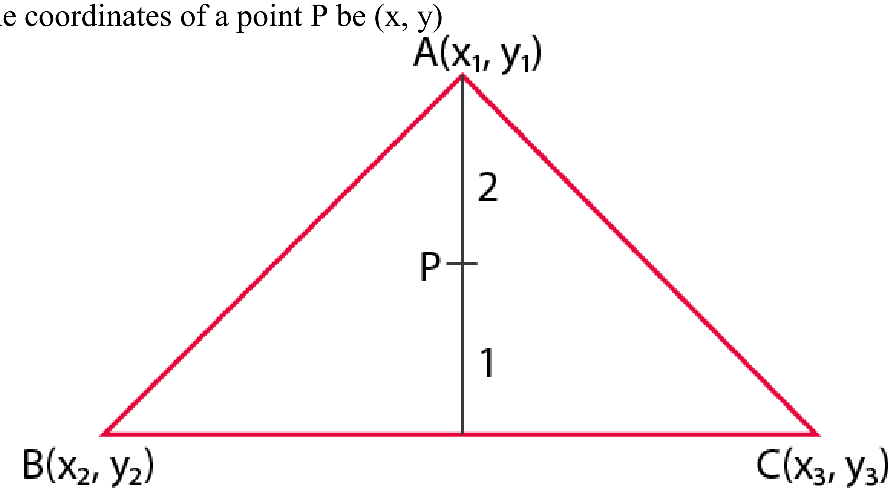
(i) As per given information D is the mid - point of BC and it bisect the line into two equal parts.

Coordinates of the mid - point of BC;

$$BC - \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

$$\Rightarrow D = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

(ii) Let the coordinates of a point P be (x, y)



Given,

The ratio in which the point P(x, y), divide the line joining,

$A(x_1, y_1)$ and $D((x_2+x_3)/2\ ,\ (y_2+y_3)/2)) = 2:1$

Then,

Coordinates of P =

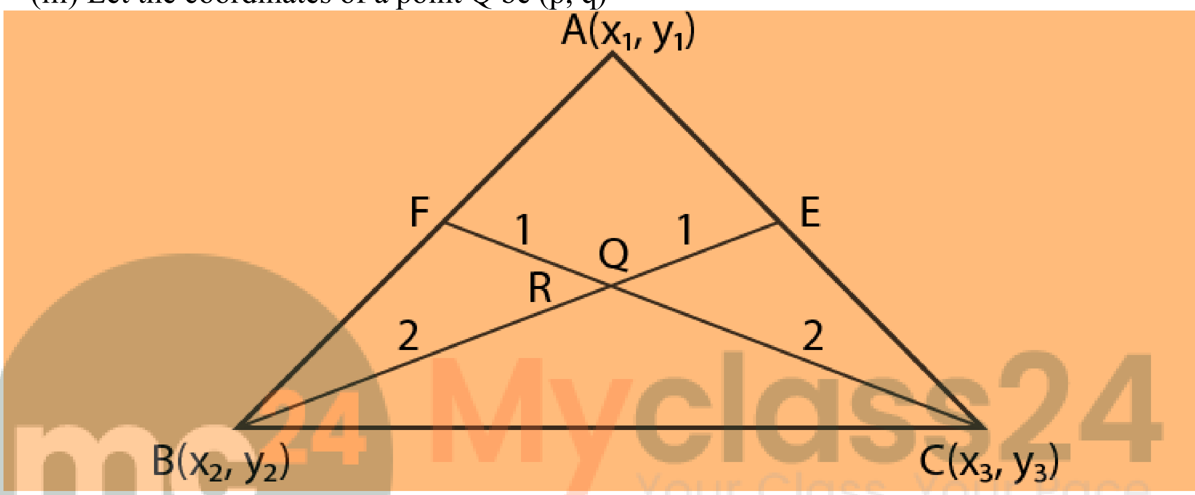
$$\left[\frac{2\times\left(\frac{x_2+x_3}{2}\right)+1\times x_1}{2+1},\frac{2\times\left(\frac{y_2+y_3}{2}\right)+1\times y_1}{2+1}\right]$$

By using internal section formula;

$$=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$=\left(\frac{x_2+x_3+x_1}{3},\frac{y_2+y_3+y_1}{3}\right)$$

(iii) Let the coordinates of a point Q be (p, q)



Given,

The point Q (p, q),

Divide the line joining $B(x_2, y_2)$ and $E\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$ in the ratio 2:1,

Then,

Coordinates of Q =

$$\left[\frac{2\times\left(\frac{x_1+x_3}{2}\right)+1\times x_2}{2+1},\frac{2\times\left(\frac{y_1+y_3}{2}\right)+1\times y_2}{2+1}\right]$$

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Since, BE is the median of side CA, So BE divides AC in to two equal parts.

∴ mid - point of AC = Coordinate of E;

$$E=\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$$

So, the required coordinate of point Q;

$$Q=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Now,

Let the coordinates of a point E be (a, β)

Given,

Point R (α, β) divide the line joining C(x3, y3) and F $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ in the ratio 2:1,

Then the coordinates of R;

$$= \left[\frac{2\times\left(\frac{x_1+x_2}{2}\right)+1\times x_3}{2+1}, \frac{2\times\left(\frac{y_1+y_2}{2}\right)+1\times y_3}{2+1}\right]$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Since, CF is the median of side AB.

So, CF divides AB in to two equal parts.

∴ mid - point of AB = Coordinate of F;

$$F = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

So, the required coordinate of point R;

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

(iv) Coordinate of the centroid of the ΔABC;

$$= \left(\frac{\text{Sum of all coordinates of all vertices}}{3}, \frac{\text{Sum of all coordinates of all vertices}}{3}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$